A NOTE ON POISSON LIE ALGEBROIDS

LIVIU POPESCU

Communicated by Izu Vaisman

Abstract. In this paper we study some properties of a Lie algebroid and its prolongation over the vector bundle projection of the dual bundle. We generalize some results on Poisson manifolds to the level of a Lie algebroid. The notions of canonical Poisson bivector and horizontal lift are studied and their compatibility conditions are pointed out.

1. Introduction

The Lie algebroid [10] is a generalization of both concepts of Lie algebra and integrable distribution, being a vector bundle (E, π, M) with a Lie bracket on his space of sections with properties very similar to those of a tangent bundle. The Poisson manifolds are the smooth manifolds equipped with a Poisson bracket on their ring of functions. I have to remark that the cotangent bundle of a Poisson manifold has the natural structure of a Lie algebroid [13]. In the last years diverse aspects of these subjects have been studied in a lot of papers (see for instance [13], [14], [12], [1] and [7]). In the present paper we study some geometrical structures on the prolongation of a Lie algebroid to its dual bundle and investigate some aspects of the Lie algebroid geometry endowed with a Poisson structure. In this way we generalize some results on Poisson manifolds.

The paper is organized as follows. In the Section 2 we recall the Cartan calculus and the Schouten-Nijenhuis bracket at the level of a Lie algebroid and present the Poisson structures on the Lie algebroid. The Section 3 deals with the prolongation of a Lie algebroid [5], [8] to its dual bundle and continue the investigation starting in [6]. We study the properties of the canonical Poisson bivector and introduce the notion of horizontal lift. Finally, the compatibility conditions of these bivectors are given. We remark that in the particular case of the standard Lie algebroid $(E = TM, \sigma = \mathrm{Id})$ some results of Mitric [12] are obtained.