JGSP **6** (2006) 47–54

JOURNAL OF
Geometry and Symmetry
in Physics

ASSOCIATED LIE ALGEBRAS AND GRADED CONTRACTIONS OF THE PAULI GRADED $\mathfrak{sl}(3,\mathbb{C})$

JIŘÍ HRIVNÁK and PETR NOVOTNÝ

Communicated by Jean-Pierre Gazeau

Abstract. We consider the Pauli grading of the Lie algebra $\mathfrak{sl}(3,\mathbb{C})$ and use a concept of graded contractions to construct non-isomorphic Lie algebras of dimension eight. We overview methods used to distinguish the results and show how associated algebras, uniquely determined by the original algebra, simplify this task. We present a short overview of resulting Lie algebras.

1. Introduction

Simple Lie algebra $A_2 = \mathfrak{sl}(3,\mathbb{C})$ as well as its real forms and its corresponding gradings, have found numerous applications in physics. Recall that a decomposition of a finite-dimensional Lie algebra $\mathcal{L}$ into a direct sum of its subspaces $\mathcal{L}_i, i \in I$

$$\mathcal{L} = \bigoplus_{i\in I} \mathcal{L}_i \tag{1}$$

is called a **grading**, when for all i, j from some set I there exists $k \in I$ such that

$$[\mathcal{L}_i, \mathcal{L}_j] \subseteq \mathcal{L}_k. \tag{2}$$

The grading $\Gamma : \mathcal{L} = \bigoplus_{i\in I} \mathcal{L}_i$ is a **refinement** of the grading $\tilde{\Gamma} : \mathcal{L} = \bigoplus_{j\in J} \tilde{\mathcal{L}}_j$ if for each $i \in I$ there exists $j \in J$ such that $\mathcal{L}_i \subseteq \tilde{\mathcal{L}}_j$. Refinement is called **proper** if the cardinality of I is greater than the cardinality of the set J. Grading which cannot be properly refined is called **fine**. The property (2) defines a binary operation on the set I. If $[\mathcal{L}_i, \mathcal{L}_j] = \{0\}$ holds, we can choose an arbitrary k. It is proved in [8] that for simple Lie algebras the index set I with this operation can always be embedded into an **Abelian group** G; then we say that the Lie algebra is graded by the group G, which is called a **grading group**. Fine gradings of simple Lie algebras are analogous of Cartan's root decomposition. On the physical side, they yield quantum observables with additive quantum numbers.