Research Article

Stability and Stabilization of Networked Control System with Forward and Backward Random Time Delays

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This paper deals with the problem of stabilization for a class of networked control systems (NCSs) with random time delay via the state feedback control. Both sensor-to-controller and controller-to-actuator delays are modeled as Markov processes, and the resulting closed-loop system is modeled as a Markovian jump linear system (MJLS). Based on Lyapunov stability theorem combined with Razumikhin-based technique, a new delay-dependent stochastic stability criterion in terms of bilinear matrix inequalities (BMIs) for the system is derived. A state feedback controller that makes the closed-loop system stochastically stable is designed, which can be solved by the proposed algorithm. Simulations are included to demonstrate the theoretical result.

1. Introduction

Feedback control systems in which the control loops are closed through a real-time network are called networked control systems (NCSs) [1]. Recently, much attention has been paid to the study of stability analysis and controller design of NCSs [2, 3] due to their low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. Consequently, NCSs have been applied to various areas such as mobile sensor networks [4], remote surgery [5], haptics collaboration over the Internet [6–8], and automated highway systems and unmanned aerial vehicles [9, 10]. However, the sampling data and controller signals are transmitted through a network, so network-induced delays in NCSs are always inevitable [11, 12].

One of the main issues in NCSs is network-induced delays, which are usually the major causes for the deterioration of system performance and potential system instability

[13]. For different scheduling protocols, the network-induced delay may be constant, or time-varying, but in most cases, it is random [14]. Hence, systems with random time delay attract considerable attention [15–18]. Based on stochastic control theory and a separation property, the effect of random delay is treated as an LQG problem in [15]. However, the network-induced random delay has to be less than one sampling interval. The results in [15] have recently been extended to the case with longer delays in [16]. It is noted that the given controller depends only on sensor-to-controller delay. In [17], a control problem for Bernoulli binary random delay is considered, and a linear matrix inequalities (LMIs) problem for the analysis of stochastic exponential mean square stability is established. The model-based NCSs with random transmission delay is studied in [18]. Sufficient conditions for almost sure stability and stochastic exponential mean square stability are presented.

On the other hand, the study of stochastic systems has attracted a great deal of attention [19–38]. Some of these results are applied to networked control systems with random time delays [39–43]. In [39, 40], the network-induced random delays are modeled as Markov chains such that the closed-loop systems are jump linear systems with one mode. It is noticed that in [39], the state feedback gain is mode independent, and in [40], the state feedback gain only depends on the delay from sensor to controller. Recently, stabilization of networked control systems with the sensor-to-controller and controller-to-actuator delays are considered in [41]. In [42, 43], a class of Markovian jump linear systems with time delays both in the system state and in the mode signal is considered. Based on Lyapunov method, a time-delayed, mode-dependent, and state feedback controller such that the closed-loop system is stochastically stable is designed. It is noticed that the time delay in the mode signal is constant in [42, 43], and the time delay in the mode signal is random. It is worth pointing out that in all of the aforementioned papers, the plant is in the discrete-time domain. To the best of the authors' knowledge, the stability and stabilization problems for NCSs with the plant being in the continuous-time domain have not been fully investigated to date. Especially for the case where both sensor-to-controller and controller-to-actuator network-induced delays are random and longer than one sampling interval, very few results related to NCSs have been available in the literature so far, which motivates the present study.

The aim of this paper is to consider a class of networked control systems with sensors and actuators connected to a controller via two communication networks in the continuous-time domain. Two Markov processes are introduced to describe sensor-to-controller transmission delay and the controller-to-actuator transmission delay. Based on Lyapunov stability theorem, a method for designing a mode-dependent state feedback controller that stabilizes this class of networked control systems is proposed. The existence of such a controller is given in terms of BMIs, which can be solved by the proposed algorithm.

This paper is organized as follows. In Section 2, the problem is stated and some useful definitions and lemmas are given, and then the main results of this paper are given in Section 3. Simulation results are presented in Section 4. Finally, the conclusions are provided in Section 5.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and I is identity matrix. A^T stands for the transpose of the corresponding matrix A. The notation $A \geq 0$ ($A > 0$) means that the matrix A is a positive semidefinite (positive definite) matrix. For an arbitrary matrix Y and two symmetric matrices X and Z, $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes a block matrix entry implied by symmetry, and $\| \cdot \|$ refers to the Euclidean norm for vectors and induced 2-norm for matrices. $\mathbb{E}(\cdot)$ stands for the mathematical expectation operator, and $\mathbb{P}(\cdot)$ for probability operator.
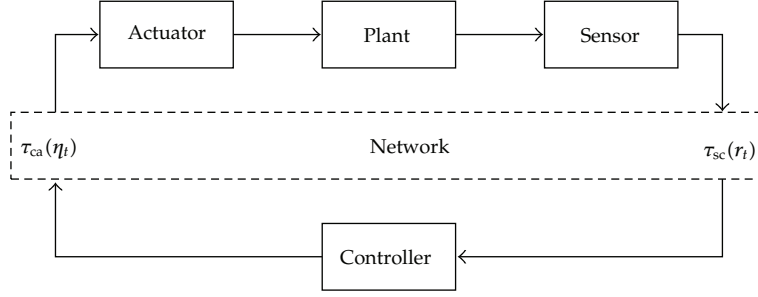
Figure 1: Illustration of NCSs over communication network.

2. Problem Formulation

Consider linear systems described by the differential equation

$$\dot{x} = Ax(t) + Bu(t), \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, and $u(t) \in \mathbb{R}^m$ is the control input. Matrices A and B are known matrices of appropriate dimensions.

The plant is interconnected by a controller over a communication network, see Figure 1. The sensor and controller are periodically sampled with the sampling interval T. We describe the sensor-to-controller transmission delay as $\tau_{sc}(r_t)$ and the controller-to-actuator transmission delay as $\tau_{ca}(\eta_t)$. The mode switching of $\tau_{sc}(r_t)$ is governed by the continuous-time discrete-state Markov process r_t taking the values in the finite set $\varsigma_r := \{1, \ldots, N_r\}$ with generator $\Lambda = (\lambda_{ij})$, $i, j \in \varsigma_r$ given by

$$\mathbb{P}[r_{t+h} = j \mid r_t = i] = \begin{cases} \lambda_{ij}h + o(h), & i \neq j, \\ 1 + \lambda_{ii}h + o(h), & i = j, \end{cases} \tag{2.2}$$

where λ_{ij} is the transition rate from mode i to j with $\lambda_{ij} \geq 0$ when $i \neq j$ and $\lambda_{ii} = -\sum_{j=1, j \neq i}^{N_r} \lambda_{ij}$, and $o(h)$ is such that $\lim_{h \to 0} o(h)/h = 0$. The mode switching of $\tau_{ca}(\eta_t)$ is governed by the continuous-time discrete-state Markov process η_t taking the values in the finite set $\varsigma_\eta := \{1, \ldots, N_\eta\}$ with generator $\Pi = (\pi_{kl})$, $k, l \in \varsigma_\eta$ given by

$$\mathbb{P}[\eta_{t+h} = l \mid \eta_t = k] = \begin{cases} \pi_{kl}h + o(h), & k \neq l, \\ 1 + \pi_{kk}h + o(h), & k = l, \end{cases} \tag{2.3}$$

with $\pi_{kl} \geq 0$ and $\pi_{kk} = -\sum_{l=1, l \neq k}^{N_\eta} \pi_{kl}$.

Throughout the paper, the following assumption is needed for the considered networked control systems.
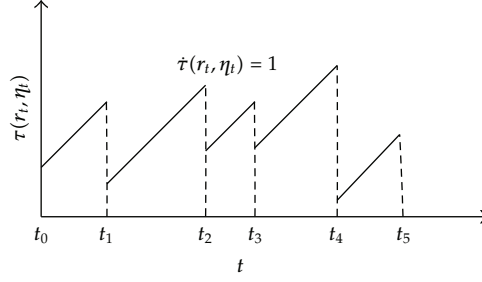
Figure 2: Illustration of the time delay.

Assumption 2.1. The switching difference of consecutive delays is less than one sampling interval, that is,

$$\mathbb{P}\big(|\tau_{\mathrm{sc}}(r_{t_{k+1}}) - \tau_{\mathrm{sc}}(r_{t_k})| \geq T\big) = 0,$$

$$\mathbb{P}\big(|\tau_{\mathrm{ca}}(\eta_{t_{k+1}}) - \tau_{\mathrm{ca}}(\eta_{t_k})| \geq T\big) = 0, \tag{2.4}$$

where $t_k = kT$ is the kth sampling instant.

Remark 2.2. Although Assumption 2.1 restricts that the switching difference of consecutive delays is less than one sampling interval T, this does not imply that the network delay $\tau_{\mathrm{sc}}(r_{t_k})$ and $\tau_{\mathrm{ca}}(\eta_{t_k})$ are less than T.

According to Figure 1, for $t_k \leq t < t_{k+1}$, the control law has the form:

$$u(t) = K(r_t, \eta_t)x\big(t_k - \tau_{\mathrm{sc}}(r_t) - \tau_{\mathrm{ca}}(\eta_t)\big). \tag{2.5}$$

Define the time delay $\tau(r_t, \eta_t)$ as follows:

$$\tau(r_t, \eta_t) = t - t_k + \tau_{\mathrm{sc}}(r_t) + \tau_{\mathrm{ca}}(\eta_t), \tag{2.6}$$

which can be illustrated by Figure 2.

Then, we have

$$u(t) = K(r_t, \eta_t)x\big(t - \tau(r_t, \eta_t)\big). \tag{2.7}$$

The associated upper bounds of $\tau(r_t, \eta_t)$ are defined as

$$\overline{\tau} = T + \max_{i \in \varsigma_r} \tau_{\mathrm{sc}}(i) + \max_{k \in \varsigma_\eta} \tau_{\mathrm{ca}}(k). \tag{2.8}$$

Applying controller (2.7) to the open-loop system (2.1) results in the closed-loop networked control system

$$\dot{x}(t) = Ax(t) + BK(r_t, \eta_t)x(t - \tau(r_t, \eta_t)),$$

$$x(\theta) = \phi(\theta), \quad \theta \in [-\overline{\tau}, 0], \tag{2.9}$$

where $\phi(\theta)$, $\theta \in [-\overline{\tau}, 0]$ is the initial function.

We have the following stochastic stability concept for system (2.9).

Definition 2.3. The system (2.9) is said to be stochastically stable if there exists a constant $\mathbb{T}(r_0, \eta_0, \phi(\cdot))$ such that

$$\mathbb{E}\left[\int_0^\infty \|x(s)\|^2 ds \mid (r_0, \eta_0, \phi(\cdot))\right] \leq \mathbb{T}(r_0, \eta_0, \phi(\cdot)), \tag{2.10}$$

for any initial condition $x(r_0, \eta_0, \phi(\cdot))$.

The following lemmas will be essential for the proofs in Section 3.

Lemma 2.4 (see [44]). *Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\epsilon > 0$ such that $\Sigma_3 = \Sigma_3^T > 0$, Then the following inequality holds:*

$$\sum_1^T \sum_2 + \sum_2^T \sum_1 \leq \epsilon \sum_1^T \sum_3 \sum_1 + \epsilon^{-1} \sum_2^T \sum_3^{-1} \sum_2. \tag{2.11}$$

For the delay functional differential equation,

$$\dot{x}(t) = f(t, x_t), \tag{2.12}$$

where

$$f : [0, +\infty) \times C([-\tau, 0], \mathbb{R}^n) \longrightarrow \mathbb{R}^n, \tag{2.13}$$

is completely continuous, $f(t, 0) = 0$, and $x_t(\theta)$ is defined as

$$x_t(\theta) = x(t + \theta), \quad \theta \in [-\tau, 0]. \tag{2.14}$$

Then we have the following Razumikhin lemma.

Lemma 2.5 (see [45]). *Suppose that $u, v, w : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ are continuous, strictly monotonous increasing functions, then $u(s)$, $v(s)$, and $w(s)$ are positive for $s > 0$, and $u(0) = v(0) = 0$. If there is a continuous function $V : [-\tau, +\infty) \times \mathbb{R}^n \rightarrow \mathbb{R}^+$ such that*

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|), \quad t \in [-\tau, +\infty), \ x \in \mathbb{R}, \tag{2.15}$$

and there is a continuous nondecreasing function $p(s) > s$ for $s > 0$, and for any $t_0 \in \mathbb{R}^+$,

$$\dot{V}(t, x) \leq -w(\|x\|) \tag{2.16}$$

if

$$V(x(t + \theta, t + \theta)) < p(V(t, x)), \quad \theta \in [-\tau, 0], \ t \geq t_0, \tag{2.17}$$

then the zero solution of (2.12) *is uniformly asymptotically stable.*

3. Main Results

The following theorem provides sufficient conditions for existence of a mode-dependent state feedback controller for the system (2.9).

Theorem 3.1. *Consider the closed-loop system* (2.9) *satisfying Assumption 2.1. If there exist symmetric matrix $Q(i, k) > 0$, matrix $Y(i, k)$, and positive scalar ϵ_1, ϵ_2 such that the following matrix inequalities hold for all $i \in \varsigma_r$ and $k \in \varsigma_\eta$,*

$$\begin{bmatrix} J(i, k) & \varphi_1(i, k) & \varphi_2(i, k) \\ * & -\psi_1 & 0 \\ * & * & -\psi_2 \end{bmatrix} < 0, \tag{3.1}$$

$$\begin{bmatrix} -\epsilon_1 Q(i, k) & AQ(i, k) \\ * & -Q(i, k) \end{bmatrix} < 0, \tag{3.2}$$

$$\begin{bmatrix} -\epsilon_2 Q(i, k) & BY(i, k) \\ * & -Q(i, k) \end{bmatrix} < 0, \tag{3.3}$$

where

$$J(i, k) = Q(i, k)A^T + AQ(i, k) + Y^T(i, k)B^T + BY(i, k) + \overline{\tau}(\epsilon_1 + 3\epsilon_2)Q(i, k) + \lambda_{ii}Q(i, k) + \pi_{kk}Q(i, k),$$

$$\varphi_1(i, k) = \left[\sqrt{\lambda_{i,1}}Q(i, k), \ldots, \sqrt{\lambda_{i,i-1}}Q(i, k), \sqrt{\lambda_{i,i+1}}Q(i, k), \ldots, \sqrt{\lambda_{i,N_r}}Q(i, k) \right],$$

$$\varphi_2(i, k) = \left[\sqrt{\pi_{k,1}}Q(i, k), \ldots, \sqrt{\pi_{k,k-1}}Q(i, k), \sqrt{\pi_{k,k+1}}Q(i, k), \ldots, \sqrt{\pi_{k,N_\eta}}Q(i, k) \right],$$

$$\psi_1 = \text{diag}[Q(1, k), \ldots, Q(i - 1, k), Q(i + 1, k), \ldots, Q(N_r, k)],$$

$$\psi_2 = \text{diag}[Q(i, 1), \ldots, Q(i, k - 1), Q(i, k + 1), \ldots, Q(i, N_\eta)], \tag{3.4}$$

with $P(i, k) = Q^{-1}(i, k)$, then the system is stochastically stable with the state feedback gain:

$$K(i, k) = Y(i, k)Q^{-1}(i, k). \tag{3.5}$$

Proof. Consider the following Lyapunov candidate:

$$V(x(t), r_t, \eta_t) = x^T(t) P(r_t, \eta_t) x(t), \tag{3.6}$$

where $P(r_t, \eta_t)$ is the positive symmetric matrix. From (3.6), it follows that

$$\beta_1 \|x(t)\|^2 \leq V(x(t), r_t, \eta_t) \leq \beta_2 \|x(t)\|^2, \tag{3.7}$$

where

$$\begin{aligned}
\beta_1 &= \min_{r_t \in \varsigma_r, \eta_t \in \varsigma_\eta} \lambda_{\min}(P(r_t, \eta_t)), \\
\beta_2 &= \max_{r_t \in \varsigma_r, \eta_t \in \varsigma_\eta} \lambda_{\max}(P(r_t, \eta_t)).
\end{aligned} \tag{3.8}$$

Note that

$$\begin{aligned}
x(t - \tau(r_r, \eta_t)) &= x(t) - \int_{-\tau(r_r, \eta_t)}^{0} \dot{x}(t + \theta) d\theta \\
&= x(t) - \int_{-\tau(r_r, \eta_t)}^{0} \left[Ax(t + \theta), +BK(r_t, \eta_t) x(t - \tau(r_r, \eta_t) + \theta) \right] d\theta.
\end{aligned} \tag{3.9}$$

Thus, the closed-loop system (2.9) can be rewritten as

$$\dot{x}(t) = \left[A + BK(r_t, \eta_t) \right] x(t) - BK(r_t, \eta_t) \int_{-\tau(r_r, \eta_t)}^{0} \left[Ax(t + \theta) + BK(r_t, \eta_t) x(t - \tau(r_r, \eta_t) + \theta) \right] d\theta. \tag{3.10}$$

Let $\mathcal{L}(\cdot)$ be the weak infinitesimal generator of $\{x(t), r_t, \eta_t, t \geq 0\}$, then for $r_t = i \in \varsigma_r, \eta_t = k \in \varsigma_\eta$, we have

$$\begin{aligned}
&\mathcal{L}V(x(t), i, k) \\
&= \dot{x}^T(t) P(i, k) x(t) + x^T(t) P(i, k) \dot{x}(t) + \sum_{j=1}^{N_r} \lambda_{ij} x^T(t) P(j, k) x(t) + \sum_{l=1}^{N_\eta} \pi_{kl} x^T(t) P(i, l) x(t) \\
&\quad + x^T(t) \left[A^T P(i, k) + P(i, k) A + K^T(i, k) B^T P(i, k) + P(i, k) BK(i, k) + \sum_{j=1}^{N_r} \lambda_{ij} P(j, k) + \sum_{l=1}^{N_\eta} \pi_{kl} P(i, l) \right] x(t) \\
&\quad - 2 \int_{-\tau(i, k)}^{0} \left\{ x^T(t) P(i, k) BK(i, k) \times \left[Ax(t + \theta) + BK(i, k) x(t - \tau(i, k) + \theta) \right] \right\} d\theta.
\end{aligned} \tag{3.11}$$

According to Lemma 2.4, we have

$$
\begin{aligned}
-2\int_{-\tau(i,k)}^{0} &\left\{ x^T(t)P(i,k)BK(i,k) \times [Ax(t+\theta) + BK(i,k)x(t-\tau(i,k)+\theta)] \right\} d\theta \\
&\leq \tau(i,k)\left[\epsilon_1^{-1}x^T(t)P(i,k)BK(i,k)AP^{-1}(i,k) \times A^T K^T(i,k)B^T P(i,k)x(t) + \epsilon_2^{-1}x^T(t) \right. \\
&\qquad\qquad \left. \times P(i,k)BK(i,k)BK(i,k)P^{-1}(i,k)K^T(i,k) \times B^T K^T(i,k)B^T P(i,k)x(t) \right] \\
&\quad + \epsilon_1 \int_{-\tau(i,k)}^{0} x^T(t+\theta)P(i,k)x(t+\theta)d\theta + \epsilon_2 \int_{-\tau(i,k)}^{0} x^T(t-\tau(i,k)+\theta)P(i,k)x(t-\tau(i,k)+\theta)d\theta.
\end{aligned}
\tag{3.12}
$$

From (3.2), (3.3), and Lemma 2.5, we can obtain

$$
\begin{aligned}
AP^{-1}(i,k)A^T &< \epsilon_1 P^{-1}(i,k), \\
BK(i,k)P^{-1}(i,k)K^T(i,k)B^T &< \epsilon_2 P^{-1}(i,k),
\end{aligned}
\tag{3.13}
$$

which yields

$$
\begin{aligned}
-2\int_{-\tau(i,k)}^{0} &\left\{ x^T(t)P(i,k)BK(i,k) \times [Ax(t+\theta) + BK(i,k)x(t-\tau(i,k)+\theta)] \right\} d\theta \\
&\leq 2\tau(i,k)\epsilon_2 x^T(t)P(i,k)x(t) + \epsilon_1 \int_{-\tau(i,k)}^{0} x^T(t+\theta)P(i,k)x(t+\theta)d\theta \\
&\quad + \epsilon_2 \int_{-\tau(i,k)}^{0} x^T(t-\tau(i,k)+\theta)P(i,k) \times x(t-\tau(i,k)+\theta)d\theta.
\end{aligned}
\tag{3.14}
$$

Following Lemma 2.5, for $-2\bar{\tau} \leq \theta \leq 0$, we assume that for any $\delta > 1$, the following inequality holds:

$$
V(x(t+\theta), r_{t+\theta}, \eta_{t+\theta}) < \delta V(x(t), r_t, \eta_t),
\tag{3.15}
$$

then we have

$$
\mathcal{L}V(x(t), i, k) \leq x^T(t)\mathcal{H}(\tau(i,k), \delta)x(t),
\tag{3.16}
$$

where $\mathcal{H}(\tau(i,k), \delta)$ is given by

$$
\begin{aligned}
\mathcal{H}(\tau(i,k), \delta) \\
= A^T P(i,k) + P(i,k)A &+ K^T(i,k)B^T P(i,k) + P(i,k)BK(i,k) + \sum_{j=1}^{N_r}\lambda_{ij}P(j,k) \\
+ \sum_{l=1}^{N_\eta}\pi_{kl}P(i,l) &+ 2\tau(i,k)\epsilon_2 P(i,k) + \tau(i,k)\epsilon_1\delta P(i,k) + \tau(i,k)\epsilon_2\delta P(i,k),
\end{aligned}
\tag{3.17}
$$

for some positive scalars ϵ_1 and ϵ_2. before and after multiplying $\mathcal{H}(\tau(i,k),\delta)$ by $Q(i,k) = P^{-1}(i,k)$ and its transpose, it gives

$$\widetilde{\mathcal{H}}(\tau(i,k),\delta)$$

$$= Q(i,k)A^T + AQ(i,k) + Q(i,k)K^T(i,k)B^T + BK(i,k)Q(i,k) + Q(i,k)\sum_{j=1}^{N_r}\lambda_{ij}P(j,k)Q(i,k)$$

$$+ Q(i,k)\sum_{l=1}^{N_\eta}\pi_{kl}P(i,l)Q(i,k) + 2\tau(i,k)\epsilon_2 Q(i,k) + \tau(i,k)\epsilon_1\delta Q(i,k) + \tau(i,k)\epsilon_2\delta Q(i,k).$$

$$(3.18)$$

Since

$$0 \leq \tau(i,k) \leq \overline{\tau}, \tag{3.19}$$

we have from (3.16) that

$$\mathcal{L}V(x(t),i,k) \leq x^T(t)\mathcal{H}(\overline{\tau},\delta)x(t). \tag{3.20}$$

From (3.1) and Lemma 2.5, it follows that

$$\widetilde{\mathcal{H}}(\overline{\tau},\delta = 1) < 0, \tag{3.21}$$

which is equivalent to

$$\mathcal{H}(\overline{\tau},\delta = 1) < 0. \tag{3.22}$$

Using the continuity properties of the eigenvalues of $\mathcal{H}$ with respect to δ, then there exists a $\delta > 1$ sufficiently small such that (3.21) still holds. Thus, for such a δ, we have

$$\mathcal{H}(\overline{\tau},\delta) < 0, \tag{3.23}$$

which yields

$$\mathcal{L}V(x(t),r_t,\eta_t) \leq -\beta\|x(t)\|^2, \tag{3.24}$$

where

$$\beta = \min_{i\in\varsigma_r,\,k\in\varsigma_\eta}[\lambda_{\min}(-\mathcal{H}(\overline{\tau},\delta))] > 0. \tag{3.25}$$

Applying Dynkin's formula, we have

$$
\begin{aligned}
\mathbb{E}[V(x(t), i, k)] - \mathbb{E}[V(x_0, r_0, \eta_0)] &= \mathbb{E}\left\{\int_0^t [\mathscr{L}V(x(s), r_s, \eta_s) ds \mid x_0, r_0, \eta_0]\right\} \\
&\leq -\beta \mathbb{E}\left\{\int_0^t [\|x(s)\|^2 ds \mid x_0, r_0, \eta_0]\right\}.
\end{aligned}
\tag{3.26}
$$

Note that

$$
\mathbb{E}[V(x(t), i, k)] \geq 0,
\tag{3.27}
$$

Then we can obtain

$$
\beta \mathbb{E}\left\{\int_0^t [\|x(s)\|^2 ds \mid x_0, r_0, \eta_0]\right\} \leq \mathbb{E}[V(x(t), i, k)] + \beta \mathbb{E}\left\{\int_0^t [\|x(s)\|^2 ds \mid x_0, r_0, \eta_0]\right\}
\tag{3.28}
$$

$$
\leq \mathbb{E}[V(x_0, r_0, \eta_0)].
$$

This completes the proof.   □

Remark 3.2. In case of constant transmission delay, that is, $\tau_{sc}(r_t) = \tau_{sc}, \tau_{ca}(\eta_t) = \tau_{ca}, \lambda_{ij} = 0$, and $\pi_{kl} = 0$, Theorem 3.1 can be directly applied to systems with constant delay.

It should be noted that the terms $\epsilon_1 Q(i, k)$ and $\epsilon_2 Q(i, k)$ in (3.1)–(3.3) are bilinear. Therefore, we propose the following algorithm to solve these bilinear matrix inequality problems.

Step 1. Set $Q_0(i, k) > 0$, and $Y_0(i, k)$ such that the following LMI holds:

$$
\begin{bmatrix}
\widetilde{J}(i, k) & \varphi_1(i, k) & \varphi_2(i, k) \\
* & -\psi_1 & 0 \\
* & * & -\psi_2
\end{bmatrix} < 0,
\tag{3.29}
$$

where

$$
\widetilde{J}(i, k) = Q(i, k)A^T + AQ(i, k) + Y^T(i, k)B^T + BY(i, k) + \lambda_{ii}Q(i, k) + \pi_{kk}Q(i, k).
\tag{3.30}
$$

Step 2. For $Q(i, k) > 0$ given in the previous step, find ϵ_{1s}, ϵ_{2s}, and $Y_s(i, k)$ by solving the following convex optimization problem:

$$
\max_{Y(i,k), \epsilon_1, \epsilon_2} \quad \overline{\tau}(Y(i, k), \epsilon_1, \epsilon_2),
\tag{3.31}
$$

$$
\text{s.t.} \quad (3.1)\text{–}(3.3) \text{ hold for } Q(i, k) > 0 \text{ fixed.}
$$

Step 3. For $Y(i,k)$, ϵ_2, and ϵ_1 given in the previous step, find $Q_s(i,k) > 0$ by solving the following quasiconvex optimization problem

$$
\begin{aligned}
&\max_{Q(i,k)>0} \quad \overline{\tau}(Q(i,k)), \\
&\text{s.t.} \quad (3.1)\text{--}(3.3) \text{ hold for } Y(i,k), \ \epsilon_2, \text{ and } \epsilon_1 \text{ fixed.}
\end{aligned}
\tag{3.32}
$$

Step 4. Return to step 2 until the convergence of $\overline{\tau}$ is attained with a desired precision.

Remark 3.3. For a given $Q(i,k)$, the considered optimization problem consists of minimizing an eigenvalue problem which is a convex one. On the other hand, for given $Y(i,k)$, ϵ_1 and ϵ_2, the considered optimization problem consists of minimizing a generalized eigenvalue problem which is a quasiconvex optimization problem. Therefore, the proposed algorithm gives a suboptimal solution.

4. Simulations

In this section, simulations of the position control for robotic manipulator ViSHaRD3 [46] are included to illustrate the effectiveness of the proposed method. Combining computed torque feedback approach [47] with friction compensation, the system is decoupled into three systems. The first and second joints of the ViSHaRD3 are

$$
\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -50 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),
\tag{4.1}
$$

and the third is

$$
\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -40 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t).
\tag{4.2}
$$

For simplicity, we only discuss the third joint of ViSHaRD3. Suppose that the sampling interval is $T = 0.01\,\text{s}$, and the Markov process r_t that governs the mode switching of the SC delay takes values in $\varsigma_r = \{1,2\}$ and has the generator

$$
\Lambda = \begin{bmatrix} -3 & 3 \\ 2 & -2 \end{bmatrix},
\tag{4.3}
$$

and the Markov process η_t that governs the mode switching of the CA delay takes values in $\eta_r = \{1,2\}$ and has the generator

$$
\Pi = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}.
\tag{4.4}
$$

Associated with modes 1 and 2, let the system have time delay $\tau_{sc}(1) = 0.03\,\text{s}$, $\tau_{ca}(1) = 0.02\,\text{s}$ and $\tau_{sc}(2) = 0.025\,\text{s}$, $\tau_{ca}(2) = 0.015\,\text{s}$, respectively. From (2.8), we have $\overline{\tau} = 0.06\,\text{s}$,
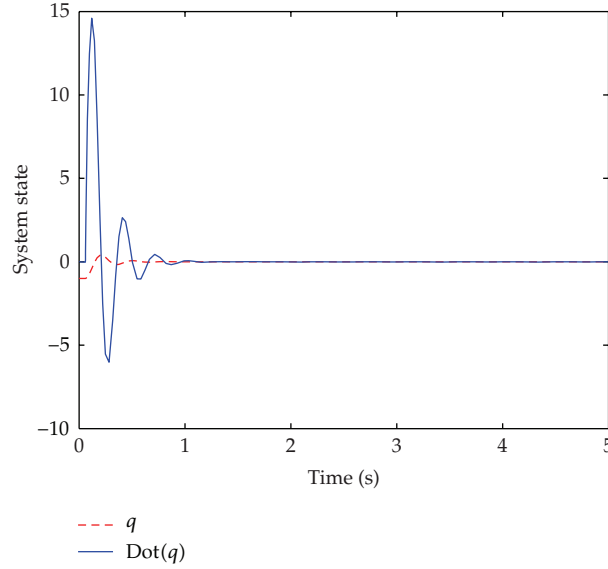
Figure 3: State response of closed-loop system.

and the initial condition is $\phi(\theta) = [-1, 0]^T$, $\theta \in [-0.06, 0]$. By the proposed algorithm and Theorem 3.1, we can obtain the controllers as follows:

$$
\begin{aligned}
K(1,1) &= \begin{bmatrix} -645.0596 & -15.9109 \end{bmatrix}, \\
K(1,2) &= \begin{bmatrix} -623.3689 & -15.4999 \end{bmatrix}, \\
K(2,1) &= \begin{bmatrix} -575.1361 & -14.2296 \end{bmatrix}, \\
K(2,2) &= \begin{bmatrix} -616.8428 & -15.3049 \end{bmatrix}.
\end{aligned}
\tag{4.5}
$$

The simulations of the state response and the control input for the closed-loop system are depicted in Figures 3 and 4, respectively, which shows that the system is stochastically stable.

5. Conclusions

In this paper, a technique of designing a mode-dependent state feedback controller for networked control systems with random time delays has been proposed. The main contribution of this paper is that both the sensor-to-controller and controller-to-actuator delays have been taken into account. Two Markov processes have been used to model these two time delays. Based on Lyapunov stability theorem combined with Razumikhin-based technique, some new delay-dependent stability criteria in terms of BMIs for the system are derived. A state feedback controller that makes the closed-loop system stochastically stable is
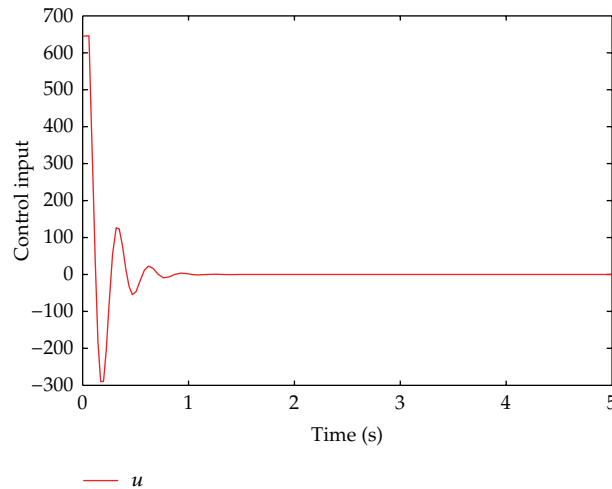
Figure 4: Control input of closed-loop system.

designed, which can be solved by the proposed algorithm. Simulations results are presented to illustrate the validity of the design methodology.

Acknowledgments

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