



## BOOK REVIEW

*Physical Components of Tensors*, by Wolf Altman and Antonio Marmo De Oliveira, CRC Press - Taylor & Francis Group, Boca Raton 2015, XXII + 209 pp., ISBN: 978-1-4822-6383-1.

In the brand new book *Physical Components of Tensors* by the prominent Brazilian scientists Wolf Altman and Antonio Marmo de Oliveira appearing in the CRC Series in *Applied and Computational Mechanics* the reader can find a concise but comprehensive account of the fundamentals of tensor calculus in holonomic and anholonomic coordinate systems and a number of its applications in continuum and structural mechanics.

Nowadays, it is hardly possible to imagine a modern description of the mechanical properties, statics and dynamics of solids and structures without using vector and tensor relations and tensor calculus. As pointed out by the authors “Anyone who lacks the knowledge of this mathematical tool is at a disadvantage in what concerns working effectively in this as well as several other fields of pure and applied mathematics”.

However, this book is not just another, although excellent, textbook on classical tensor calculus. Definitely, it provides an exhaustive presentation of the theory of physical and nonholonomic components of tensors with application to the continuum mechanics and shell theory. In fact, this is what makes Altman and de Oliveira’s book unique. Doubtless, each graduate student, professor or researcher working in the field of mechanical or civil engineering, theoretical or experimental physics or applied mathematics would find it useful.

The book is organized in five chapters. The first three chapters, which may be thought of as a self-contained introductory part of the book, namely, Chapter 1. *Finite-Dimensional Vector Spaces*, Chapter 2. *Vector and Tensor Algebras* and Chapter 3. *Tensor Calculus* introduce the mathematical background of the theory developed in Chapter 4. *Physical and Anholonomic Components of Tensors* and applied in Chapter 5. *Deformation of Continuous Media*. Here, it should be noted

that the book assumes some basic knowledge and skills in linear algebra and elementary calculus, although all the relevant concepts and results from these subjects can be found in the introductory part.

Chapter 1 introduces the basic concepts related with the finite-dimensional vector spaces: the notions of inner products, norms, metrics, Cartesian and curvilinear systems of coordinates as well as the transformation of coordinate systems. The main algebraic operations for tensors, the notions of tensor products, duality and component representation of vectors and tensors are presented in Chapter 2. Chapter 3 presents the classical tensor calculus: scalar, vector and tensor fields, directional derivative, Christoffel symbols, covariant differentiation and integral theorems of Gauss and of Stokes.

In Chapter 4 the authors expose their theory of physical and anholonomic components of tensors applying the basic principles of dimensional analysis to tensors, connections and covariant derivatives. Actually, Altman and de Oliveira have a long-standing interest in this subject (see [1–3, 5, 6]) and have devoted about forty years to developing of a more general, expanded or anholonomic tensor calculus. It should be remarked that the idea of physical components was introduced for the first time more than one hundred years ago, in 1901, by Ricci and Levi-Civita [12]. Later on, several authors have defined and dealt with physical components of vectors and tensors (see [4, 7–11, 13–15]). Here, the theory is built up using two different methods to determine the physical components of tensors which are shown, however, to lead to the same result. In the first one, based on the so-called “Truesdell’s hypothesis” [15], each tensor is regarded as a linear transformation between two properly chosen vector spaces. Within the second method, this components are obtained by virtue of the invariance of the tensorial representation and decomposition of the tensor in anholonomic and natural bases. It is noteworthy that such an exhaustive and consistent theory as that presented here cannot be found elsewhere. The relationship and compatibility of this theory with various aspects of the aforementioned studies on the subject are also shown.

Chapter 5 comprises two important applications of the theory developed in the previous Chapter. The first one concerns representation of the Cauchy and Piola-Kirchhoff stress tensors, strain-displacement relations and equations of motion and equilibrium for continuous bodies. All equations are given both in tensor and physical components. The same holds true for the second application. It concerns the shell theory. Here, the reader can find presented the essentials of the theory of thin elastic shells derived from the three-dimensional equations of continuum mechanics: strain-displacement relation, kinematic and governing equations for shells. Constitutive equations are given too, but only in the special case of thermoelastic shells.

An important feature of the book is the large number of examples and exercises with solution which not only helps the reader to improve his or her understanding of the subject, but also complements the materials introduced in the text.

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