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A Problem Related to the Hall Effect in a Semiconductor with an Electrode of an Arbitrary Shape

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A problem on electric current in a semiconductor film from an electrode of an arbitrary shape is studied in the presence of a magnetic field. This situation describes the Hall effect, which indicates the deflection of electric current from electric field in a semiconductor. From mathematical standpoint we consider the skew derivative problem for harmonic functions in the exterior of an open arc in a plane. By means of potential theory the problem is reduced to the Cauchy singular integral equation and next to the Fredholm equation of the 2nd kind which is uniquely solvable. The solution of the integral equation can be computed by standard codes by discretization and inversion of the matrix. The uniqueness and existence theorems are formulated.

Keywords: Hall effect; Electrode in a semiconductor; Electric current

1 INTRODUCTION

It is well-known [18–21] that the direction of an electric current and the direction of an electric field do not coincide in a semiconductor in the presence of a magnetic field. This effect has been found by Hall [17]. Mathematically the Hall effect leads to a skew derivative boundary value problem [5,8,9,23–25]. An electrode in a semiconductor can be modeled by an open arc of an arbitrary shape. So, the problem on an

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electric current from such an electrode in the presence of a magnetic field results in the skew derivative problem in the exterior of an open arc in a plane. Similar problems were not treated before, while Dirichlet and Neumann problems in the exterior of an open arc as well as skew derivative problems in domains bounded by closed curves were actively treated. In general, boundary value problems with an open boundary are different from those with closed boundary, because different methods are used in their analysis, so that the number of rigorous results is much less in case of an open boundary. Nevertheless, Dirichlet and Neumann problems in the exterior of an open arc were considered, for example, in [1,4,10–13,15,16] with the help of classical single and double layer potentials.

The skew derivative problems for an open boundary are too complicated to be effectively studied by a classical approach. To solve the 2-D skew derivative problem outside an open arc by a classical approach, we must look for a solution of this problem by a linear combination of single and double layer potentials, because the problem cannot be solved by only one of them. In this way we arrive at a very complicated system of boundary integro-differential equations. The system contains hypersingular integrals, Cauchy singular integrals, compact operators and the derivative of the density of the double layer potential. Clearly, the system is too complicated to be studied by standard methods. The basic lack of the classical approach is so that single and double layer potentials have different orders of singularities at the boundary. In the present paper we suggest to solve the 2-D skew derivative problem outside an open arc in another way, namely, with the help of the nonclassical angular potential, which has the same order of singularity as a single layer potential [2,6,7]. Looking for a solution of the problem as a sum of angular and single layer potentials, we reduce the problem to a Cauchy singular integral equation with some additional conditions. By inversion of Cauchy singular operator, we obtain the uniquely solvable Fredholm equation of the second kind. Therefore the solution of the problem can be computed by standard codes.

2 FORMULATION OF THE PROBLEM

In Cartesian coordinates $x=(x_1,x_2)\in R^2$ we consider plane semiconductor film. Suppose, that the constant magnetic field acts in the normal direction to the plane (x_1, x_2). The projection of the magnetic induction onto the Ox_3 axis is M. The equations for the electric current in the semiconductor film in a linear case [5,18–21] are

$$\operatorname{div}\mathbf{J} = 0, \quad \mathbf{J} = \Lambda\mathbf{E}, \quad \mathbf{E} = -\nabla u.$$

Here $\mathbf{J} = (J_1, J_2)$ is the current density, $\mathbf{E}$ is the intensity of an electric field, u is the electric field potential, Λ is the conductivity tensor

$$\Lambda = \frac{\eta}{1+\beta^2}\begin{pmatrix} 1 & \beta \\ -\beta & 1 \end{pmatrix},$$

where η is the conductivity of the semiconductor, if the magnetic field is absent, $\beta = \alpha M$, α is the mobility of the carriers. Suppose, that η is a positive constant and β is a real constant.

We note, that our equations describe the Hall effect [17–21], that is the directions of $\mathbf{E}$ and $\mathbf{J}$ do not coincide in the presence of a magnetic field.

By a simple open curve we mean a nonclosed smooth arc of finite length without self-intersections [11].

We consider an electrode placed in the unbounded semiconductor film. The electrode is modeled by a simple open curve $\Gamma \in C^{2,\lambda}$, where the Hölder index $\lambda \in (0, 1]$.

We assume that the curve Γ is parametrized by the arc length s: $\Gamma = \{x\colon\ x = x(s) = (x_1(s),\ x_2(s)),\ s \in [a, b]\}$. Therefore points $x \in \Gamma$ and values of the parameter s are in one-to-one correspondence.

We denote the tangent vector to Γ at the point $x(s)$ by $\tau_x = (\cos\alpha(s), \sin\alpha(s))$, where $\cos\alpha(s) = x_1'(s)$, $\sin\alpha(s) = x_2'(s)$. Let $\mathbf{n}_x = (\sin\alpha(s), -\cos\alpha(s))$ be a normal vector to Γ at $x(s)$. The direction of $\mathbf{n}_x$ is chosen such that it will coincide with the direction of τ_x if $\mathbf{n}_x$ is rotated anticlockwise through an angle of $\pi/2$.

Suppose that the normal current density is specified at the electrode Γ. According to equations written above, this leads to the skew derivative boundary condition in terms of the electric potential $u(x)$:

$$\begin{aligned}(\mathbf{J}, \mathbf{n}_x)|_{x(s)\in\Gamma} &= -\frac{\sigma}{1+\beta^2}\left(\frac{\partial u}{\partial \mathbf{n}_x} + \beta\frac{\partial u}{\partial \tau_x}\right)\Bigg|_{x(s)\in\Gamma} \\ &= -\frac{\sigma}{1+\beta^2} f(s),\end{aligned}$$

where $f(s)$ is a function, specified on $[a, b]$.

The equations presented above are transformed to the Laplace equation with respect to $u(x)$. To give the rigorous mathematical formulation of the skew derivative problem for the Laplace equation we introduce the definition of the appropriate smoothness class.

We say, that the function $u(x)$ belongs to the smoothness class **K** if

(1) $u \in C^0(\overline{R^2\backslash\Gamma}) \cap C^2(R^2\backslash\Gamma)$, and u is continuous at the ends of Γ,
(2) $\nabla u \in C^0(\overline{R^2\backslash\Gamma}\backslash X)$, where X is a point-set, consisting of the end-points of Γ, i.e. $X = \{x(a) \cup x(b)\}$,
(3) in the neighborhood of any point $x(d) \in X$ for some constants $\mathcal{C} > 0$, $\epsilon > -1$ the inequality holds

$$|\nabla u| \leq \mathcal{C}|x - x(d)|^{\epsilon}, \tag{1}$$

where $x \to x(d)$ and $d = a$ or $d = b$.

Remark In the definition of the class **K** we consider functions, which are continuously extended on Γ from the left and right, but their values on Γ from the left and right can be different, so that the functions may have a jump on Γ.

On the basis of our model we arrive at the skew derivative problem for the Laplace equation in $R^2\backslash\Gamma$.

Problem U *To find a function $u(x)$ of the class* **K** *which satisfies the Laplace equation*

$$\Delta u(x) = 0, \;\; x \in R^2\backslash\Gamma; \quad \Delta = \partial^2_{x_1} + \partial^2_{x_2}, \tag{2a}$$

the boundary condition

$$\left(\frac{\partial}{\partial \mathbf{n}_x} u(x(s)) + \beta \frac{\partial}{\partial \tau_x} u(x(s))\right)\bigg|_{\Gamma} = f(s) \tag{2b}$$

and the following conditions at infinity

$$\begin{gathered} |u(x)| \leq Const, \quad |\nabla u(x)| = o(|x|^{-1}), \\ |x| = \sqrt{x_1^2 + x_2^2} \to \infty. \end{gathered} \tag{2c}$$

We suppose, that β is a real constant. All conditions of the problem **U** must be satisfied in the classical sense.

The Neumann problem for the Laplace equation in the exterior of an open curve is a particular case of our problem when $\beta = 0$.

On the basis of the energy equalities [14] we can easily prove the following assertion.

THEOREM 1 *Let* $\Gamma \in C^{2,\lambda}$, $\lambda \in (0, 1]$. *If the solution of the problem* **U** *exists, then it is defined up to an arbitrary additive constant.*

Proof Let us show, that if $u_0(x)$ is a solution of the homogeneous problem **U**, then $u_0(x) \equiv const$. To prove this with the help of the energy equalities, we envelope Γ by a closed contour, tend this contour to Γ and use the smoothness of the solution of the problem **U**. In this way we obtain

$$\begin{aligned}\|\nabla u_0\|^2_{L_2(R^2\setminus\Gamma)} &= \lim_{r\to\infty} \|\nabla u_0\|^2_{L_2(C_r\setminus\Gamma)} \\ &= \int_a^b u_0^+ \left(\frac{\partial u_0}{\partial \mathbf{n}_x}\right)^+ \mathrm{d}s - \int_a^b u_0^- \left(\frac{\partial u_0}{\partial \mathbf{n}_x}\right)^- \mathrm{d}s,\end{aligned}$$

where the conditions (1), (2c) are taken into account and C_r is the circle of the radius r with the center in the origin. Besides, in the latter formula we consider Γ as a cut. The side of Γ which is on the left, when the parameter s increases, we denote by Γ^+ and the opposite side we denote by Γ^-. In a similar manner, by the superscripts " + " and " − " we denote the limit values of functions on Γ^+ and Γ^- respectively.

Using the homogeneous boundary condition (2b) we obtain from the latter formula

$$\begin{aligned}\|\nabla u_0\|^2_{L_2(R^2\setminus\Gamma)} &= -\beta\left\{\int_a^b u_0^+ \left(\frac{\partial u_0}{\partial \tau_x}\right)^+ \mathrm{d}s - \int_a^b u_0^- \left(\frac{\partial u_0}{\partial \tau_x}\right)^- \mathrm{d}s\right\} \\ &= -\beta\frac{1}{2}\left\{\left(\left[u_0^+(x(b))\right]^2 - \left[u_0^+(x(a))\right]^2\right)\right. \\ &\qquad \left. - \left(\left[u_0^-(x(b))\right]^2 - \left[u_0^-(x(a))\right]^2\right)\right\} = 0,\end{aligned}$$

since $u_0^+(x(b)) = u_0^-(x(b))$, $u_0^+(x(a)) = u_0^-(x(a))$, in accordance with the smoothness properties of the function u_0, which belongs to the class **K**. Thus, $\nabla u_0 \equiv 0$, $u_0 \equiv const$ and the theorem is proved due to the linearity of the problem **U**.

3 INTEGRAL EQUATIONS AT THE BOUNDARY

Below we assume that $f(s)$ from (2b) is an arbitrary function from the Banach space $C^{0,\lambda}[a,b]$, where the Hölder index $\lambda \in (0,1]$.

We consider the angular potential from [2] for the Eq. (2a) on Γ

$$v[\mu](x) = -\frac{1}{2\pi}\int_a^b \mu(\sigma)V(x,\sigma)\,\mathrm{d}\sigma. \qquad (3)$$

The kernel $V(x,\sigma)$ is defined (up to indeterminacy $2\pi m$, $m = \pm 1, \pm 2, \dots$) by the formulae

$$\cos V(x,\sigma) = \frac{x_1 - y_1(\sigma)}{|x - y(\sigma)|}, \quad \sin V(x,\sigma) = \frac{x_2 - y_2(\sigma)}{|x - y(\sigma)|},$$

where

$$y(\sigma) = (y_1(\sigma), y_2(\sigma)) \in \Gamma,$$
$$|x - y(\sigma)| = \sqrt{(x_1 - y_1(\sigma))^2 + (x_2 - y_2(\sigma))^2}.$$

One can see, that $V(x,\sigma)$ is the angle between the vector $\overrightarrow{y(\sigma)x}$ and the direction of the Ox_1 axis. More precisely, $V(x,\sigma)$ is a many-valued harmonic function of x connected with $\ln|x - y(\sigma)|$ by the Cauchy–Riemann relations.

Below by $V(x,\sigma)$ we denote an arbitrary fixed branch of this function, which varies continuously with σ along the curve Γ for given fixed $x \notin \Gamma$.

Under this definition of $V(x,\sigma)$, the potential $v[\mu](x)$ is a many-valued function. In order that the potential $v[\mu](x)$ be single-valued it is necessary to impose the following additional condition:

$$\int_a^b \mu(\sigma)\,\mathrm{d}\sigma = 0. \qquad (4)$$

Below we suppose that the density $\mu(\sigma)$ belongs to the Banach space $C_q^{\omega}[a,b]$, $\omega \in (0,1]$, $q \in [0,1)$ and satisfies condition (4).

We say, that $\mu(s) \in C_q^{\omega}[a,b]$ if

$$\mu(s)|s-a|^q|s-b|^q \in C^{0,\omega}[a,b],$$

where $C^{0,\omega}[a,b]$ is a Hölder space with the index ω and

$$\|\mu(s)\|_{C_q^\omega[a,b]} = \|\mu(s)|s-a|^q|s-b|^q\|_{C^{0,\omega}[a,b]}.$$

As shown in [2,6] for such $\mu(\sigma)$ the angular potential $v[\mu](x)$ belongs to the class **K**. In particular, the inequality (1) holds with $\epsilon = -q$, if $q \in (0,1)$. Moreover, integrating $v[\mu](x)$ by parts and using (4) we express the angular potential in terms of a double layer potential

$$v[\mu](x) = \frac{1}{2\pi}\int_a^b \rho(\sigma)\frac{\partial}{\partial \mathbf{n}_y}\ln|x-y(\sigma)|\,\mathrm{d}\sigma, \tag{5}$$

with the density

$$\rho(\sigma) = \int_a^\sigma \mu(\xi)\,\mathrm{d}\xi, \quad \sigma \in [a,b]. \tag{6}$$

Consequently, $v[\mu](x)$ satisfies both Eq. (2a) outside Γ and the conditions at infinity (2c).

Let us construct a solution of the problem **U**. This solution can be obtained with the help of potential theory for Eq. (2a). We seek a solution of the problem in the following form:

$$u[\mu](x) = v[\mu](x) - \beta w[\mu](x) + C, \tag{7}$$

where C is an arbitrary constant, $v[\mu](x)$ is given by (3), (5) and

$$w[\mu](x) = -\frac{1}{2\pi}\int_a^b \mu(\sigma)\ln|x-y(\sigma)|\,\mathrm{d}\sigma.$$

As mentioned above, we will seek $\mu(s)$ from the Banach space $C_q^\omega[a,b]$, $\omega \in (0,1]$, $q \in [0,1)$. We note, that $\mu(s)$ must satisfy condition (4).

For such $\mu(s)$ the function (7) belongs to the class **K** and satisfies all conditions of the problem **U** except the boundary condition (2b). In particular, conditions at infinity hold due to (4).

To satisfy the boundary condition we put (7) in (2b), use the limit formulas for the angular potential from [2,6] and arrive at the singular integral equation [11] for the density $\mu(s)$:

$$-\frac{1+\beta^2}{2\pi}\int_a^b \mu(\sigma)\frac{\sin\varphi_0(x(s),y(\sigma))}{|x(s)-y(\sigma)|}\,\mathrm{d}\sigma = f(s), \quad s \in [a,b], \tag{8}$$

where $\varphi_0(x,y)$ is the angle between the vector $\overrightarrow{xy}$ and the direction of the normal $\mathbf{n}_x$. The angle $\varphi_0(x,y)$ is taken to be positive if it is measured anticlockwise from $\mathbf{n}_x$ and negative if it is measured clockwise from $\mathbf{n}_x$. Besides, $\varphi_0(x,y)$ is continuous in $x, y \in \Gamma$ if $x \neq y$.

Thus, if $\mu(s)$ is a solution of Eqs. (4), (8) from the space $C_q^{\omega}[a,b]$, $\omega \in (0,1]$, $q \in [0,1)$, then the potential (7) satisfies all conditions of the problem **U**. The following theorem holds.

THEOREM 2 *If* $\Gamma \in C^{2,\lambda}$, $f(s) \in C^{0,\lambda}[a,b]$, $\lambda \in (0,1]$, *Eq.* (8) *has a solution* $\mu(s)$ *from the Banach space* $C_q^{\omega}[a,b]$, $\omega \in (0,1]$, $q \in [0,1)$ *and condition* (4) *holds, then the function* (7) *is a solution of the problem* **U**.

Our further treatment will be aimed to the proof of the solvability of the system (4), (8) in the Banach space $C_q^{\omega}[a,b]$. Moreover, we reduce the system (4), (8) to a Fredholm equation of the second kind, which can easily be computed by classical methods.

It can easily be proved that

$$Y(s,\sigma) = \frac{1}{\pi}\left(\frac{\sin\varphi_0(x(s),y(\sigma))}{|x(s)-y(\sigma)|} - \frac{1}{\sigma - s}\right) \in C^{0,\lambda}([a,b]\times[a,b])$$

(see [6,7] for details). Therefore we can rewrite (8) in the form

$$\frac{1}{\pi}\int_a^b \mu(\sigma)\frac{\mathrm{d}\sigma}{\sigma - s} + \int_a^b \mu(\sigma)\,Y(s,\sigma)\,\mathrm{d}\sigma = -\frac{2}{1+\beta^2}f(s), \quad s \in [a,b]. \tag{9}$$

4 THE FREDHOLM INTEGRAL EQUATION AND THE SOLUTION OF THE PROBLEM

Inverting the singular integral operator in (9) we arrive at the following integral equation of the second kind [11]:

$$\mu(s) + \frac{1}{\sqrt{s-a}\sqrt{b-s}}\int_a^b \mu(\sigma)A(s,\sigma)\,\mathrm{d}\sigma + \frac{1}{\sqrt{s-a}\sqrt{b-s}}G = \frac{1}{\sqrt{s-a}\sqrt{b-s}}\Phi(s), \quad s \in [a,b], \tag{10}$$

where

$$A(s,\sigma) = -\frac{1}{\pi}\int_a^b \frac{Y(\xi,\sigma)}{\xi - s}\sqrt{\xi - a}\sqrt{b-\xi}\,\mathrm{d}\xi,$$

$$\Phi(s) = \frac{1}{1+\beta^2}\frac{1}{\pi}\int_a^b \frac{2\sqrt{\sigma - a}\sqrt{b-\sigma}f(\sigma)}{\sigma - s}\,\mathrm{d}\sigma,$$

and G is an arbitrary constant. We mean arithmetic values of square roots in all formulas where they are used.

To find the constant G we substitute $\mu(s)$ from (10) in the condition (4) and use the explicit formulas for integrals from [22]:

$$\int_a^b \frac{\mathrm{d}\sigma}{\sqrt{\sigma - a}\sqrt{b-\sigma}} = \pi,$$

$$\int_a^b \frac{1}{\sqrt{\sigma - a}\sqrt{b-\sigma}}\frac{\mathrm{d}\sigma}{\sigma - s} = 0, \quad s \in [a,b].$$

Then we obtain that $G=0$. We substitute G in (10) and arrive at the integral equation for $\mu(s)$ on $[a,b]$

$$\mu(s) + \frac{1}{\sqrt{s-a}\sqrt{b-s}}\int_a^b \mu(\sigma)A(s,\sigma)\,\mathrm{d}\sigma = \frac{1}{\sqrt{s-a}\sqrt{b-s}}\Phi(s), \quad s \in [a,b]. \tag{11}$$

It can be shown using the properties of singular integrals [3,11], that $\Phi(s), A(s,\sigma)$ are Hölder functions if $s \in [a,b]$, $\sigma \in [a,b]$. Consequently, any solution of (11) belongs to $C^{\omega}_{1/2}[a,b]$ and below we look for $\mu(s)$ on $[a,b]$ in this space. Moreover, it follows from our treatment, that any solution of (11) meets condition (4).

Instead of $\mu(s) \in C^{\omega}_{1/2}[a,b]$ we introduce the new unknown function

$$\mu_*(s) = \mu(s)\sqrt{s-a}\sqrt{b-s} \in C^{0,\omega}[a,b]$$

and rewrite (11) in the form

$$\mu_*(s) + \int_a^b \mu_*(\sigma)\frac{A(s,\sigma)}{\sqrt{\sigma - a}\sqrt{b-\sigma}}\,\mathrm{d}\sigma = \Phi(s), \quad s \in [a,b]. \tag{12}$$

Thus, the system of equations (4), (8) for $\mu(s)$ has been reduced to the Eq. (12) for the function $\mu_*(s)$. It is clear from our consideration that any solution of (12) gives a solution of system (4), (8).

As noted above, $\Phi(s)$ and $A(s,\sigma)$ are Hölder functions if $s\in[a,b]$, $\sigma\in[a,b]$. More precisely (see [7,11]), $\Phi(s)\in C^{0,p}[a,b]$, $p=\min\{1/2,\lambda\}$ and $A(s,\sigma)$ belongs to $C^{0,p}[a,b]$ in s uniformly with respect to $\sigma\in[a,b]$.

We arrive at the following assertion.

LEMMA 1 *If* $\Gamma\in C^{2,\lambda}$, $\lambda\in(0,1]$, $\Phi(s)\in C^{0,p}[a,b]$, $p=\min\{\lambda,1/2\}$, *and* $\mu_*(s)$ *from* $C^0[a,b]$ *satisfies the Eq.* (12), *then* $\mu_*(s)\in C^{0,p}[a,b]$.

The condition $\Phi(s)\in C^{0,p}[a,b]$ holds if $f(s)\in C^{0,\lambda}[a,b]$. Hence below we will seek $\mu_*(s)$ from $C^0[a,b]$. Since $A(s,\sigma)\in C^0([a,b]\times[a,b])$, the integral operator from (12):

$$(\mathbf{A}\mu_*)(s)=\int_a^b \mu_*(\sigma)\frac{A(s,\sigma)}{\sqrt{\sigma-a}\sqrt{b-\sigma}}\,\mathrm{d}\sigma$$

is a compact operator mapping $C^0[a,b]$ into itself. Therefore, (12) is a Fredholm equation of the second kind in the Banach space $C^0[a,b]$.

Let us show that homogeneous equation (12) has only a trivial solution. Then, according to Fredholm's theorems, the inhomogeneous equation (12) has a unique solution for any right-hand side. We will prove this by a contradiction. Let $\mu_*^0(s)\in C^0[a,b]$ be a nontrivial solution of the homogeneous equation (12). According to the Lemma 1 $\mu_*^0(s)\in C^{0,p}[a,b], p=\min\{\lambda,1/2\}$. Therefore the function

$$\mu^0(s)=\frac{\mu_*^0(s)}{\sqrt{s-a}\sqrt{b-s}}\in C^p_{1/2}[a,b]$$

converts the homogeneous equation (11) into identity. Using the homogeneous identity (11) we check, that $\mu^0(s)$ satisfies condition (4). Besides, acting on the homogeneous identity (11) with a singular operator with the kernel $(s-t)^{-1}$ we find that $\mu^0(s)$ satisfies the homogeneous equation (9). Consequently, $\mu^0(s)$ satisfies the homogeneous equation (8). On the basis of Theorem 2, $u[\mu^0](x)$ is a solution of the homogeneous problem **U**. According to Theorem 1: $u[\mu^0](x)\equiv const$, $x\in R^2\backslash\Gamma$. Using the limit formulas for tangent and normal derivatives of potentials

[2,6], we obtain

$$\lim_{x\to x(s)\in(\Gamma)^+}\left\{\beta\frac{\partial}{\partial \mathbf{n}_x}u[\mu^0](x)-\frac{\partial}{\partial\tau_x}u[\mu^0](x)\right\}$$
$$-\lim_{x\to x(s)\in(\Gamma)^-}\left\{\beta\frac{\partial}{\partial \mathbf{n}_x}u[\mu^0](x)-\frac{\partial}{\partial\tau_x}u[\mu^0](x)\right\}$$
$$=-(1+\beta^2)\mu^0(s)\equiv 0,\quad s\in[a,b].$$

By Γ^+ we denote the side of Γ which is on the left as a parameter s increases and by Γ^- we denote the other side.

Consequently, if $s\in[a,b]$, then $\mu^0(s)\equiv 0$,

$$\mu_*^0(s)=\frac{\mu^0(s)}{\sqrt{s-a}\sqrt{b-s}}\equiv 0$$

and we arrive at the contradiction to the assumption that $\mu_*^0(s)$ is a nontrivial solution of the homogeneous equation (12). Thus, the homogeneous Fredholm equation (12) has only a trivial solution in $C^0[a,b]$.

We have proved the following assertion.

THEOREM 3 *If* $\Gamma\in C^{2,\lambda}$, $\lambda\in(0,1]$, *then* (12) *is a Fredholm equation of the second kind in the space* $C^0[a,b]$. *Moreover, Eq.* (12) *has a unique solution* $\mu_*(s)\in C^0[a,b]$ *for any* $\Phi(s)\in C^0[a,b]$.

As a consequence of the Theorem 3 and the Lemma 1 we obtain the corollary.

COROLLARY *If* $\Gamma\in C^{2,\lambda}$, $\lambda\in(0,1]$ *and* $\Phi(s)\in C^{0,p}[a,b]$, *where* $p=\min\{\lambda,1/2\}$, *then the unique solution of* (12) *in* $C^0[a,b]$, *ensured by Theorem* 3, *belongs to* $C^{0,p}[a,b]$.

We recall that $\Phi(s)$ belongs to the class of smoothness required in the corollary if $f(s)\in C^{0,\lambda}[a,b]$. As mentioned above, if $\mu_*(s)\in C^{0,p}[a,b]$ is a solution of (12), then

$$\mu(s)=\frac{\mu_*(s)}{\sqrt{s-a}\sqrt{b-s}}\in C^p_{1/2}[a,b]$$

is a solution of system (4), (8). We obtain the following statement.

PROPOSITION *If* $\Gamma \in C^{2,\lambda}$, $f(s) \in C^{0,\lambda}[a,b]$, $\lambda \in (0,1]$, *then the system of equations* (4), (8) *has a solution* $\mu(s) \in C^{p}_{1/2}[a,b]$, $p = \min\{1/2, \lambda\}$, *which is expressed by the formula*

$$\mu(s) = \frac{\mu_*(s)}{\sqrt{s-a}\sqrt{b-s}},$$

where $\mu_*(s) \in C^{0,p}[a,b]$ *is the unique solution of the Fredholm equation* (12) *in* $C^0[a,b]$.

Thus, the system (4), (8) is solvable for any $f(s) \in C^{0,\lambda}[a,b]$. On the basis of the Theorem 2 we arrive at the final result.

THEOREM 4 *If* $\Gamma \in C^{2,\lambda}$, $f(s) \in C^{0,\lambda}[a,b]$, $\lambda \in (0,1]$, *then the solution of the problem* **U** *exists and is given by* (7), *where* $\mu(s)$ *is a solution of equations* (4), (8) *from* $C^{p}_{1/2}[a,b]$, $p = \min\{1/2, \lambda\}$ *ensured by the proposition.*

It can be checked directly that the solution of the problem **U** satisfies condition (1) with $\epsilon = -1/2$. Explicit expressions for singularities of the solution gradient at the end-points of Γ can easily be obtained with the help of formulas presented in [6,7].

Theorem 4 ensures existence of a classical solution of the problem **U** when $\Gamma \in C^{2,\lambda}$, $f(s) \in C^{0,\lambda}[a,b]$. On the basis of our consideration we suggest the following scheme for solving the problem **U**. First, we find the unique solution $\mu_*(s)$ of the Fredholm equation (12) from $C^0[a,b]$. This solution automatically belongs to $C^{0,p}[a,b]$, $p = \min\{\lambda, 1/2\}$. Second, we construct the solution of Eqs. (4), (8) from $C^{p}_{1/2}[a,b]$ by the formula

$$\mu(s) = \frac{\mu_*(s)}{\sqrt{s-a}\sqrt{b-s}}.$$

Finally, substituting $\mu(s)$ in (7) we obtain the solution of the problem **U**.

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