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Research Article

Adaptive Neural Control for a Class of Outputs Time-Delay Nonlinear Systems

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This paper considers an adaptive neural control for a class of outputs time-delay nonlinear systems with perturbed or no. Based on RBF neural networks, the radius basis function (RBF) neural networks is employed to estimate the unknown continuous functions. The proposed control guarantees that all closed-loop signals remain bounded. The simulation results demonstrate the effectiveness of the proposed control scheme.

1. Introduction

The study of the time-delay systems has been one of the most active research topics in recent years [1–15]. The time-delay systems can be divided into four types: systems with input delay [1–5], systems with state delay [6–9, 16–18], systems with both input and state delays, and systems with both input and output delays [19]. The effect of time delay on stability and asymptotic performance has been investigated in [20]. In [21], Lyapunov-Krasovskii functionals were used with backstepping to obtain a robust controller for a class of single-input single-output (SISO) nonlinear time-delay systems with known bounds on the functions of delayed states, but it was commented that results could not be constructively obtained in [22]. In [23], the problem of the adaptive neural-networks control for a class of nonlinear state-delay systems with unknown virtual control coefficients is considered. In [24], An adaptive control scheme combined with radius basis function (RBF) neural networks, backstepping, and adaptive control is proposed for the output tracking control problem of a class of MIMO nonlinear system with input delay and disturbances. Neural networks are employed to estimate the unknown continuous functions; the control scheme ensures that the closed-loop system is semiglobally uniformly ultimately bounded (SGUUB). In [11] A control scheme combined with backstepping, radius basis function (RBF) neural networks, and adaptive control is proposed for the stabilization of nonlinear system with input and state delay.

In this paper, we present an adaptive neural controller design procedure for a class of output time-delay nonlinear systems with perturbed, based on backstepping, adaptive control, and neural networks. RBF neural network is employed to the unknown continuous function. A numerical example is provided to show the effectiveness of the control scheme.

2. Problem Formulation and Preliminaries

Consider the nonlinear time-delay system is described as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + g_i(y) + f_i(y(t-\tau)) + \nu_i(t), \quad 1 \le i \le n-1, \\ \dot{x}_n &= u(t) + g_n(y) + f_n(y(t-\tau)) + \nu_n(t), \\ y &= x_1, \end{aligned} \tag{2.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is state, $u \in R$ is control and $y \in R$ is output vectors, respectively. $\nu_i(t)$ $(i = 1, 2, \ldots, n)$ is a time-varying disturbance. $f_i(y(t-\tau))$, $(i = 1, 2, \ldots, n)$, $g_i(y)$, $(i = 1, 2, \ldots, n)$ are unknown continuous functions.

Assumption 2.1. The unknown function $f_i(y)$ satisfies $f_i^2(y) \le k_i$, where k_i $(i = 1, 2, \ldots, n)$ is a known constant.

Assumption 2.2. The time-varying disturbance $\nu_i(t)$ satisfies $|\nu_i(t)| \le d_i < 1$, $1 \le i \le n$, where d_i $(i = 1, 2, \ldots, n)$ is a known constant.

Lemma 2.3. *$x(t) \in \Omega_M, t \in [-\tau_{\max}, \tau]$, where $\Omega_M = \{x \mid \|x\| \le M\}$, M is an unknown constant.*

3. RBF NN Approximation

In this paper, for a given $\delta > 0$ and any continuous function $H_i(\eta_i)$ defined on Ω_i, there is a perfect RBF neural network, which satisfies

$$F_i(\zeta_i) = W_i^T S_i(\zeta_i) + \delta_i(\zeta_i), \tag{3.1}$$

where $|\delta_i(\zeta_i)| \le \delta W_i \in R^{m_i}$ is the weight vector of the neural networks, m_i is the number of the NN nodes, $\zeta_i \in \Omega_i$ is the input vector, $S_i(\zeta_i) = [s_{i1}, \ldots, s_{im_i}]^T$ is defined by

$$s_{ij}(\zeta_i) = \exp\left[-\frac{(\zeta_i - \mu_{ij})^T(\zeta_i - \mu_{ij})}{\xi^2}\right]. \tag{3.2}$$

According to the discussion in [21, 22], denote the best weight vector as follows:

$$W_i^* = \arg \min_{w_i \in R^{m_i}} \sup_{\zeta_i \in \Omega_i} \left| W_i^T S_i(\zeta_i) - H_i(\zeta_i) \right| \tag{3.3}$$

which is unknown and needs to be estimated in control design. Let $\widehat{W}_i$ be the estimate of W_i^*, and define $\widetilde{W}_i = \widehat{W}_i - W_i^*$.

4. Main Result

In this section, we will consider system (2.1).

(I) when $v_i(t) = 0$, $i = 1, 2, \ldots, n$,

Let us define error variables z_i assistant functions and the virtual control α_i, respectively, as follows:

$$\begin{aligned} z_1 &= x_1, \\ z_i &= x_i - \alpha_{i-1}, \quad 2 \le i \le n. \end{aligned} \tag{4.1}$$

Define the following sets:

$$\Omega_{z_i} = \{ z_i \in R \mid |z_i| < \lambda_i \}, \qquad \Omega_{z_i}^o = \{ z_i \in R \mid |z_i| \ge \lambda_i \}, \tag{4.2}$$

where λ_i is a small constant. Define assistant functions as

$$\begin{aligned} F_1 &= g_1(x) + \frac{1}{2\lambda_1^2} U_1(x_1 z_1), \\ F_i &= g_i(x) + \frac{1}{2\lambda_i^2} z_i \sum_{j=1}^{i-1} U_j(x_1) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \dot{\widehat{W}}_j \\ &\quad - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + g_j(x_1) - \frac{1}{2} \frac{\partial \alpha_{i-1}}{\partial x_j} z_j \right), \quad 2 \le i \le n. \end{aligned} \tag{4.3}$$

Define the virtual control as

$$\alpha_i = -k_{i0} z_i - \widehat{W}_i^T S_i(\xi_i), \quad 1 \le i \le n, \tag{4.4}$$

where

$$
\begin{aligned}
k_{10} &= c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2}\left(\lambda_2^2 + \tau_{\max} k_1\right),\\
k_{i0} &= c_i + \frac{b_i}{2} + 2 + \frac{1}{2\lambda_i^2}\left(\lambda_{i+1}^2 + \tau_{\max}\sum_{j=1}^{i-1} k_j\right), \quad 2 \le i \le n-1,\\
k_{n0} &= c_n + \frac{b_n}{2} + 1 + \frac{1}{2\lambda_n^2}\left(\lambda_{i+1}^2 + \tau_{\max}\sum_{j=1}^{n-1} k_j\right),\\
\xi_1 &= \left[x_1, \widehat{x}_1\right]^T,\\
\xi_i &= \left[\overline{x}_i^T, \overline{\widehat{x}}_i^T, \alpha_{i-1}, \left(\frac{\partial_{i-1}}{\partial \overline{x}_{i-1}}\right)^T \left(\frac{\partial_{i-1}}{\partial \overline{\widehat{x}}_{i-1}}\right)^T, \varphi_{i-1}\right]^T, \quad 2 \le i \le n,\\
\varphi_{i-1} &= \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \partial \dot{\widehat{W}}_j, \quad 2 \le i \le n.
\end{aligned}
\tag{4.5}
$$

Theorem 4.1. *System* (2.1) *with both input delay and state delay satisfies Assumptions 2.1 and 2.2. The virtual control can be selected as* (4.4). *If the control law and the adaptive law are selected as follows:*

$$u(t) = \alpha_n, \tag{4.6}$$

$$\dot{\widehat{W}}_i = \Gamma_i\left(z_i S_i(\xi_i) - \sigma_i \widehat{W}_i\right), \quad 1 \le i \le n, \tag{4.7}$$

then the closed-loop system is semi-globally uniformly ultimately bounded.

Proof. Define the Lyapunov-Kresovskii functional $V(t)$ as

$$V_{z_i}(t) = \frac{1}{2} z_i^2(t), \qquad V_{W_i}(t) = \frac{1}{2} \widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i, \tag{4.8}$$

$$V_{U_i}(t) = \frac{1}{2} \sum_{j=1}^{i} \int_{t-\tau_h}^{t} U_j\left(y(\sigma)\right) d\sigma, \tag{4.9}$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + V_{W_i}(t), \quad 1 \le i \le n, \tag{4.10}$$

$$V(t) = \sum_{i=1}^{n} V_i(t). \tag{4.11}$$

Step 1. For the first differential equation of the the first subsystem, by (4.1), (4.3), we can get

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 = x_2 + g_1(x_1) + f_1(x_1(t-\tau_h)) \\ &= z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + g_1(x_1(t-\tau_h)). \end{aligned} \tag{4.12}$$

By

$$z_1 f_1(x_1(t-\tau_h)) \le \frac{1}{2}z_1^2 + \frac{1}{2}f_1^2(x_1(t-\tau_h)) \tag{4.13}$$

differentiating (4.10) and using (4.12), the inequality below can be obtained easily.

$$\begin{aligned} \dot{V}_1 &= z_1\dot{z}_1 + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_h)) \\ &= z_1\left(z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + f_1(x_1(t-\tau_h))\right) \\ &\quad + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_h)) \\ &\le z_1z_2 + z_1\alpha_1 + z_1F_1(\zeta_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1\left(1 - \frac{z_1^2}{\lambda_1^2}\right) + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1. \end{aligned} \tag{4.14}$$

(1) If $z_1 \in \Omega_{z_1}^o$, then $|z_1| \ge \lambda_1$. Thus, substituting (4.4) and (4.7) into (4.14) results in

$$\begin{aligned} \dot{V}_1 &\le z_1z_2 + z_1\left(-k_{10}z_1 - \widehat{W}_1^TS_1(\zeta_1)\right) + z_1F_1(\zeta_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1\left(1 - \frac{z_1^2}{\lambda_1^2}\right) + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 \\ &\le \frac{1}{2}\|z_1\|^2 + \frac{1}{2}\|z_2\|^2 - \left(c_1 + \frac{b_1}{2} + \frac{3}{2}\right)z_1^2 + z_1F_1(\zeta_1) - \frac{z_1^2}{2\lambda_1}\left(\tau_{\max}k_1 + \lambda_2^2\right) \\ &\quad - z_1\widehat{W}_1^TS_1(\zeta_1) + \frac{1}{2}z_1^2 + \frac{1}{2}\|z_1\|^2 + \widetilde{W}\left(z_1S_1(\zeta_1) - \sigma_1\widehat{W}_1\right) \\ &\le \frac{1}{2}z_2^2 - \left(c_1 + \frac{b_1}{2}\right)z_1^2 - \frac{z_1^2}{2\lambda_1^2}\left(\tau_{\max}k_1 + \lambda_2^2\right) + z_1\varepsilon_1(\xi_1) - \sigma_1\widetilde{W}_1^T\widehat{W}_1 \\ &\le \frac{1}{2}z_2^2 - \left(c_1 + \frac{b_1}{2}\right)z_1^2 + \frac{b_1}{2}z_1^2 + \frac{1}{2b_1}(\varepsilon_1^*)^2 - \frac{1}{2}\left(\tau_{\max}k_1 + \lambda_2^2\right) - \sigma_1\widetilde{W}_1^T\widehat{W}_1 \\ &\le -c_1z_1^2 - V_{U_1(T)} - \frac{\sigma_1}{2}\left\|\widetilde{W}_1\right\|^2 + \frac{\sigma_1}{2}\|W_1^*\|^2 + \frac{1}{2b_1}(\varepsilon_1^*)^2 + \frac{1}{2}\left(z_2^2 - \lambda_2^2\right) \\ &\le -k_1V_1 + b_{v_1} + \Theta_1, \end{aligned} \tag{4.15}$$

where $k_1 = \min\{2c_1, \sigma_1/\lambda_{\max}(\Gamma_1^{-1}), 1\}$, $b_{v_1} = (\sigma_1/2)\|W_1^*\|^2 + (1/2b_1)(\varepsilon_1^*)^2$. $\Theta_1 = (1/2)(z_2^2 - \lambda_2^2)$.

If there is no item Θ_1 in (4.15), then

$$0 \le V_1(t) \le (V_1(0) - \delta_v)e^{-k_1 t} + \delta_v, \tag{4.16}$$

where $\delta_v = b_{v1}/k_1$. Thus V_1 is bounded.

(2) If $z_1 \in \Omega_{z_1}$, then $|z_1| < \lambda_1$, z_1 is bounded. By the integral median theorem, we can obtain

$$\frac{1}{2}\int_{t-\tau_h}^{t} U_1(y(\sigma))d\sigma = \frac{1}{2}\tau_h U_1(y(\theta)), \quad \theta \in (t-\tau_h, t). \tag{4.17}$$

By Assumption 2.1 and (4.9), it can be concluded that V_{U_1} is bounded.

Differentiating V_{w_1},

$$\begin{aligned} V_{w_1} &= \widetilde{W}_1^T\left(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1\right) \\ &\le -\frac{1}{2}(\sigma_1 - k_{w_1})\left\|\widetilde{W}_1\right\|^2 + \frac{1}{2}\left(\sigma_1\left\|W_1^*\right\|^2 + \frac{1}{k_{w_1}}z_1^2\|S_1(\zeta_1)\|^2\right) \\ &\le -k_1 V_{w_1} + b_1, \end{aligned} \tag{4.18}$$

where

$$k_1 = \frac{\sigma_1 - k_{w_1}}{\lambda_{\max}\left(\Gamma_1^{-1}\right)}, \qquad b_1 = \frac{1}{2}\left(\sigma_1\left\|W_1^*\right\|^2 + \frac{m_1}{k_{w_1}}\lambda_1^2\right). \tag{4.19}$$

m_1 is the number of neurons of the neural networks. Choose the parameter so that $k_{w_1} < \sigma_1, k_1 > 0$. Therefore

$$0 \le V_{w_1}(t) \le (V_{w_1}(0) - \delta_{w_1})e^{-k_1 t} + \delta_{w_1}, \tag{4.20}$$

where $\delta_{w_1} = b_1/k_1$. Thus $V_{w_1}(t)$ is bounded. Because $V_{z_1}, V_{U_1}, V_{w_1}$ are all bounded, V_1 is bounded when $z_1 \in \Omega_{z_1}$.

Step i. For the ith $(2 \le i \le n-1)$ subsystem, by utilizing (4.1)(4.3), we have

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= x_{i+1} + g_i(x_1) + f_i(x_1((t-\tau_h))) - \dot{\alpha}_{i-1} \\ &= z_{i+1} + \alpha_i + g_i(x_1) + f_i(x_1(t-\tau_h)) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\left(x_{j+1} + g_i(x_1) + f_i(x_1(t-\tau_h))\right) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial \widehat{w}_j}\dot{\widehat{W}}_j. \end{aligned} \tag{4.21}$$

Differentiating (4.10) along track (4.21), we have

$$\begin{aligned}\dot{V}_i &\le z_i(z_{i+1}+\alpha_i+F_i(\zeta_i))+\frac{1}{2}z_i^2+\frac{1}{2}\sum_{j=1}^{i}f_j^2(x_1(t-\tau_h))\\&\quad-\frac{z_i^2}{2\lambda_i^2}\sum_{j=1}^{i-1}U_j(x_1)+\frac{1}{2}\sum_{j=1}^{i-1}(U_j(x_1)-U_j(x_1(t-\tau_h)))\\&\le z_1\left(z_2+\alpha_1+F_1(\zeta_1)-\frac{1}{2\lambda_1^2}U_1(x_1)z_1\right)\\&\quad+f_1(x_1(t-\tau_h))+\widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1\\&\quad+\frac{1}{2}U_1(x_1)-\frac{1}{2}U_1(x_1(t-\tau_h))+\widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i+z_iv_i(t)\\&\le z_1z_2+z_1\alpha_1+z_1F_1(\zeta_1)+\frac{1}{2}z_1^2+\frac{1}{2}U_1\left(1-\frac{z_1^2}{\lambda_1^2}\right)\\&\quad+\widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i+z_iv_i(t)\\&\le z_1z_2+z_1\alpha_1+z_1F_1(\zeta_1)+\frac{1}{2}z_1^2+\frac{1}{2}z_i^2\\&\quad+\sum_{j=1}^{i-1}U_j(x_1)\left(1-\frac{z_i^2}{\lambda_i^2}\right)+\widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i.\end{aligned}\tag{4.22}$$

(1) If $z_i\in\Omega_{z_i}^o$, then $|z_i|\ge\lambda_i$. Thus, substituting (4.4) and (4.7) into (4.22) results in

$$\begin{aligned}\dot{V}_i &\le z_iz_{i+1}+z_iF_i(\zeta_i)\frac{1}{2}z_i^2-\left(c_i+\frac{b_i}{2}+2\right)z_i^2\\&\quad-\frac{z_i^2}{2\lambda_i^2}\left[\tau_{\max}\sum_{j=1}^{i-1}k_j+\lambda_{i+1}^2\right]-z_i\widehat{W}_i^TS_i(\zeta_i)\\&\quad+\widetilde{W}_i^T\left(z_iS_i(\zeta_i)-\sigma_i\widehat{W}_i\right)\\&\le z_iz_{i+1}-\left(c_i+\frac{b_i}{2}+1\right)z_i^2-\frac{1}{2}\left[\tau_{\max}\sum_{j=1}^{i-1}k_j+\lambda_{i+1}^2\right]+z_i\varepsilon_i(\xi_i)-\sigma_i\widetilde{W}_i^T\widehat{W}_i+\frac{1}{2}d_i^2\\&\le -c_iz_i^2-\frac{\tau_{\max}}{2}\sum_{j=1}^{i-1}k_j-\frac{\sigma}{2}\left\|\widetilde{W}_i\right\|^2+\frac{\sigma}{2}\|W_i^*\|^2\\&\quad+\frac{1}{2b_i}(\varepsilon_i^*)^2+\frac{1}{2}\left(-z_i^2+z_{i+1}^2-\lambda_{i+1}^2\right)\\&\le -k_iV_i+b_{vi}+\Theta_i,\end{aligned}\tag{4.23}$$

where $k_i=\min\{2c_i,\sigma_i/(\lambda_{\max}(\Gamma_i^{-1})),1\}$, $b_{v_i}=(\sigma_i/2)\|W_i^*\|^2+(1/2b_i)(\varepsilon_i^*)^2$. $\Theta_i=(1/2)(-z_i^2+z_{i+1}^2-\lambda_{i+1}^2)$.

If there is no item Θ_i in (4.23), then

$$0 \leq V_i(t) \leq (V_i(0) - \delta_v)e^{-k_i t} + \delta_v, \tag{4.24}$$

where $\delta_v = b_{vi}/k_i$. Thus V_i is bounded.

(2) If $z_i \in \Omega_{z_i}$, similar to step 1, we have V_i is bounded.

Step n. This is the last step for the nth subsystem, similarly to the ith subsystem, if $z_n \in \Omega^o_{z_n}$, then $|z_n| \geq \lambda_n$. Thus we have

$$\begin{aligned}\dot{V}_n \leq & -c_n z_n^2 - \frac{\tau_{\max}}{2}\sum_{j=1}^{n-1} k_j - \frac{\sigma}{2}\left\|\widetilde{W}_n\right\|^2 + \frac{\sigma}{2}\|W_n^*\|^2 + \frac{1}{2b_n}(\varepsilon_n^*)^2 - \frac{1}{2}z_n^2 \\ \leq & -k_n V_n + b_{v_n} + \Theta_n,\end{aligned} \tag{4.25}$$

where $k_n = \min\{2c_n, \sigma_n/\lambda_{\max}(\Gamma_n^{-1}), 1\}$, $b_{v_n} = (\sigma_n/2)\|W_n^*\|^2 + (1/2b_n)(\varepsilon_n^*)^2$. $\Theta_n = -(1/2)z_n^2$.

By (4.25), it is easy to have

$$0 \leq V_n(t) \leq (V_n(0) - \delta_v)e^{-k_n t} + \delta_v, \tag{4.26}$$

where $\delta_v = b_{v_n}/k_n$. Thus V_n is bounded.

(1) If $z_n \in \Omega_{z_n}$, similar to step 1, we have V_n is bounded.

The V_i, $(1 \leq i \leq n)$ is bounded when $z_i \in \Omega_{z_i}$, $(1 \leq i \leq n)$. In $z_i \in \Omega^o_{z_i}$, $(1 \leq i \leq n)$:

$$\begin{aligned}\dot{V}(t) &\leq \sum_{i=1}^{n}\dot{V}_i \\ &\leq -\sum_{i=1}^{n}k_i V_i + \sum_{i=1}^{n}b_{vi} + \sum_{i=1}^{n}\Theta_i \\ &\leq -k_v V(t) + b_v,\end{aligned} \tag{4.27}$$

where $k_v = \min\{k_1, k_2, \ldots, k_n\}$, $b_v = \sum_{i=1}^{n} b_{vi}$.

Then

$$0 \leq V(t) \leq (V(0) - \delta_v)e^{-k_v t} + \delta_v, \tag{4.28}$$

where $\delta_v = b_v/k_v$. Thus $V(t)$ is bounded. □

(II) When $\nu_i(t) \neq 0, i = 1, 2, \ldots, n$.

Let us define error variables z_i assistant functions and the virtual control α_i, respectively, as follows:

$$\begin{aligned} z_1 &= x_1, \\ z_i &= x_i - \alpha_{i-1}, \quad 2 \leq i \leq n.\end{aligned} \tag{4.29}$$

Define the following sets:

$$\Omega_{z_i} = \{z_i \in R \mid |z_i| < \lambda_i\}, \qquad \Omega_{z_i}^{o} = \{z_i \in R \mid |z_i| \geq \lambda_i\}, \tag{4.30}$$

where λ_i is a small constant. Define assistant functions as

$$F_1 = g_1(x) + \frac{1}{2\lambda_1^2} U_1(x_1 z_1),$$

$$F_i = g_i(x) + \frac{1}{2\lambda_i^2} z_i \sum_{j=1}^{i-1} U_j(x_1) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \dot{\widehat{W}}_j - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(x_{j+1} + g_j(x_1) - \frac{1}{2} \frac{\partial \alpha_{i-1}}{\partial x_j} z_j \right), \quad 2 \leq i \leq n. \tag{4.31}$$

Define the virtual control as

$$\alpha_i = -k_{i0} z_i - \widehat{W}_i^T S_i(\xi_i), \quad 1 \leq i \leq n, \tag{4.32}$$

where

$$k_{10} = c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2}\left(\lambda_2^2 + \tau_{\max} k_1 + d_1^2\right),$$

$$k_{i0} = c_i + \frac{b_i}{2} + 2 + \frac{1}{2\lambda_i^2}\left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{i-1} k_j + d_i^2\right), \quad 2 \leq i \leq n-1 \tag{4.33}$$

$$k_{n0} = c_n + \frac{b_n}{2} + 1 + \frac{1}{2\lambda_n^2}\left(\lambda_{i+1}^2 + \tau_{\max} \sum_{j=1}^{n-1} k_j + d_n^2\right),$$

$$\xi_1 = [x_1, \hat{x}_1]^T$$

$$\xi_i = \left[\overline{x}_i^T, \overline{\hat{x}}_i^T, \alpha_{i-1}, \left(\frac{\partial_{i-1}}{\partial \overline{x}_{i-1}}\right)^T \left(\frac{\partial_{i-1}}{\partial \overline{\hat{x}}_{i-1}}\right)^T, \psi_{i-1}\right]^T, \quad 2 \leq i \leq n, \tag{4.34}$$

$$\psi_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \widehat{W}_j} \partial \dot{\widehat{W}}_j, \quad 2 \leq i \leq n.$$

Theorem 4.2. *System* (2.1) *with both input delay and state delay satisfies Assumptions 2.1 and 2.2. The virtual control can be selected as* (4.32)*. If the control law and the adaptive law are selected as follow:*

$$u(t) = \alpha_n, \tag{4.35}$$

$$\dot{\widehat{W}}_i = \Gamma_i\left(z_i S_i(\xi_i) - \sigma_i \widehat{W}_i\right), \quad 1 \leq i \leq n, \tag{4.36}$$

then the closed-loop system is semi-globally uniformly ultimately bounded.

Proof. Define the Lyapunov-Kresovskii functional $V(t)$ as

$$V_{z_i}(t) = \frac{1}{2}z_i^2(t), \qquad V_{W_i}(t) = \frac{1}{2}\widetilde{W}_i^T\Gamma_i^{-1}\widetilde{W}_i, \tag{4.37}$$

$$V_{U_i}(t) = \frac{1}{2}\sum_{j=1}^{i}\int_{t-\tau_h}^{t} U_j(y(\sigma))d\sigma, \tag{4.38}$$

$$V_i(t) = V_{z_i}(t) + V_{U_i}(t) + V_{W_i}(t), \quad 1 \le i \le n, \tag{4.39}$$

$$V(t) = \sum_{i=1}^{n} V_i(t). \tag{4.40}$$

Step 1. For the first differential equation of the first subsystem, by (4.29), (4.31), We can get

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 = x_2 + g_1(x_1) + f_1(x_1(t-\tau_h)) + \omega_1(t)\\ &= z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + h_1(x_1(t-\tau_h)) + \omega_1(t).\end{aligned} \tag{4.41}$$

By

$$z_1 f_1(x_1(t-\tau_h)) \le \frac{1}{2}z_1^2 + \frac{1}{2}f_1^2(x_1(t-\tau_h)) \tag{4.42}$$

differentiating (4.39) and using (4.41), the inequality below can be obtained easily.

$$\begin{aligned}\dot{V}_1 &= z_1\dot{z}_1 + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_h))\\ &= z_1\left(z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + \omega_1(t) + f_1(x_1(t-\tau_h))\right) + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1\\ &\quad + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_h))\\ &\le z_1z_2 + z_1\alpha_1 + z_1F_1(\zeta_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1\left(1 - \frac{z_1^2}{\lambda_1^2}\right) + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 + z_1\omega_1(t).\end{aligned} \tag{4.43}$$

(1) If $z_1 \in \Omega_{z_1}^o$, then $|z_1| \geq \lambda_1$. Thus, substituting (4.32) and (4.36) into (4.43) results in

$$\begin{aligned}
\dot{V}_1 \leq\ & z_1 z_2 + z_1\left(-k_{10} z_1 - \widehat{W}_1^T S_1(\zeta_1)\right) + z_1 F_1(\zeta_1) + \frac{1}{2} z_1^2 + \frac{1}{2} U_1\left(1 - \frac{z_1^2}{\lambda_1^2}\right) + \widetilde{W}_1^T \Gamma_1^{-1} \dot{\widehat{W}}_1 + z_1 \nu_1(t) \\
\leq\ & \frac{1}{2}\|z_1\|^2 + \frac{1}{2}\|z_2\|^2 - \left(c_1 + \frac{b_1}{2} + \frac{3}{2}\right) z_1^2 + z_1 F_1(\zeta_1) - \frac{z_1^2}{2\lambda_1}\left(\tau_{\max} k_1 + \lambda_2^2 + d_1^2\right) - z_1 \widehat{W}_1^T S_1(\zeta_1) \\
& + \frac{1}{2} z_1^2 + \frac{1}{2}\|z_1\|^2 + \widetilde{W}\left(z_1 S_1(\zeta_1) - \sigma_1 \widehat{W}_1\right) + \frac{1}{2} d_1^2 \\
\leq\ & \frac{1}{2} z_2^2 - \left(c_1 + \frac{b_1}{2}\right) z_1^2 - \frac{z_1^2}{2\lambda_1^2}\left(\tau_{\max} k_1 + \lambda_2^2\right) + z_1 \varepsilon_1(\xi_1) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\
\leq\ & \frac{1}{2} z_2^2 - \left(c_1 + \frac{b_1}{2}\right) z_1^2 + \frac{b_1}{2} z_1^2 + \frac{1}{2b_1}\left(\varepsilon_1^*\right)^2 + \frac{1}{2} d_1^2 - \frac{1}{2}\left(\tau_{\max} k_1 + \lambda_2^2 + \frac{1}{2} d_1^2\right) - \sigma_1 \widetilde{W}_1^T \widehat{W}_1 \\
\leq\ & -c_1 z_1^2 - V_{U_1(T)} - \frac{\sigma_1}{2}\left\|\widetilde{W}_1\right\|^2 + \frac{\sigma_1}{2}\left\|W_1^*\right\|^2 + \frac{1}{2b_1}\left(\varepsilon_1^*\right)^2 + \frac{1}{2}\left(z_2^2 - \lambda_2^2\right) \\
\leq\ & -k_1 V_1 + b_{v_1} + \Theta_1,
\end{aligned} \tag{4.44}$$

where $k_1 = \min\{2c_1, \sigma_1/(\lambda_{\max}(\Gamma_1^{-1})), 1\}$, $b_{v_1} = (\sigma_1/2)\|W_1^*\|^2 + (1/2b_1)(\varepsilon_1^*)^2$. $\Theta_1 = (1/2)(z_2^2 - \lambda_2^2)$.

If there is no item Θ_1 in (4.44), then

$$0 \leq V_1(t) \leq (V_1(0) - \delta_v) e^{-k_1 t} + \delta_v, \tag{4.45}$$

where $\delta_v = b_{v1}/k_1$. Thus V_1 is bounded.

(2) If $z_1 \in \Omega_{z_1}$, then $|z_1| < \lambda_1$, z_1 is bounded. By the integral median theorem, we can obtain

$$\frac{1}{2}\int_{t-\tau_h}^{t} U_1(y(\sigma))\, d\sigma = \frac{1}{2}\tau_h U_1(y(\theta)), \quad \theta \in (t - \tau_h, t). \tag{4.46}$$

By Assumption 2.1 and (4.38), it can be concluded that V_{U_1} is bounded.

Differentiating V_{w_1},

$$\begin{aligned}
V_{w_1} =\ & \widetilde{W}_1^T\left(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1\right) \\
\leq\ & -\frac{1}{2}(\sigma_1 - k_{w_1})\left\|\widetilde{W}_1\right\|^2 + \frac{1}{2}\left(\sigma_1\left\|W_1^*\right\|^2 + \frac{1}{k_{w_1}} z_1^2\|S_1(\zeta_1)\|^2\right) \\
\leq\ & -k_1 V_{w_1} + b_1,
\end{aligned} \tag{4.47}$$

where

$$k_1 = \frac{\sigma_1 - k_{w_1}}{\lambda_{\max}\left(\Gamma_1^{-1}\right)}, \qquad b_1 = \frac{1}{2}\left(\sigma_1\left\|W_1^*\right\|^2 + \frac{m_1}{k_{w_1}}\lambda_1^2\right). \tag{4.48}$$

m_1 is the number of neurons of the neural networks. Choose the parameter so that $k_{w_1} < \sigma_1, k_1 > 0$. Therefore

$$0 \le V_{w_1}(t) \le (V_{w_1}(0) - \delta_{w_1})e^{-k_1 t} + \delta_{w_1}, \tag{4.49}$$

where $\delta_{w_1} = b_1/k_1$. Thus $V_{w_1}(t)$ is bounded. Because $V_{z_1}, V_{U_1}, V_{w_1}$are all bounded, V_1 is bounded when $z_1 \in \Omega_{z_1}$.

Step i. For the ith $(2 \le i \le n-1)$ subsystem, by utilizing (4.29), (4.31), we have

$$\begin{aligned}
\dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} \\
&= x_{i+1} + g_i(x_1) + f_i(x_1((t-\tau_h))) - \dot{\alpha}_{i-1} + \nu_i(t) \\
&= z_{i+1} + \alpha_i + g_i(x_1) + f_i(x_1(t-\tau_h)) + \nu_i(t) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(x_{j+1} + g_i(x_1) + f_i(x_1(t-\tau_h))\right) \\
&\quad - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{w}_j}\dot{\widehat{w}}_j.
\end{aligned} \tag{4.50}$$

Differentiating (4.39) along track (4.50),we have

$$\begin{aligned}
\dot{V}_i &\le z_i(z_{i+1} + \alpha_i + F_i(\zeta_i)) + \frac{1}{2}z_i^2 + \frac{1}{2}\sum_{j=1}^{i} f_j^2(x_1(t-\tau_h)) \\
&\quad - \frac{z_i^2}{2\lambda_i^2}\sum_{j=1}^{i-1}U_j(x_1) + \frac{1}{2}\sum_{j=1}^{i-1}\left(U_j(x_1) - U_j(x_1(t-\tau_h))\right) \\
&\le z_1\left(z_2 + \alpha_1 + F_1(\zeta_1) - \frac{1}{2\lambda_1^2}U_1(x_1)z_1 + \nu_i(t)\right) \\
&\quad + f_1(x_1(t-\tau_h)) + \widetilde{W}_1^T\Gamma_1^{-1}\dot{\widehat{W}}_1 \\
&\quad + \frac{1}{2}U_1(x_1) - \frac{1}{2}U_1(x_1(t-\tau_h)) + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i + z_i\nu_i(t) \\
&\le z_1z_2 + z_1\alpha_1 + z_1F_1(\zeta_1) + \frac{1}{2}z_1^2 + \frac{1}{2}U_1\left(1 - \frac{z_1^2}{\lambda_1^2}\right) \\
&\quad + \widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i + z_i\nu_i(t)
\end{aligned}$$

$$\begin{aligned}&\leq z_1z_2+z_1\alpha_1+z_1F_1(\zeta_1)+\frac{1}{2}z_1^2+\frac{1}{2}z_i^2+\frac{1}{2}d_i^2\\&\quad+\sum_{j=1}^{i-1}U_j(x_1)\left(1-\frac{z_i^2}{\lambda_i^2}\right)+\widetilde{W}_i^T\Gamma_i^{-1}\dot{\widehat{W}}_i.\end{aligned}\tag{4.51}$$

(1) If $z_i\in\Omega_{z_i}^o$, then $|z_i|\geq\lambda_i$. Thus, substituting (4.32) and (4.36) into (4.51) results in

$$\begin{aligned}\dot{V}_i&\leq z_iz_{i+1}+z_iF_i(\zeta_i)\frac{1}{2}z_i^2-\left(c_i+\frac{b_i}{2}+2\right)z_i^2\\&\quad-\frac{z_i^2}{2\lambda_i^2}\left[d_i^2+\tau_{\max}\sum_{j=1}^{i-1}k_j+\lambda_{i+1}^2\right]-z_i\widehat{W}_i^TS_i(\zeta_i)+\widetilde{W}_i^T\left(z_iS_i(\zeta_i)-\sigma_i\widehat{W}_i\right)\\&\leq z_iz_{i+1}-\left(c_i+\frac{b_i}{2}+1\right)z_i^2-\frac{1}{2}\left[\tau_{\max}\sum_{j=1}^{i-1}k_j+d_i^2+\lambda_{i+1}^2\right]+\frac{1}{2}d_i^2+z_i\varepsilon_i(\xi_i)-\sigma_i\widetilde{W}_i^T\widehat{W}_i+\frac{1}{2}d_i^2\\&\leq -c_iz_i^2-\frac{\tau_{\max}}{2}\sum_{j=1}^{i-1}k_j-\frac{\sigma}{2}\left\|\widetilde{W}_i\right\|^2+\frac{\sigma}{2}\|W_i^*\|^2+\frac{1}{2b_i}(\varepsilon_i^*)^2+\frac{1}{2}\left(-z_i^2+z_{i+1}^2-\lambda_{i+1}^2\right)\\&\leq -k_iV_i+b_{vi}+\Theta_i,\end{aligned}\tag{4.52}$$

where $k_i=\min\{2c_i,\sigma_i/\lambda_{\max}(\Gamma_i^{-1}),1\}$, $b_{v_i}=(\sigma_i/2)\|W_i^*\|^2+(1/2b_i)(\varepsilon_i^*)^2$. $\Theta_i=(1/2)(-z_i^2+z_{i+1}^2-\lambda_{i+1}^2)$.

If there is no item Θ_i in (4.52), then

$$0\leq V_i(t)\leq(V_i(0)-\delta_v)e^{-k_it}+\delta_v,\tag{4.53}$$

where $\delta_v=b_{vi}/k_i$. Thus V_i is bounded.

(2) If $z_i\in\Omega_{z_i}$, similar to step 1, we have V_i is bounded.

Step n. This is the last step for the nth subsystem, similarly to the ith subsystem, If $z_n\in\Omega_{z_n}^o$, then $|z_n|\geq\lambda_n$. Thus we have

$$\begin{aligned}\dot{V}_n&\leq -c_nz_n^2-\frac{\tau_{\max}}{2}\sum_{j=1}^{n-1}k_j-\frac{\sigma}{2}\left\|\widetilde{W}_n\right\|^2+\frac{\sigma}{2}\|W_n^*\|^2+\frac{1}{2b_n}(\varepsilon_n^*)^2-\frac{1}{2}z_n^2\\&\leq -k_nV_n+b_{v_n}+\Theta_n,\end{aligned}\tag{4.54}$$

where $k_n=\min\{2c_n,\sigma_n/\lambda_{\max}(\Gamma_n^{-1}),1\}$, $b_{v_n}=(\sigma_n/2)\|W_n^*\|^2+(1/2b_n)(\varepsilon_n^*)^2$. $\Theta_n=-(1/2)z_n^2$.

By (4.54), it is easy to have

$$0\leq V_n(t)\leq(V_n(0)-\delta_v)e^{-k_nt}+\delta_v,\tag{4.55}$$

where $\delta_v=b_{v_n}/k_n$. Thus V_n is bounded. □

(II) If $z_n \in \Omega_{z_n}$, similar to step 1, we have V_n is bounded.

The V_i, $(1 \le i \le n)$ is bounded. When $z_i \in \Omega_{z_i}$, $(1 \le i \le n)$. In $z_i \in \Omega_{z_i}^o$, $(1 \le i \le n)$:

$$\begin{aligned} \dot{V}(t) &\le \sum_{i=1}^{n} \dot{V}_i \\ &\le -\sum_{i=1}^{n} k_i V_i + \sum_{i=1}^{n} b_{vi} + \sum_{i=1}^{n} \Theta_i \\ &\le -k_v V(t) + b_v, \end{aligned} \tag{4.56}$$

where $k_v = \min\{k_1, k_2, \ldots, k_n\}$, $b_v = \sum_{i=1}^{n} b_{vi}$.

Then

$$0 \le V(t) \le (V(0) - \delta_v)e^{-k_v t} + \delta_v, \tag{4.57}$$

where $\delta_v = b_v / k_v$. Thus $V(t)$ is bounded.

Simulation Example

Consider the nonlinear system with input and state delays as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 + 0.3y^2 + \sin\big(2y(t-0.2)\big) + \omega_1, \\ \dot{x}_2 &= u(t-0.20) + 0.2y^2 - \sin\big(2y(t-0.2)\big) + \omega_2, \\ y &= x_1. \end{aligned} \tag{4.58}$$

Define virtual control as

$$\begin{aligned} \alpha_1 &= -\left[c_1 + \frac{b_1}{2} + \frac{3}{2} + \frac{1}{2\lambda_1^2}\Big(\tau_{\max} k_1 + \lambda_2^2 + d_1^2\Big)\right] z_1 - \widehat{W}_1^T S_1(\xi_1), \\ \alpha_2 &= -\left[c_2 + \frac{b_2}{2} + 1 + \frac{1}{2\lambda_2^2}\Big(\tau_{\max} k_1 + d_2^2\Big)\right] z_2 - \widehat{W}_2^T S_2(\xi_2), \end{aligned} \tag{4.59}$$

where $c_1 = c_2 = 30$, $b_1 = b_2 = 2$, $\lambda_1 = \lambda_2 = 2$, $\tau_{\max} = 0.6$, $k_1 = 2$. $d_1 = d_2 = 0.6$, $\Gamma_1 = \Gamma_2 = 600$, $\sigma_1 = \sigma_2 = 0.006$, $\omega_1 = 0.05\sin(2\pi t)$, $\omega_2 = 0.05\cos(2\pi t)$.

$$\begin{aligned} \dot{\widehat{W}}_1 &= \Gamma_1\Big(z_1 S_1(\xi_1) - \sigma_1 \widehat{W}_1\Big), \\ \dot{\widehat{W}}_2 &= \Gamma_2\Big(z_2 S_2(\xi_2) - \sigma_2 \widehat{W}_2\Big), \\ z_1 &= x_1, \\ z_2 &= x_2 - \alpha_1. \end{aligned} \tag{4.60}$$

The result of control scheme is in Figures 1 and 2.

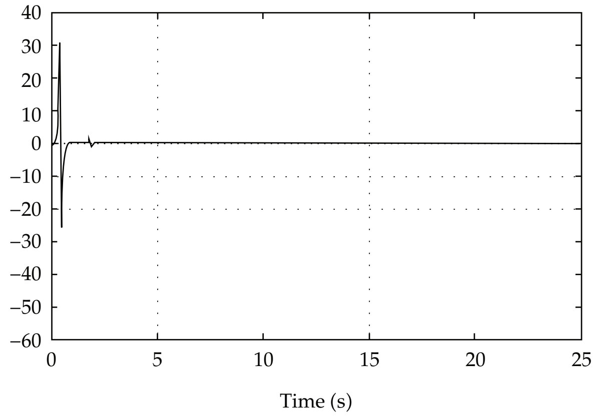

Figure 1: The control input $u(t)$.

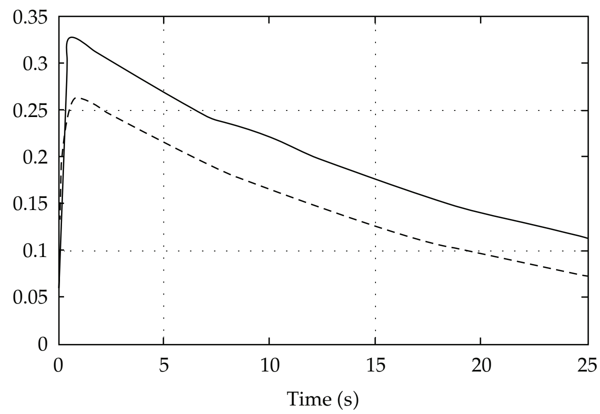

Figure 2: System state $x_1(t)$ ("−") and $x_2(t)$ ("− .−").

5. Conclusion

For a class of outputs time-delay nonlinear systems with perturbed or not, a control scheme combined with adaptive control, backstepping, and neural network is proposed. The radius basis function (RBF) neural networks is employed to estimate the unknown continuous functions. It is shown that the proposed method guarantees the semi-globally uniformly ultimately boundedness of all signals in the adaptive closed-loop systems. Simulation results are provided to illustrate the performance of the proposed approach.

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