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ERRATUM

Erratum to: On Kazhdan–Lusztig cells in type *B*

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In [2], we have found, using brute force computations, some (not all) Kazhdan–Lusztig relations (let us call them the *elementary relations*) between very particular elements of a Weyl group of type B. This shows in particular that the equivalence classes generated by the elementary relations are contained in Kazhdan–Lusztig cells.

It was announced in [6, Theorems 1.2 and 1.3] that the elementary relations generate the equivalence classes defined by the domino insertion algorithm (let us call them the *combinatorial cells*). As a consequence, we “deduced” that the combinatorial cells are contained in the Kazhdan–Lusztig cells [2, Theorem 1.5], thus confirming conjectures of Geck, Iancu, Lam and the author [3, Conjectures A and B]. However, as was explained in a revised version of [6] (see [7]), the equivalence classes generated by the elementary relations are in general strictly contained in the combinatorial cells. This has no consequence on most of the intermediate results in [2], but changes the scope of validity of [2, Theorem 1.5]. Indeed, for some special cases of the parameters, T. Pietraho [5] has found that the elementary relations generate the combinatorial cells. So part of [2, Theorem 1.5] can be saved: the aim of this note is to explain precisely what is proved and what remains to be proved.

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Remark The fact that [6, Theorems 1.2 and 1.3] is false does not imply that the result stated in [2, Theorem 1.5] is also false: it just means that its proof is not complete and we still expect the statement to be correct (as conjectured in [3, Conjectures A and B]).

1 Proved and unproved results from [2]

Unproved results We keep the notation of [2]. First of all, the proof of the Theorem stated in the introduction of [2], so its statement remains a conjecture (and similarly for the Corollary stated at the end of this introduction). Also, [2, Theorem 1.5(a)] is still a conjecture. However, [2, Theorem 1.5(b)] is still correct: its proof must only be adapted, using Pietraho's results [5].

Theorem 1 *Let $r \geq 0$ and assume that $b = ra > 0$. Let $? \in \{L, R, LR\}$ and $x, y \in W_n$ be such that $x \approx_?^r y$. Then $x \sim_? y$.*

The proof of Theorem 1 will be given in the next section. It must also be noted that [2, Theorem 1.5] is also valid if $b > (n-1)a$ (see [4, Theorem 7.7] and [1, Corollaries 3.6 and 5.2]).

Proved results Apart from the above mentioned results, all other intermediate results (about computations of Kazhdan–Lusztig polynomials, structure constants, elementary relations) are correct.

2 Proof of Theorem 1

In [2, Sect. 7.1], we have introduced, following [6], three elementary relations $\smile_1$, $\smile_2^r$ and $\smile_3^r$: for adapting our argument to the setting of [5], we shall need to introduce another relation, which is slightly stronger than $\smile_3^r$.

Definition 2 If w and w' are two elements of W_n, we shall write $w \frown_3^r w'$ whenever $w' = tw$ and $|w(1)| > |w(2)| > \cdots > |w(r+2)|$. If $r \geq n-1$, then, by convention, the relation $\frown_3^r$ never occurs.

Using this definition, Pietraho's Theorem [5, Theorem 3.11] can be stated as follows:

Pietraho's Theorem *The relation $\approx_R^r$ is the equivalence relation generated by $\smile_1$, $\smile_2^r$ and $\frown_3^{r-1}$.*

It is easy to check that, if $w \frown_3^r w'$, then $w \smile_3^r w'$. Therefore, Theorem 1 follows from [2, Lemmas 7.1, 7.2 and 7.3] and the argument in [2, Sect. 7.2].

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