



## EIGENVECTORS OF THE $SO(3, \mathbb{R})$ MATRICES

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**Abstract.** Let  $R = (r_{ij}) \in SO(3, \mathbb{R})$ . We give several different proofs of the fact that the vector

$$V := \left( \frac{1}{r_{23} + r_{32}}, \frac{1}{r_{13} + r_{31}}, \frac{1}{r_{12} + r_{21}} \right)^t$$

if it exists, is an eigenvector of  $R$  corresponding to the eigenvalue one.

MSC:15A18, 15-01, 97Axx

Keywords: Eigenvectors, orthogonal matrices, rotations in  $\mathbb{R}^3$

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