



## SOME STRANGE FEATURES OF THE GALILEI GROUP

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**Abstract.** Discussed are certain strange properties of the Galilei group, connected first of all with the property of mechanical energy-momentum covector to be an affine object, rather than the linear one. Its affine transformation rule is interesting in itself and dependent on the particle mass. On the quantum level this means obviously that we deal with the projective unitary representation of the group rather than with the usual representation. The status of mass is completely different than in relativistic theory, where it is a continuous eigenvalue of the Casimir invariant. In Galilei framework it is a parameter characterizing the factor of the projective representation, in the sense of V. Bargmann. This “pathology” from the relativistic point of view is nevertheless very interesting and it underlies the Weyl-Wigner-Moyal-Ville approach to quantum mechanics.

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### 1. Introduction to the Galilei Group

It was shown that when some natural postulates are accepted, then there are only two natural flat space-time structures: Galilean and Minkowskian. And, respectively, there are two alternative symmetry groups: Galilei and Poincare groups. The peculiarity of Minkowski-Poincare structure is the existence of some fixed universal physical constant-velocity of light  $c$ . In a sense the traditional Galilei structure is obtained in the limit transition  $c \rightarrow \infty$ . And in fact there are numerous formulae in which this limit is smoothly achieved. But as usual when it is a “true” limit transition to some singular value, certain discontinuities in the limit appear and the resulting theory changes drastically. The asymptotic form of the theory is not its merely special case and important qualitative discontinuities appear. The same, and even more catastrophically, happens in the  $\hbar \rightarrow 0$  asymptotics of quantum mechanics. Classical mechanics asymptotically corresponds to quantum-mechanical results, but nevertheless it is a qualitatively different theory. In the space-time theory the main peculiarity of the limit transition  $c \rightarrow \infty$  is the catastrophic change of the structure of the four-momentum obtained on the basis of the four-dimensional Legendre transformation. In this sense the relativistic theory