



## FINITE-DIFFERENCE IMPLEMENTATION OF CONSERVED PROPERTIES OF VECTOR NLSE

MICHAIL TODOROV

Communicated by Jean-Francois Ganghoffer

---

**Abstract.** In this work we consider conserved properties of the vector Nonlinear Schrödinger Equations for linearly polarized solitons in the initial configuration. We derive analytic formulae for the mass, pseudomomentum and energy and compare results with the discrete formulae based on a conservative fully implicit finite-difference scheme in complex arithmetic.

---

### 1. Introduction

The investigation of soliton supporting systems is of great importance both in view of applications and for the fundamental understanding of the phenomena associated with propagation of solitons. Recently, elaborate models such as vector Nonlinear Schrödinger Equation (NLSE) appeared in the literature (see, for example [4, 6]). They involve more parameters and have richer phenomenology but, as a rule, they are not fully integrable and require numerical approaches. The non-fully-integrable models possess as a rule three conservation laws: (wave) “mass”, (wave) momentum, and energy and these have to be faithfully represented by the numerical schemes.

An implicit scheme of Crank-Nicolson type was first proposed for the single NLS in the extensive numerical treatise [9]. The concept of the internal iterations was first applied to vector NLSE in [3] and extended in [7] in order to ensure the implementation of the conservation laws on difference level within the round-off error of the calculations. Here, we follow generally the works [3, 7] but focus on the conservative properties of the conservative scheme.

The comparison of both analytic and numerical calculations shows a significant advantage in the efficiency of the finite difference scheme and algorithm.