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SOME APPLICATIONS OF THE LORENTZIAN HOLONOMY ALGEBRAS

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Abstract. It is shown how one can apply the classification of the holonomy algebras of Lorentzian manifolds to solve some problems. In particular, a new proof to the classification of Lorentzian manifolds with recurrent curvature tensor is given and the classification of two-symmetric Lorentzian manifolds is explained. Then the conformally recurrent Lorentzian manifolds are classified and the recurrent symmetric bilinear forms on these manifolds are described.

1. Introduction

While the classification of the Riemannian holonomy algebras is a classical result that has many applications both to geometry and physics, see e.g. [4, 15], the classification of the Lorentzian holonomy algebras has been achieved only recently [10, 17]. We review it in Section 3. The holonomy algebra of a pseudo-Riemannian manifold is an important invariant of the Levi-Civita connection. It provides information about parallel and recurrent tensors on the manifold. Using that property, we solve some problems in Lorentzian geometry. As a first illustration, in Section 6 we give a new and modern proof to the classification of Lorentzian manifolds (M, g) with recurrent curvature tensor R, i.e., satisfying the condition

$$\nabla_X R = \theta(X) R \tag{1}$$

for all vector fields X and a one-form θ. Originally this classification is achieved in [24]. In Section 7 we discuss the Lorentzian symmetric spaces. As a new result, in Section 9 we obtain a classification of Lorentzian manifolds with recurrent conformal Weyl tensor W. This generalizes a result from [8, 9] that gives classification of Lorentzian manifolds with parallel W. In Section 10 we explain the result from [2] about the classification of two-symmetric Lorentzian manifolds (M, g), i.e., manifolds satisfying the condition

$$\nabla^2 R = 0, \qquad \nabla R \neq 0. \tag{2}$$