JGSP **10** (2007) 29–40

JOURNAL OF
Geometry and Symmetry in Physics

TUBE FORMULA, BEREZINIANS, AND DWORK FORMULA

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Communicated by Theodore Voronov

Abstract. We consider an example of tubes of hypersurfaces in Euclidean space and generalise the tube formula to supercase. By this we assign to a point of the hypersurface in superspace a rational characteristic function. Does this rational function appear when we calculate the ζ-function of an arithmetic variety?

Introduction

I would like to make a remark on relations between tube formula and Dwork formula for ζ-function for arithmetic varietes. I have been thinking about this relation and have discussed it for several years with many colleagues. In particular I spoke about it in Białowieża last summer. Recently a very interesting paper [1] appeared in the web which touches on a related circle of ideas.

1. Tubes of Hypersurfaces

Recall some simple facts concerning tubes of hypersurfaces in Euclidean space.

Let M be a surface in Euclidean space $\mathbb{E}^{n+1}$. By a tube we shall understand the set of points in $\mathbb{E}^{n+1}$ that are at distance h from M, $h \geq 0$. If M is an orientable hypersurface (surface of codimension 1), then a direction of normal vector can be chosen. This defines sign of the distance between a point and the surface. In such a case the tube of radius h is the disconnected union of two *half-tubes* M_h and M_{-h}. We consider here only oriented hypersurfaces and later denote by M_h a half-tube for any $h \in \mathbb{R}$. The n-dimensional volumes of tubes and half-tubes are polynomials in h if h is small enough. These formulae can be traced to Steiner (1840), who derived them for a polygon and a polyhedron. In 1939 Weyl gave general formulae for polynomials expressing volumes of tubes and half-tubes. The coefficients of these polynomials are integrals of expressions which are formed from the second quadratic form at n-dimensional surface. For