Research Article

Dynamic Feedback Backstepping Control for a Class of MIMO Nonaffine Block Nonlinear Systems

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For a class of MIMO nonaffine block nonlinear systems, a neural network- (NN-) based dynamic feedback backstepping control design method is proposed to solve the tracking problem. This problem is difficult to be dealt with in the control literature, mainly because the inverse controls of block nonaffine systems are not easy to resolve. To overcome this difficulty, dynamic feedback, backstepping design, sliding mode-like technique, NN, and feedback linearization techniques are incorporated to deal with this problem, in which the NNs are used to approximate and adaptively cancel the uncertainties. It is proved that the whole closed-loop system is stable in the sense of Lyapunov. Finally, simulations verify the effectiveness of the proposed scheme.

1. Introduction

In the last two decades, a number of efforts have been made on developing systematic design tools for control of uncertain nonlinear systems. Among the obtained results, feedback linearization techniques [1], adaptive backstepping design [2], and NN control [3, 4] are the representative theoretical achievements. The common assumptions made in most of the researches are that the systems to be controlled are affine and the nonlinearities are linearly parameterized by unknown parameters [2, 5]. NN-based adaptive control has relaxed the assumption on linear parameterized nonlinearities mostly in affine systems [4, 6, 7], which can deal with nonlinear parameterized nonlinearities. But some systems, such as chemical reactions [8] and flight control systems [9], cannot be expressed in an affine form.

There are three kinds of methods to deal with the controller design for nonaffine systems.

The idea of the first method is to transform a nonaffine system into an affine system with respect to a new control input by introducing an integrator [9–12]. In these attempts, an augmented system affine in $\dot{u}$ is derived for control design by differentiation [13].

The second method directly controls a nonaffine system without transformation to an affine system [14–19]. Under the assumption that a control Lyapunov function (CLF) was available, Moulay and Perruquetti [16] obtained a sufficient condition to guarantee the existence of a continuous stabilizing control for nonaffine systems. Lin [14, 15] presented how nonaffine passive systems theory, together with the techniques of feedback equivalence and bounded control, could be used to explicitly construct a smooth state feedback control law that solved the problem of global stabilization for nonaffine nonlinear systems. New state feedback stabilizing controllers and sufficient conditions of asymptotic stability were proposed by Shiriaev and Fradkov [18] under assumptions similar to those in [14]. But it is difficult to find a CLF and to deal with controller design for systems with uncertainties.

The third one employs NNs, PI, or fuzzy-neural models to approximate the inverse system or the uncertainties in controller design for nonaffine nonlinear systems [20–28]. For a class of general nonaffine nonlinear systems, virtual-linearized-system- (VLS-) based design methods were proposed, in which the T-S fuzzy-neural model was employed to approximate a VLS of a real system with modelling errors and external disturbances [20, 26, 27]. Teo et al. [25] constructed a proportional-integral (PI) controller for the minimum-phase nonaffine system, which was an equivalent realization of an approximate dynamic inversion controller. Ge and Zhang [21] suggested using NNs as emulators of inverse systems for controller design of general nonlinear systems. Using the implicit function theory and the mean value theorem, an NN was employed to approximate an ideal control signal which solved the tracking problem in [22]. In [29–32], instead of seeking a direct solution to the inverse problem, a solution was obtained by introducing an analytically invertible model and then employing an NN to compensate inversion error. By using implicit function theorem and Taylor series expansion, an observer-based adaptive fuzzy-neural control scheme was presented for the nonaffine nonlinear system in the presence of unknown structure of nonlinearities [33]. A neural synthesis method was considered for a class of multivariable nonaffine uncertain systems [28]. The method extended the previous approach developed in a single-input single-output system to a multi-input multi output system without resorting to a fixed-point assumption or boundedness assumption on the time derivative of a control effectiveness term. The difficulty associated with these methods for nonaffine control systems is that an explicit inverting control design is, in general, not possible even if the inverse exists by the implicit function theorem [28]. Moreover, this kind of method relies on the approximation ability of NN.

Backstepping method is one of the breakthroughs in design of nonlinear control systems. Therefore, it has become one of the important and popular approaches for nonlinear systems [2]. This approach is based on a systematic procedure for the design of feedback control strategies suitable for the design of a large class of nonlinear systems with unmatched uncertainties, and it guarantees global regulation and tracking for the class of nonlinear systems transformable into the strict-feedback form. Developing a systematic synthesis method for general nonaffine systems still remains a challenging problem.

In this paper, we discussed the NN-based backstepping design for a class of uncertain nonaffine systems in block control form. The main contributions of this paper can be summarized as follows: (1) the proposed method avoids the difficulties to solve the inverse control in most literatures; (2) it does not rely on implicit function theorem and Taylor series expansion which makes the output tracking difficult; (3) it can deal with the systems with unmatched uncertainties; (4) introducing the sliding mode surface-like variables into backstepping procedure makes the design and stability analysis clear and simple; (5) a

systematic procedure is proposed for tracking control design for a class of block nonlinear systems that are nonaffine in the control inputs.

The rest of the work is organized as follows. The problem formulation is introduced in Section 2. The controller design and stability analysis are given in Section 3. Simulation example is given in Section 4 and followed by Section 5 which concludes the work.

2. Problem Formulation

The uncertain block nonaffine system considered in this paper is given by

$$\dot{x}_1 = f_1(x_1, x_2),$$

$$\dot{x}_2 = f_2(\overline{x}_2, x_3),$$

$$\vdots \tag{2.1}$$

$$\dot{x}_n = f_n(\overline{x}_n, u),$$

$$y = x_1,$$

where $x_i = [x_{i1} \cdots x_{im}]^T \in R^m$, $\overline{x}_i = [x_1^T \cdots x_i^T]^T$, $i = 1, 2, \ldots, n$, $u \in R^m$, and $y \in R^m$, are state variables, input and output, respectively.

Remark 2.1. Although we assume $x_i \in R^m$, the proposed method is easy to extend to the other cases by using pseudoinversion.

Assumption 2.2. $f_i(\overline{x}_{i+1}) = [f_{i1}(\overline{x}_{i+1}), f_{i2}(\overline{x}_{i+1}), \ldots, f_{im}(\overline{x}_{i+1})]^T$, $i = 1, 2, \ldots, n$, is an unknown smooth function vector.

The control objective is to design an adaptive NN controller for the system (2.1) such that the output tracks the desired signal y_d and all signals in the closed-loop system remain bounded. Let $\| \cdot \|$ denote the 2-norm, and let $\| \cdot \|_F$ denote the Frobenius norm.

3. Controller Design and Stability Analysis

In the following, introducing sliding mode-like technique, a systematic design method is proposed for a class of the uncertain block nonaffine systems.

Consider the following NN:

$$h(x) = W^T S(V^T x), \tag{3.1}$$

where $W = [w_1 \ w_2 \cdots w_l]^T \in R^{p \times l}$ and $V = [v_1 v_2 \cdots v_l]^T \in R^{N \times l}$ are the first-to-second layer and the second-to-third layer weights, respectively, $h(x) \in R^p$, $p \geq 1$, $x \in R^N$ is the input vector, and the node number is $l > 1$:

$$S(V^T x) = [s_v(v_1^T x) \ s_v(v_2^T x) \ \cdots \ s_v(v_{l-1}^T x) \ 1]^T, \tag{3.2}$$

where $s_v(x_a) = 1/(1 + e^{-\gamma x_a})$ with the constant $\gamma > 0$.

Before designing the controller, we make the following assumptions.

Assumption 3.1. One has a function vector $h(x) : \Omega \rightarrow R^p$; for any $\sigma > 0$, there always exist a Gauss function array $S : R^N \rightarrow R^l$ and an optimal weight matrix W^* such that $\|h(x) - W^{*T}S(V^{*T}x)\| \leq \sigma$, for all $x \in \Omega$, where Ω is a compact subset of R^N, $W^{*T}S(V^{*T}x)$ is the optimal approximation of $h(x)$ using NN, and $h(x) - W^{*T}S(V^{*T}x) \overset{\text{def}}{=} \Delta h(x)$ is called reconstruction error. Define $\widetilde{W} = \widehat{W} - W^*$ and $\widetilde{V} = \widehat{V} - V^*$, where $\widehat{W}$ and $\widehat{V}$ are the estimated values of W^* and V^*.

Assumption 3.2. $J_{x_i0}^{-1}$ and J_{u0}^{-1}, $i = 2,\ldots,n$, exist. $J_{x_i}(\overline{x}_{i-1},x_i) = \partial f_{i-1}(\overline{x}_{i-1},x_i)/\partial x_i$ and $J_u(\overline{x}_n,u) = \partial f_n(\overline{x}_n,u)/\partial u$ denote the Jacobians with respect to x_i and u, respectively. Let $f_{i-1}(\overline{x}_{i-1},x_i) = f_{(i-1)0}(\overline{x}_{i-1},x_i) + \Delta f_{(i-1)0}(\overline{x}_{i-1},x_i)$, $f_n(\overline{x}_n,u) = f_{n0}(\overline{x}_n,u) + \Delta f_{n0}(\overline{x}_n,u)$, $J_{x_i}(\overline{x}_{i-1},x_i) = J_{x_i0}(\overline{x}_{i-1},x_i) + \Delta J_{x_i0}(\overline{x}_{i-1},x_i)$, and $J_u(\overline{x}_n,u) = J_{u0}(\overline{x}_n,u) + \Delta J_{u0}(\overline{x}_n,u)$, where $f_{(i-1)0}(\overline{x}_{i-1},x_i)$, $f_{n0}(\overline{x}_n,u)$, $J_{x_i0}(\overline{x}_{i-1},x_i)$, and $J_{u0}(\overline{x}_n,u)$ are the nominal parts of the functions $f_{i-1}(\overline{x}_{i-1},x_i)$, $f_n(\overline{x}_n,u)$, $J_{x_i}(\overline{x}_{i-1},x_i)$, and $J_u(\overline{x}_n,u)$, respectively, and $\Delta f_{(i-1)0}(\overline{x}_{i-1},x_i)$, $\Delta f_{n0}(\overline{x}_n,u)$, $\Delta J_{x_i0}(\overline{x}_{i-1},x_i)$, and $\Delta J_{u0}(\overline{x}_n,u)$ are the unknown parts.

Remark 3.3. Assumption 3.2 is not a strong condition imposed on the system. In fact, because $J_{x_i0}(\overline{x}_{i-1},x_i)$ and $J_{u0}(\overline{x}_n,u)$ are the nominal parts of the functions $J_{x_i}(\overline{x}_{i-1},x_i)$ and $J_u(\overline{x}_n,u)$, respectively, we can modify the values of the elements of $J_{x_i0}(\overline{x}_{i-1},x_i)$ and $J_{u0}(\overline{x}_n,u)$ such that $J_{x_i0}^{-1}$, and J_{u0}^{-1}, $i = 2,\ldots,n$, exist.

Lemma 3.4 (see [31, 34]). *For the* NN *approximator, the approximation error can be described as*

$$\widehat{W}^T S\left(\widehat{V}^T X\right) - W^{*T} S\left(V^{*T} X\right) = \widetilde{W}^T\left(\widehat{S} - \widehat{S}'\widehat{V}^T X\right) + \widehat{W}^T \widehat{S}'\widetilde{V}^T X + d_u, \tag{3.3}$$

where $\widehat{S} = S(\widehat{V}^T X)$, $\widehat{S}' = \text{diag}\{s'_{v1}, s'_{v2}, \ldots, s'_{vl}\}\widehat{s}'_{vi} = s'_v(\widehat{v}_i^T X) = d[s_v(x_a)]/dx_a|_{x_a=\widehat{v}_i^T X}$, *and the residual term* d_u *satisfies the following inequality:*

$$\|d_u\| \leq \|V^*\|_F \left\|\widehat{W}^T \widehat{S}'\right\|_F \|X\| + \|W^*\|_F \left\|\widehat{S}'\widehat{V}^T X\right\|_F + \|W^*\|_F \sqrt{l}. \tag{3.4}$$

Step 1. Consider the first subsystem of (2.1) $\dot{x}_1 = f_1(x_1, x_2)$. Taking its derivative gives

$$\ddot{x}_1 = J_{x_1}(x_1, x_2) f_1(x_1, x_2) + J_{x_2}(x_1, x_2)\dot{x}_2, \tag{3.5}$$

where $J_{x_1}(x_1, x_2) = \partial f_1(x_1, x_2)/\partial x_1$ denotes the Jacobian with respect to x_1. Equation (3.5) can be rewritten as

$$\ddot{x}_1 = J_{x_10}(x_1, x_2) f_{10}(x_1, x_2) + J_{x_20}(x_1, x_2)\dot{x}_2 + \Delta f_1(x_1, x_2), \tag{3.6}$$

where

$$\Delta f_1(x_1, x_2) = \Delta J_{x_10}(x_1, x_2) f_{10}(x_1, x_2) + J_{x_10}(x_1, x_2)\Delta f_{10}(x_1, x_2)$$
$$+ \Delta J_{x_10}(x_1, x_2)\Delta f_{10}(x_1, x_2) + \Delta J_{x_20}(x_1, x_2)\dot{x}_2. \tag{3.7}$$

Let $z_1 = x_1 - x_{1d}$ and $s_1 = z_1 + c_1\dot{z}_1$, where $c_1 > 0$ is a constant, s_1 is a sliding mode surface-like vector, and x_{1d} is the reference signal of x_1. Taking the time derivative of s_1, we can obtain

$$
\begin{aligned}
\dot{s}_1 &= \dot{z}_1 + c_1\ddot{z}_1 \\
&= \dot{z}_1 + c_1 J_{x_10}(x_1, x_2) f_{10}(x_1, x_2) + c_1 J_{x_20}(x_1, x_2)\dot{x}_2 + c_1 \Delta f_1(x_1, x_2) - c_1\ddot{x}_{1d} \\
&= \dot{z}_1 + c_1 J_{x_10}(x_1, x_2) f_{10}(x_1, x_2) + c_1 J_{x_20}(x_1, x_2)\dot{x}_{2d} \\
&\quad - J_{x_20}(x_1, x_2)z_2 + c_1 \Delta f_1(x_1, x_2) - c_1\ddot{x}_{1d} + J_{x_20}(x_1, x_2)s_2.
\end{aligned}
\tag{3.8}
$$

Let $z_2 = x_2 - x_{2d}$ and $s_2 = z_2 + c_1\dot{z}_2$, where x_{2d} is the desired signal of x_2 and s_2 is a sliding mode surface-like vector.

Choose the virtual control as

$$
\dot{x}_{2d} = -[c_1 J_{x_20}(x_1, x_2)]^{-1}\left[c_1 J_{x_10}(x_1, x_2) f_{10}(x_1, x_2) + \hat{\dot{z}}_1 - J_{x_20}(x_1, x_2)z_2 + v_1\right],
\tag{3.9}
$$

where $v_i = k_i s_i - c_i\ddot{x}_{id} + v_{\mathrm{NN}i} - v_{ir}$, $i = 1, 2, \ldots, n$, v_{1r} will be defined in (3.20), k_i is a diagonal matrix with its elements positive, and $\hat{\dot{z}}_i$ is the output of a tracking differentiator [35] with z_i as its input. The error between $\hat{\dot{z}}_i$ and $\dot{z}_i$ can be approximated by a neural network. $v_{\mathrm{NN}i}$ is the NN compensator, which is used to overcome the influence of the uncertainties in the system. According the approximation ability, we can assume that

$$
c_i \Delta f_i(\overline{x}_i, x_{i+1}) + \Delta_i = W_i^{*T} S_i\left(V_i^{*T} X_i\right) + \varepsilon_i,
\tag{3.10}
$$

where $W_i^{*T} S_i(V_i^{*T} X_i) + \varepsilon_i$ is the optimal approximation of $\Delta f_i(\overline{x}_i, x_{i+1}) + \Delta_i$, Δ_i is the uncertainty induced by the error between the output of the tracking differentiator $\hat{\dot{z}}_i$ and $\dot{z}_i$, namely, $\Delta_i = \dot{z}_i - \hat{\dot{z}}_i$, and $X_i = [\overline{x}_i^T, x_{i+1}^T, \hat{\dot{x}}_i^T, 1]^T$ is the input of NN. $\|\varepsilon_i\| \le \varepsilon_{iu}$ is the approximation error with constant $\varepsilon_{iu} > 0$.

Let

$$
v_{\mathrm{NN}i} = \widehat{W}_i^T S_i\left(\widehat{V}_i^T X_i\right), \quad i = 1, 2, \ldots, n.
\tag{3.11}
$$

Substituting (3.9) into (3.8) leads to

$$
\begin{aligned}
\dot{s}_1 &= c_1 \Delta f_1(x_1, x_2) + \Delta_1 - c_1\ddot{x}_{1d} + J_{x_20}(x_1, x_2)s_2 - v_1 \\
&= -c_1\ddot{x}_{1d} + J_{x_20}(x_1, x_2)s_2 - v_1 + W_1^{*T} S_1\left(V_1^{*T} X_1\right) + \varepsilon_1.
\end{aligned}
\tag{3.12}
$$

Substituting the expressions of v_1 and $v_{\mathrm{NN}1}$ into (3.12) gives

$$
\dot{s}_1 = -k_1 s_1 - \widehat{W}_1^T S_1\left(\widehat{V}_1^T X_1\right) + W_1^{*T} S_1\left(V_1^{*T} X_1\right) + \varepsilon_1 + J_{x_20}(x_1, x_2)s_2 + v_{1r}.
\tag{3.13}
$$

According to Lemma 3.4, (3.13) can be transformed into

$$
\begin{aligned}
\dot{s}_1 = &-k_1 s_1 + J_{x_2 0}(x_1, x_2) s_2 \\
&- \widetilde{W}_1^T \left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) - \widehat{W}_1^T \widehat{S}_1' \widetilde{V}_1^T X_1 - d_{u1} + \varepsilon_1 + v_{1r}.
\end{aligned}
\tag{3.14}
$$

We choose Lyapunov function as

$$
V_1 = \frac{1}{2} s_1^T s_1 + \frac{1}{2} \operatorname{tr}\left\{ \widetilde{W}_1^T \Gamma_{W1}^{-1} \widetilde{W}_1 \right\} + \frac{1}{2} \operatorname{tr}\left\{ \widetilde{V}_1^T \Gamma_{V1}^{-1} \widetilde{V}_1 \right\},
\tag{3.15}
$$

where $\Gamma_{W1} = \Gamma_{W1}^T > 0$ and $\Gamma_{V1} = \Gamma_{V1}^T > 0$ are constant design parameters. Taking the derivative of V_1, we have

$$
\begin{aligned}
\dot{V}_1 =\ & s_1^T \dot{s}_1 + \operatorname{tr}\left\{ \widetilde{W}_1^T \Gamma_{W1}^{-1} \dot{\widehat{W}}_1 \right\} + \operatorname{tr}\left\{ \widetilde{V}_1^T \Gamma_{V1}^{-1} \dot{\widehat{V}}_1 \right\} \\
=\ & s_1^T \left[-k_1 s_1 + J_{x_2 0}(x_1, x_2) s_2 - \widetilde{W}_1^T \left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) - \widehat{W}_1^T \widehat{S}_1' \widetilde{V}_1^T X_1 - d_{u1} + \varepsilon_1 + v_{1r} \right] \\
& + \operatorname{tr}\left\{ \widetilde{W}_1^T \Gamma_{W1}^{-1} \dot{\widehat{W}}_1 \right\} + \operatorname{tr}\left\{ \widetilde{V}_1^T \Gamma_{V1}^{-1} \dot{\widehat{V}}_1 \right\} \\
=\ & -k_1 \|s_1\|^2 + \operatorname{tr}\left\{ \widetilde{W}_1^T \Gamma_{W1}^{-1} \dot{\widehat{W}}_1 \right\} + \operatorname{tr}\left\{ \widetilde{V}_1^T \Gamma_{V1}^{-1} \dot{\widehat{V}}_1 \right\} + s_1^T J_{x_2 0}(x_1, x_2) s_2 \\
& - s_1^T \left[\widetilde{W}_1^T \left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) + \widehat{W}_1^T \widehat{S}_1' \widetilde{V}_1^T X_1 \right] + s_1^T (-d_{u1} + \varepsilon_1 + v_{1r}).
\end{aligned}
\tag{3.16}
$$

Choose the following adaptive tuning laws as

$$
\begin{aligned}
\dot{\widehat{W}}_1 &= \Gamma_{W1} \left[\left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) s_1^T - \sigma_{W1} \widehat{W}_1 \right], \\
\dot{\widehat{V}}_1 &= \Gamma_{V1} \left(X_1 s_1^T \widehat{W}_1^T \widehat{S}_1' - \sigma_{V1} \widehat{V}_1 \right),
\end{aligned}
\tag{3.17}
$$

where $\sigma_{W1} > 0$ and $\sigma_{V1} > 0$ are small design parameters. Substituting (3.17) into (3.16) results in

$$
\begin{aligned}
\dot{V}_1 =\ & -k_1 \|s_1\|^2 + \operatorname{tr}\left\{ \widetilde{W}_1^T \left[\left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) s_1^T - \sigma_{W1} \widehat{W}_1 \right] \right\} \\
& + \operatorname{tr}\left\{ \widetilde{V}_1^T \left(X_1 s_1^T \widehat{W}_1^T \widehat{S}_1' - \sigma_{V1} \widehat{V}_1 \right) \right\} - s_1^T (d_{u1} - \varepsilon_1 - v_{1r}) \\
& - s_1^T \left[\widetilde{W}_1^T \left(\widehat{S}_1 - \widehat{S}_1' \widehat{V}_1^T X_1 \right) + \widehat{W}_1^T \widehat{S}_1' \widetilde{V}_1^T X_1 \right] + s_1^T J_{x_2 0}(x_1, x_2) s_2.
\end{aligned}
\tag{3.18}
$$

With the property $\operatorname{tr}\{ y x^T \} = x^T y$, (3.18) can be simplified as

$$
\dot{V}_1 = -k_1 \|s_1\|^2 - \sigma_{W1} \operatorname{tr}\left\{ \widetilde{W}_1^T \widehat{W}_1 \right\} - \sigma_{V1} \operatorname{tr}\left\{ \widetilde{V}_1^T \widehat{V}_1 \right\} - s_1^T (d_{u1} - \varepsilon_1 - v_{1r}) + s_1^T J_{x_2 0}(x_1, x_2) s_2.
\tag{3.19}
$$

Design the robust term v_{ir} as

$$v_{ir} = -\frac{s_i\left(\left\|\widehat{W}_i^T \widehat{S}_i'\right\|_F^2 \|X_i\|^2 + \left\|\widehat{S}_i' \widehat{V}_i^T X_i\right\|_F^2 + 2\right)}{\eta_i}, \quad i = 1, 2, \ldots, n, \tag{3.20}$$

where $\eta_i > 0$ is a small constant. After applying Lemma 3.4 and substituting (3.20) into (3.19), $\dot{V}_1$ is upper bounded by

$$\dot{V}_1 \leq -k_1 \|s_1\|^2 - \sigma_{W1} \operatorname{tr}\left\{\widetilde{W}_1^T \widehat{W}_1\right\} - \sigma_{V1} \operatorname{tr}\left\{\widetilde{V}_1^T \widehat{V}_1\right\}$$
$$- \frac{\|s_1\|^2 \left(\left\|\widehat{W}_1^T \widehat{S}_1'\right\|_F^2 \|X_1\|^2 + \left\|\widehat{S}_1' \widehat{V}_1^T X_1\right\|_F^2 + 2\right)}{\eta_1} + s_1^T J_{x_20}(x_1, x_2) s_2 \tag{3.21}$$
$$+ \left\|s_1^T\right\| \left(\|V_1^*\|_F \left\|\widehat{W}_1^T \widehat{S}_1'\right\|_F \|X_1\| + \|W_1^*\|_F \left\|\widehat{S}_1' \widehat{V}_1^T X_1\right\|_F + \sqrt{l} \|W_1^*\|_F + \varepsilon_{1u}\right).$$

With the following properties [31]:

$$-\sigma_{W1} \operatorname{tr}\left\{\widetilde{W}_1^T \widehat{W}_1\right\} \leq \frac{\sigma_{W1}}{2} \|W_1^*\|_F^2 - \frac{\sigma_{W1}}{2} \left\|\widetilde{W}_1\right\|_F^2,$$
$$-\sigma_{V1} \operatorname{tr}\left\{\widetilde{V}_1^T \widehat{V}_1\right\} \leq \frac{\sigma_{V1}}{2} \|V_1^*\|_F^2 - \frac{\sigma_{V1}}{2} \left\|\widetilde{V}_1\right\|_F^2,$$
$$\left\|s_1^T\right\| \|V_1^*\|_F \left\|\widehat{W}_1^T \widehat{S}_1'\right\|_F \|X_1\| \leq \frac{\|s_1^T\|^2}{\eta_1} \left\|\widehat{W}_1^T \widehat{S}_1'\right\|_F^2 \|X_1\|^2 + \frac{\eta_1}{4} \|V_1^*\|_F^2, \tag{3.22}$$
$$\left\|s_1^T\right\| \|W_1^*\|_F \left\|\widehat{S}_1' \widehat{V}_1^T X_1\right\|_F \leq \frac{\|s_1^T\|^2}{\eta_1} \left\|\widehat{S}_1' \widehat{V}_1^T X_1\right\|_F^2 + \frac{\eta_1}{4} \|W_1^*\|_F^2,$$
$$\left\|s_1^T\right\| \left(\sqrt{l} \|W_1^*\|_F + \varepsilon_{1u}\right) \leq \frac{2\|s_1^T\|^2}{\eta_1} + \frac{l\eta_1}{4} \|W_1^*\|_F^2 + \frac{\eta_1}{4} \varepsilon_{1u}^2,$$

Equation (3.21) can be simplified as

$$\dot{V}_1 \leq -k_1 \|s_1\|^2 - \frac{\sigma_{W1}}{2} \left\|\widetilde{W}_1\right\|_F^2 - \frac{\sigma_{V1}}{2} \left\|\widetilde{V}_1\right\|_F^2 + b_1 + s_1^T J_{x_20}(x_1, x_2) s_2, \tag{3.23}$$

where $b_1 = [\eta_1((1/4)\|W_1^*\|_F^2 + (1/4)\|V_1^*\|_F^2 + (l/4)\|W_1^*\|_F^2 + \varepsilon_{1u}^2) + (\sigma_{W1}/2)\|W_1^*\|_F^2 + (\sigma_{V1}/2)\|V_1^*\|_F^2]$ is a bounded constant.

Step 2. Let us consider the subsystem $\dot{x}_2 = f_2(\overline{x}_2, x_3)$. Taking its derivative leads to

$$\ddot{x}_2 = J_{\overline{x}_2}(\overline{x}_2, x_3) f_2(\overline{x}_2, x_3) + J_{x_3}(\overline{x}_2, x_3) \dot{x}_3, \tag{3.24}$$

where $J_{\overline{x}_2}(\overline{x}_2, x_3) = \partial f_2(\overline{x}_2, x_3)/\partial \overline{x}_2$ denotes the Jacobian with respect to $\overline{x}_2$. Equation (3.24) can be rewritten as

$$\ddot{x}_2 = J_{\overline{x}_20}(\overline{x}_2, x_3) f_{20}(\overline{x}_2, x_3) + J_{x_30}(\overline{x}_2, x_3)\dot{x}_3 + \Delta f_2(\overline{x}_2, x_3), \tag{3.25}$$

where

$$\begin{aligned}
\Delta f_2(\overline{x}_2, x_3) &= \Delta J_{\overline{x}_20}(\overline{x}_2, x_3) f_{20}(\overline{x}_2, x_3) + J_{\overline{x}_20}(\overline{x}_2, x_3)\Delta f_{20}(\overline{x}_2, x_3) \\
&\quad + \Delta J_{\overline{x}_20}(\overline{x}_2, x_3)\Delta f_{20}(\overline{x}_2, x_3) + \Delta J_{\overline{x}_20}(\overline{x}_2, x_3)\dot{x}_3.
\end{aligned} \tag{3.26}$$

Taking its time derivative of s_2, we can obtain

$$\begin{aligned}
\dot{s}_2 &= \dot{z}_2 + c_1 J_{\overline{x}_20}(\overline{x}_2, x_3) f_{20}(\overline{x}_2, x_3) + c_1 J_{x_30}(\overline{x}_2, x_3)\dot{x}_{3d} \\
&\quad - J_{x_30}(\overline{x}_2, x_3)z_3 + c_1\Delta f_2(\overline{x}_2, x_3) - c_1\ddot{x}_{2d} + J_{x_30}(\overline{x}_2, x_3)s_3.
\end{aligned} \tag{3.27}$$

Let $z_3 = x_3 - x_{3d}$ and $s_3 = z_3 + c_1\dot{z}_3$, where x_{3d} is the desired signal of x_3 and s_3 is a sliding mode surface-like vector.

Design a virtual control signal as

$$\begin{aligned}
\dot{x}_{3d} &= -[c_1 J_{x_30}(\overline{x}_2, x_3)]^{-1} \\
&\quad \cdot \left\{ c_1 J_{\overline{x}_20}(\overline{x}_2, x_3) f_{20}(\overline{x}_2, x_3) + \hat{\dot{z}}_2 - J_{x_30}(\overline{x}_2, x_3)z_3 + J_{x_20}^T(x_1, x_2)s_1 + v_2 \right\}.
\end{aligned} \tag{3.28}$$

Choose Lyapunov function as

$$V_2 = V_1 + \frac{1}{2}s_2^T s_2 + \frac{1}{2}\operatorname{tr}\left\{ \widetilde{W}_2^T \Gamma_{W2}^{-1}\widetilde{W}_2 \right\} + \frac{1}{2}\operatorname{tr}\left\{ \widetilde{V}_2^T \Gamma_{V2}^{-1}\widetilde{V}_2 \right\}, \tag{3.29}$$

where $\Gamma_{W2} = \Gamma_{W2}^T > 0$ and $\Gamma_{V2} = \Gamma_{V2}^T > 0$ are constant design parameters.

Choose the following adaptive tuning laws:

$$\begin{aligned}
\hat{\dot{W}}_2 &= \Gamma_{W2}\left[\left(\hat{S}_2 - \hat{S}_2'\hat{V}_2^T X_2 \right)s_2^T - \sigma_{W2}\widehat{W}_2 \right], \\
\hat{\dot{V}}_2 &= \Gamma_{V2}\left(X_2 s_2^T \widehat{W}_2^T \hat{S}_2' - \sigma_{V2}\hat{V}_2 \right),
\end{aligned} \tag{3.30}$$

where $\sigma_{W2} > 0$ and $\sigma_{V2} > 0$ are small constants.

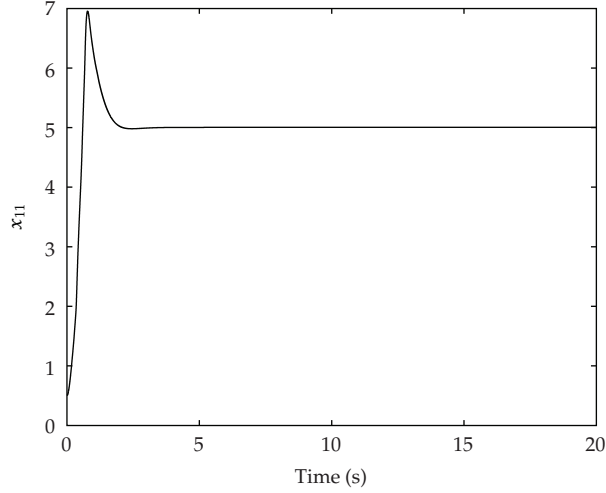**Figure 1:** Curve of x_{11} in case of the system without uncertainties.

Taking (3.25)–(3.30) into account, we have

$$\dot{V}_2 \le -k_1\|s_1\|^2 - k_2\|s_2\|^2 - \frac{\sigma_{W1}}{2}\left\|\widetilde{W}_1\right\|_F^2 - \frac{\sigma_{V1}}{2}\left\|\widetilde{V}_1\right\|_F^2 - \sigma_{W2}\,\mathrm{tr}\left\{\widetilde{W}_2^T\widehat{W}_2\right\}$$

$$- \sigma_{V2}\,\mathrm{tr}\left\{\widetilde{V}_2^T\widehat{V}_2\right\}$$

$$- \frac{\|s_2\|^2\left(\left\|\widehat{W}_2^T\widehat{S}_2'\right\|_F^2\|X_2\|^2 + \left\|\widehat{S}_2'\widehat{V}_2^TX_2\right\|_F^2 + 2\right)}{\eta_2} + s_2^T J_{x_{30}}(\overline{x}_2, x_3)s_3$$

$$+ \|s_2\|\left(\|V_2^*\|_F\left\|\widehat{W}_2^T\widehat{S}_2'\right\|_F\|X_2\| + \|W_2^*\|_F\left\|\widehat{S}_2'\widehat{V}_2^TX_2\right\|_F + \sqrt{l}\|W_2^*\|_F + \varepsilon_{2u}\right) + b_1. \tag{3.31}$$

Similar to Step 1, (3.31) can be simplified as

$$\dot{V}_2 \le -k_1\|s_1\|^2 - k_2\|s_2\|^2 - \frac{\sigma_{W1}}{2}\left\|\widetilde{W}_1\right\|_F^2 - \frac{\sigma_{V1}}{2}\left\|\widetilde{V}_1\right\|_F^2$$

$$- \frac{\sigma_{W2}}{2}\left\|\widetilde{W}_2\right\|_F^2 - \frac{\sigma_{V1}}{2}\left\|\widetilde{V}_2\right\|_F^2 + b_1 + b_2 + s_2^T J_{x_{30}}(\overline{x}_2, x_3)s_3, \tag{3.32}$$

where $b_2 = [\eta_2((1/4)\|W_2^*\|_F^2 + (1/4)\|V_2^*\|_F^2 + (l/4)\|W_2^*\|_F^2 + \varepsilon_{2u}^2) + (\sigma_{W2}/2)\|W_2^*\|_F^2 + (\sigma_{V2}/2)\|V_2^*\|_F^2]$ is a bounded constant.

Steps 3 to $n-1$ are similar to Step 2, which are omitted here.

Step n. Let us consider the system $\dot{x}_n = f_n(\overline{x}_n, u)$. Taking its derivative leads to

$$\ddot{x}_n = J_{\overline{x}_n}(\overline{x}_n, u)f_n(\overline{x}_n, u) + J_u(\overline{x}_n, u)\dot{u}, \tag{3.33}$$
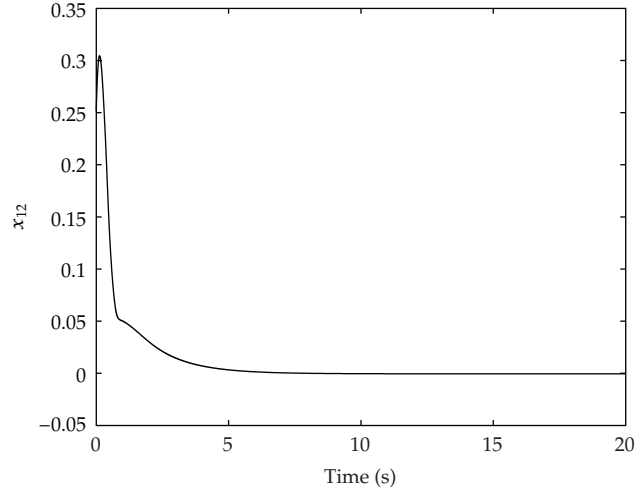
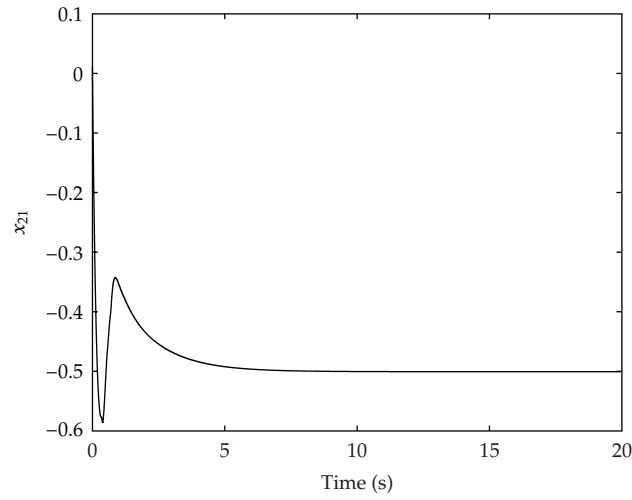**Figure 2:** Curve of x_{12} in case of the system without uncertainties.

Figure 3: Curve of x_{21} in case of the system without uncertainties.

where $J_{\overline{x}_n}(\overline{x}_n, u) = \partial f_n(\overline{x}_n, u)/\partial \overline{x}_n$ denotes the Jacobian with respect to $\overline{x}_n$. Equation (3.33) can be rewritten as

$$\ddot{x}_n = J_{\overline{x}_n 0}(\overline{x}_n, u) f_{n0}(\overline{x}_n, u) + J_{u0}(\overline{x}_n, u)\dot{u} + \Delta f_n(\overline{x}_n, u), \tag{3.34}$$

where

$$\begin{aligned} \Delta f_n(\overline{x}_n, u) = \Delta J_{\overline{x}_n 0}(\overline{x}_n, u) f_{n0}(\overline{x}_n, u) + J_{\overline{x}_n 0}(\overline{x}_n, u) \Delta f_{n0}(\overline{x}_n, u) \\ + \Delta J_{\overline{x}_n 0}(\overline{x}_n, u) \Delta f_{n0}(\overline{x}_n, u) + \Delta J_{\overline{x}_n 0}(\overline{x}_n, u)\dot{u}. \end{aligned} \tag{3.35}$$
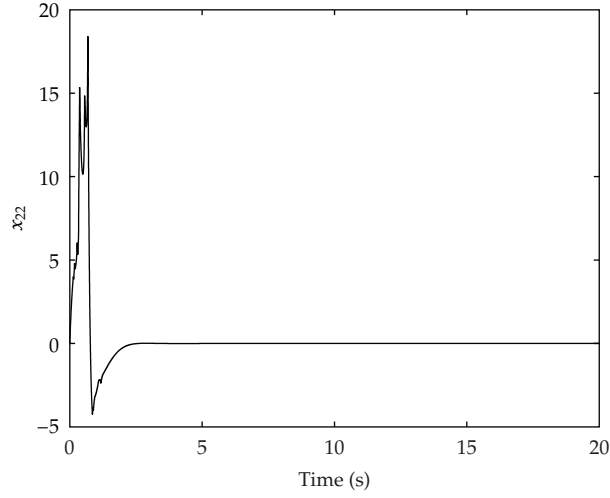
Figure 4: Curve of x_{22} in case of the system without uncertainties.

Let $z_n = x_n - x_{nd}$ and $s_n = z_n + c_1 \dot{z}_n$, where x_{nd} is the desired signal of x_n and s_n is a sliding mode surface-like vector. Taking its time derivative, we can obtain

$$
\begin{aligned}
\dot{s}_n &= \dot{z}_n + c_1 \ddot{z}_n \\
&= \dot{z}_n + c_1 J_{\bar{x}_n 0}(\bar{x}_n, u) f_{20}(\bar{x}_n, u) \\
&\quad + c_1 J_u(\bar{x}_n, u)\dot{u} + c_1 \Delta f_n(\bar{x}_n, u) - c_1 \ddot{x}_{nd}.
\end{aligned}
\tag{3.36}
$$

We choose the control law as

$$
\dot{u} = -[c_1 J_{u0}(\bar{x}_n, u)]^{-1} \left[c_1 J_{\bar{x}_n 0}(\bar{x}_n, u) f_{n0}(\bar{x}_n, u) + \widehat{\dot{z}}_n + J_{x_{n-1}0}^T(\bar{x}_{n-1}, x_n) s_{n-1} + v_n \right].
\tag{3.37}
$$

Let

$$
v_{\mathrm{NN}n} = \widehat{W}_n^T S_n \left(\widehat{V}_n^T X_n \right).
\tag{3.38}
$$

Let us consider the following Lyapunov function:

$$
V_n = \sum_{i=1}^{n-1} V_i + \frac{1}{2} s_n^T s_n + \frac{1}{2} \operatorname{tr}\left\{ \widetilde{W}_n^T \Gamma_{Wn}^{-1} \widetilde{W}_n \right\} + \frac{1}{2} \operatorname{tr}\left\{ \widetilde{V}_n^T \Gamma_{Vn}^{-1} \widetilde{V}_n \right\},
\tag{3.39}
$$

where $\Gamma_{Wn} = \Gamma_{Wn}^T > 0$ and $\Gamma_{Vn} = \Gamma_{Vn}^T > 0$ are constant design parameters.
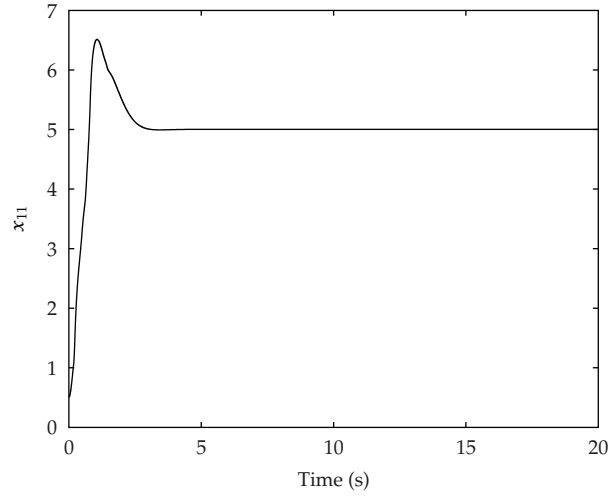
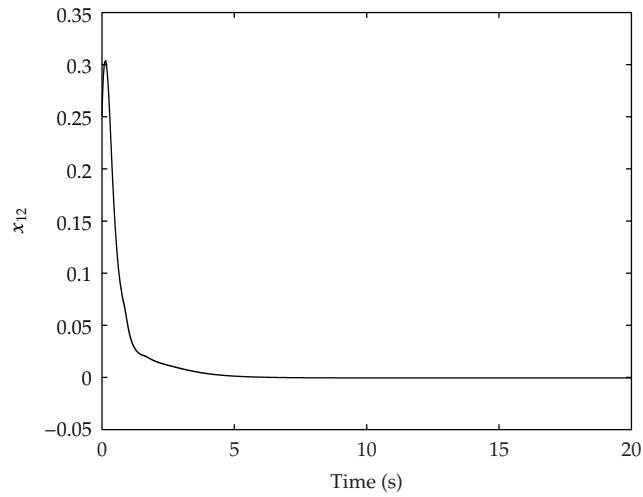**Figure 5:** Curve of x_{11} in case of the system with uncertainties.

Figure 6: Curve of x_{12} in case of the system with uncertainties.

Choose the adaptive tuning law as

$$\dot{\widehat{W}}_n = \Gamma_{Wn} \left[\left(\widehat{S}_n - \widehat{S}'_n \widehat{V}_n^T X_n \right) s_n^T - \sigma_{Wn} \widehat{W}_n \right],$$

$$\dot{\widehat{V}}_n = \Gamma_{Vn} \left(X_n s_n^T \widehat{W}_n^T \widehat{S}'_n - \sigma_{Vn} \widehat{V}_n \right), \tag{3.40}$$

where $\sigma_{Wn} > 0$ and $\sigma_{Vn} > 0$ are small parameters.

Similar to the derivation process in Step 1, we have

$$\dot{V}_n \le -\sum_{i=1}^{n}\left[k_i\|s_i\|^2 + \frac{\sigma_{Wi}}{2}\left\|\widetilde{W}_i\right\|_F^2 + \frac{\sigma_{Vi}}{2}\left\|\widetilde{V}_i\right\|_F^2 - b_i\right]$$

$$\le -kV_n + b, \tag{3.41}$$

where $b_n = [\eta_n((1/4)\|W_n^*\|_F^2+(1/4)\|V_n^*\|_F^2+(1/4)\|W_n^*\|_F^2+\varepsilon_{nu}^2)+(\sigma_{Wn}/2)\|W_n^*\|_F^2+(\sigma_{Vn}/2)\|V_n^*\|_F^2]$ is a bounded constant, $k = \min_{j=1,\dots,n}\{2k_i, \sigma_{Wj}/\lambda_{\max}(\Gamma_{Wj}^{-1}), \sigma_{Vj}/\lambda_{\max}(\Gamma_{Vj}^{-1})\}$, and $b = \sum_{j=1}^{n} b_j$.

Integrating (3.41) over $[0, t]$, it can be shown that

$$V_n(t) \le V_n(0)e^{-kt} + \frac{1}{k}b \le V_n(0) + \frac{b}{k}, \quad \forall t \ge 0. \tag{3.42}$$

Defining $\lambda_{\min}(\Gamma_W^{-1}) = \min_{j=1,2,\cdots n}\{\lambda_{\min}(\Gamma_{Wj}^{-1})\}$, and $\lambda_{\min}(\Gamma_V^{-1}) = \min_{j=1,2,\cdots n}\{\lambda_{\min}(\Gamma_{Vj}^{-1})\}$, and from (3.39), it can be shown that

$$\sum_{j=1}^{n}\left\|\widetilde{W}_j\right\|_F^2 \le \frac{2V_n(t)}{\lambda_{\min}(\Gamma_W^{-1})}, \qquad \sum_{j=1}^{n}\left\|\widetilde{V}_j\right\|_F^2 \le \frac{2V_n(t)}{\lambda_{\min}(\Gamma_V^{-1})}. \tag{3.43}$$

It is clear that

$$V_n(t) \ge \frac{1}{2}\sum_{j=1}^{n}\|s_j\|^2. \tag{3.44}$$

It can be seen from (3.41)–(3.44) that all the closed-loop signals are uniformly ultimately bounded. Inequality (3.41) implies that $\dot{V}_n(t) \le (-k/2)\sum_{j=1}^{n}\|s_j\|^2 + b$ holds. Integrating it yields

$$\int_0^t \|s_j(\tau)\|^2 d\tau \le \frac{2[V_n(0) + tb]}{k}, \quad j = 1,\dots,n. \tag{3.45}$$

Summarizing the previous discussion, we have the following results.

Theorem 3.5. *Considering the system (2.1), if Assumptions 2.2–3.2 hold, the* NN *weights are updated according to (3.17), (3.30), (3.40), and the control $\dot{u}$ is given in (3.37), and then the following results hold.*
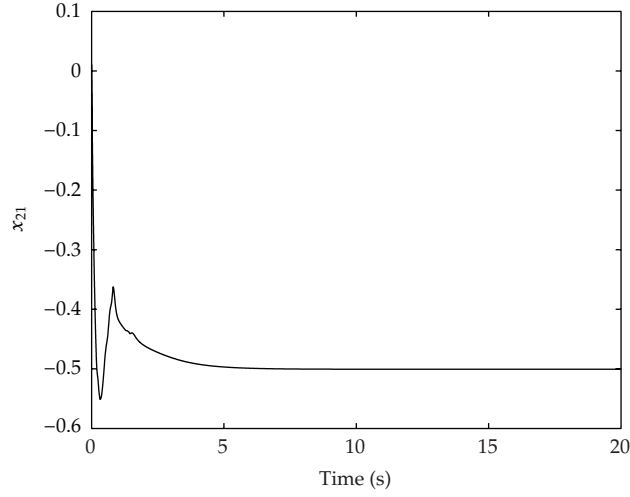
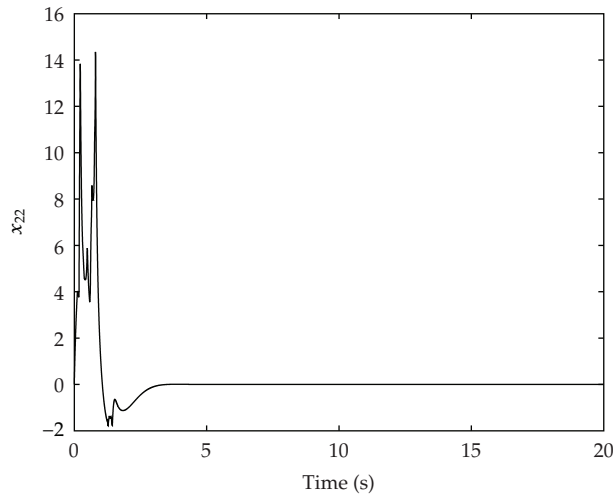**Figure 7:** Curve of x_{21} in case of the system with uncertainties.

Figure 8: Curve of x_{22} in case of the system with uncertainties.

(1) *The sliding surfaces s_j and the estimated parameter errors of* NN *are bounded and converge to the neighbourhoods of the origins exponentially:*

$$\Omega_j = \left\{ s_j, \widetilde{W}_j, \widetilde{V}_j \, \middle| \, \sum_{j=1}^{n} \|s_j\|^2 \le 2\left[V_n(0) + (b/k)\right], \sum_{j=1}^{n} \left\|\widetilde{W}_j\right\|^2 \le \frac{2\left[V_n(0) + (b/k)\right]}{\lambda_{\min}\left(\Gamma_W^{-1}\right)}, \right.$$

$$\left. \sum_{j=1}^{n} \left\|\widetilde{V}_j\right\|^2 \le \frac{2\left[V_n(0) + (b/k)\right]}{\lambda_{\min}\left(\Gamma_V^{-1}\right)}, \, j = 1, \ldots, n \right\}. \tag{3.46}$$
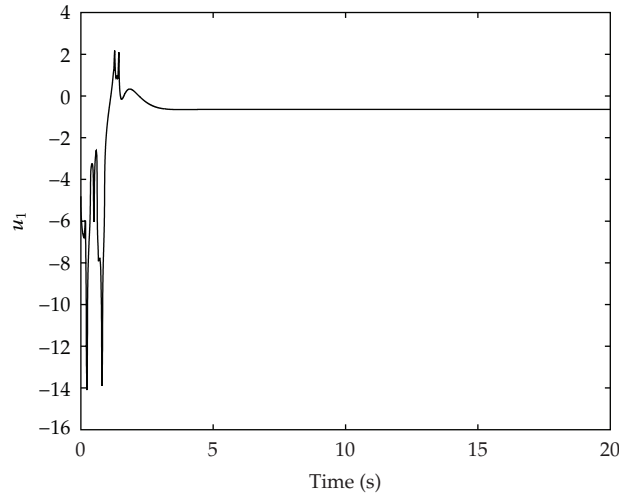
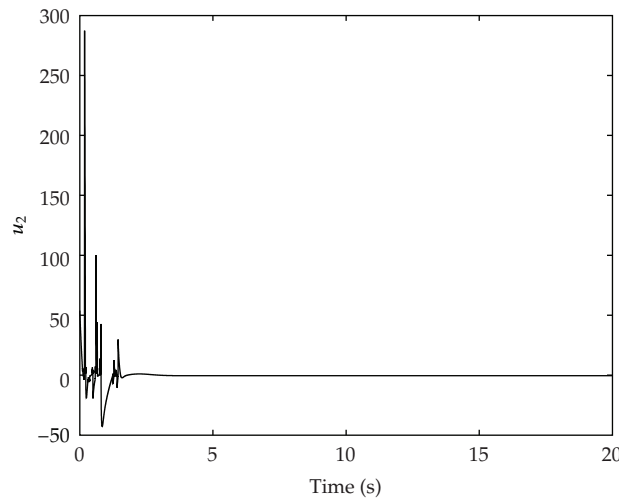**Figure 9:** Control signal u_1 in case of the system with uncertainties.

Figure 10: Control signal u_2 in case of the system with uncertainties.

(2) *The following inequality holds:*

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t \|s_j(\tau)\|^2 d\tau \le \frac{2b}{k}. \tag{3.47}$$

Remark 3.6. It is obvious that the bounded s_j, $j = 1, \ldots, n$, implies the bounded z_j and x_j. Furthermore, if $s_j \to 0$ as $t \to \infty$, we also can conclude that $z_j \to 0$ and $x_j \to x_{jd}$ as $t \to \infty$.

Remark 3.7. The result (1) of Theorem 3.5 indicates that adjusting the values of k_i, Γ_{W_i}, Γ_{V_i}, σ_{W_i}, and σ_{V_i} can control the convergence rate and the size of the convergence region. It is shown from the expression (3.46) that larger gains k_i, Γ_{W_i}, Γ_{V_i}, σ_{W_i}, and σ_{V_i} may result

in smaller convergence region. However, in practice, we do not suggest the use of high adaptation gains because such a choice may cause large oscillations in the control outputs [36].

4. Simulation Study

In order to check the effectiveness of the algorithm, consider the following system:

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1, x_2), \\
\dot{x}_2 &= f_2(\overline{x}_2, u), \\
y &= x_1,
\end{aligned}
\tag{4.1}
$$

where $x_1 = [x_{11}\ x_{12}]^T$, $x_2 = [x_{21}\ x_{22}]^T$,

$$
f_1(x_1, x_2) = \left[x_{12} + x_{22}\quad x_{11}x_{12}^2 - x_{11}x_{12} + (2 + 0.3\sin x_{12})x_{21} + \cos(0.1x_{21})\right]^T,
$$
$$
f_2(\overline{x}_2, u) = \left[x_{22} + 0.8\cos(u_1)\quad -x_{21} + x_{22} + u_2 + \left(x_{21}^2 + x_{22}^2\right)\frac{(1 - e^{-u_2})}{(1 + e^{-u_2})} - x_{21}^2 x_{22}\right]^T.
\tag{4.2}
$$

Let the desired output of the system be $y_d = [x_{11d}\ x_{12d}]^T = [5\ 0]^T$, and let the initial conditions be $[\,x_{11}(0)\ x_{12}(0)\ x_{21}(0)\ x_{22}(0)\,] = [0.5\ 0.25\ 0.01\ 0]$, $W_1(0) = W_2(0) = [0]_{11 \times 2}$, and $V_1(0) = V_2(0) = [0]_{5 \times 11}$.

According to Remark 3.7, we choose the parameters of the controller as follows: $k_{111} = 6.6$, $k_{122} = 6.6$, $k_{211} = 12.4$, $k_{222} = 12.4$, $\Gamma_{W_1} = 0.01$, $\Gamma_{W_2} = 0.28$, $\sigma_{W_1} = \sigma_{W_2} = 0.05$, $\Gamma_{V_1} = 0.11$, $\Gamma_{V_2} = 0.07$, $\sigma_{V_1} = \sigma_{V_2} = 0.052$, $\eta_1 = \eta_2 = 0.001$, and $c_1 = 0.69$.

The following two cases will be considered.

Case 1. All the parameters in (4.1) are known.

Case 2. One has the system with uncertainties $f_{10}(x_1, x_2) = 0.8f_1(x_1, x_2)$, $\Delta f_{10}(x_1, x_2) = 0.2f_1(x_1, x_2)$, $f_{20}(\overline{x}_2, u) = 0.8f_2(\overline{x}_2, u)$, and $\Delta f_{20}(\overline{x}_2, u) = 0.2f_2(\overline{x}_2, u)$.

The simulation results are shown in Figures 1–8. Figures 1–4 show the state responses in case of the system without uncertainties (Case 1), where Figure 1 shows the tracking response curve of the state x_{11}, Figure 2 shows the response curve of the state x_{12}, Figure 3 shows the response curve of the state x_{21}, and Figure 4 shows the response curve of the state x_{22}. Figures 5–8 show the state responses in case of the system with uncertainties (Case 2), where Figure 5 shows the tracking response curve of the state x_{11}, Figure 6 shows the response curve of the state x_{12}, Figure 7 shows the response curve of the state x_{21}, and Figure 8 shows the response curve of the state x_{22}. The control signals are shown in Figures 9 and 10.

Although no exact model of the plant is available and the initial NN weights are set to zero, through the NN learning phase and the action of the robust term, it can be seen that the output tracking performance shown in Figure 5 is quite well and the output tracking error converges to a quite small set after 4 s in Case 2.

From the figures, one can conclude that the proposed control method presents a good quality control in both cases.

5. Conclusions

In this paper, an NN-based sliding mode-like controller is presented for a class of uncertain block nonaffine systems. The controller is designed using NN control, dynamic feedback, backstepping design, sliding mode-like technique, and feedback linearization techniques, which makes the stability analysis simple for block nonaffine systems and guarantees the stability of the closed-loop system. The sliding mode-like technique can be applied to other classes of nonlinear systems in strict feedback form. The simulation results show the effectiveness of the proposed scheme.

Acknowledgment

References

[1] A. Isidori, *Nonlinear Control Systems*, Springer, Berlin, Germany, 3rd edition, 1995.

[2] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, NY, USA, 1995.

[3] S. S. Ge, T. H. Lee, and C. J. Harris, *Adaptive Neural Network Control of Robotic Manipulators*, World Scientific, London, UK, 1998.

[4] F. L. Lewis, S. Jagannathan, and A. Yesildirek, *Neural Network Control of Robot Manipulators and Nonlinear Systems*, Taylor and Francis, London, UK, 1999.

[5] R. Marino and P. Tomei, *Nonlinear Control Design: Geometric, Adaptive, &Robust*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1995.

[6] A. Rovithakis and M. A. Christodoulou, *Adaptive Control with Recurrent High-Order Neural Networks*, Springer, London, UK, 2000.

[7] T. Spooner, M. Maggiore, R. Ordóñez, and K. M. Passino, *Stable Adaptive Control and Estimation for Nonlinear Systems-Neural and Fuzzy Approximator Techniques*, Wiley, New York, NY, USA, 2002.

[8] S. S. Ge, C. C. Hang, and T. Zhang, "Nonlinear adaptive control using neural network and its application to cstr systems," *Journal of Process Control*, vol. 9, no. 4, pp. 313–323, 1998.

[9] J. D. Boskovic, L. Chen, and R. K. Mehra, "Adaptive control design for nonaffine models arising in flight control," *Journal of Guidance, Control, and Dynamics*, vol. 27, no. 2, pp. 209–217, 2004.

[10] G. Bartolini and E. Punta, "Reduced-order observer in the sliding-mode control of nonlinear nonaffine systems," *Institute of Electrical and Electronics Engineers*, vol. 55, no. 10, pp. 2368–2373, 2010.

[11] S. H. Lane and R. F. Stengel, "Flight control design using nonlinear inverse dynamics," *Automatica*, vol. 24, no. 4, pp. 471–483, 1988.

[12] H. Nijmeijer and A. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer, New York, NY, USA, 1990.

[13] B. J. Yang and A. J. Calise, "Adaptive control of a class of nonaffine systems using neural networks," *IEEE Transactions on Neural Networks*, vol. 18, no. 4, pp. 1149–1159, 2007.

[14] W. Lin, "Feedback stabilization of general nonlinear control systems: a passive system approach," *Systems & Control Letters*, vol. 25, no. 1, pp. 41–52, 1995.

[15] W. Lin, "Global asymptotic stabilization of general nonlinear systems with stable free dynamics via passivity and bounded feedback," *Automatica*, vol. 32, no. 6, pp. 915–924, 1996.

[16] E. Moulay and W. Perruquetti, "Stabilization of nonaffine systems: a constructive method for polynomial systems," *Institute of Electrical and Electronics Engineers*, vol. 50, no. 4, pp. 520–526, 2005.

[17] R. Sepulchre, M. Jankovic, and P. V. Kokotovic, *Constructive Nonlinear Control*, Springer, New York, NY, USA, 1997.

[18] A. S. Shiriaev and A. L. Fradkov, "Stabilization of invariant sets for nonlinear non-affine systems," *Automatica*, vol. 36, no. 11, pp. 1709–1715, 2000.

[19] E. D. Sontag, "A "universal" construction of Artstein's theorem on nonlinear stabilization," *Systems*

& *Control Letters*, vol. 13, no. 2, pp. 117–123, 1989.

[20] Y. H. Chien, W. Y. Wang, Y. G. Leu, and T. T. Lee, "Robust adaptive controller design for a class of uncertain nonlinear systems using online T-S fuzzy-neural modeling approach," *IEEE Transactions on Systems, Man, and Cybernetics B*, vol. 41, no. 2, pp. 542–552, 2011.

[21] S. S. Ge and J. Zhang, "Neural-network control of nonaffine nonlinear system with zero dynamics by state and output feedback," *IEEE Transactions on Neural Networks*, vol. 14, no. 4, pp. 900–918, 2003.

[22] C. J. Goh, "Model reference control of non-linear systems via implicit function emulation," *International Journal of Control*, vol. 60, no. 1, pp. 91–115, 1994.

[23] S. Labiod and T. M. Guerra, "Indirect adaptive fuzzy control for a class of nonaffine nonlinear systems with unknown control directions," *International Journal of Control, Automation, and Systems*, vol. 8, no. 4, pp. 903–907, 2010.

[24] E. Lavretsky and N. Hovakimyan, "Adaptive dynamic inversion for nonaffine-in-control uncertain systems via time-scale separation. II," *Journal of Dynamical and Control Systems*, vol. 14, no. 1, pp. 33–41, 2008.

[25] J. Teo, J. P. How, and E. Lavretsky, "Proportional-integral controllers for minimum-phase nonaffine-in-control systems," *Institute of Electrical and Electronics Engineers*, vol. 55, no. 6, pp. 1477–1483, 2010.

[26] W. Y. Wang, Y. H. Chien, Y. G. Leu, and T. T. Lee, "Adaptive T-S fuzzy-neural modeling and control for general MIMO unknown nonaffine nonlinear systems using projection update laws," *Automatica*, vol. 46, no. 5, pp. 852–863, 2010.

[27] W. Y. Wang, Y. H. Chien, and T. T. Lee, "Observer-based T-S fuzzy control for a class of general nonaffine nonlinear systems using generalized projection-update laws," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 3, pp. 493–504, 2011.

[28] B. J. Yang and A. J. Calise, "Adaptive control of a class of multivariable nonaffine systems," in *Proceedings of the 46th IEEE Conference on Decision and Control*, pp. 4809–4814, New Orleans, LA, USA, December 2007.

[29] A. J. Calise, M. Sharma, and S. Lee, "Adaptive autopilot design for guided munitions," *Journal of Guidance, Control, and Dynamics*, vol. 23, no. 5, pp. 837–843, 2000.

[30] H. B. Du and X. C. Chen, "NN-based output feedback adaptive variable structure control for a class of non-affine nonlinear systems: a nonseparation principle design," *Neurocomputing*, vol. 72, no. 7–9, pp. 2009–2016, 2009.

[31] Y. Q. Jin, *Research on the robust adaptive control of block control uncertain nonliear systems*, Ph.D. thesis, Naval Aeronautical and Astronautical University, 2006.

[32] B. S. Kim and A. J. Calise, "Nonlinear flight control using neural networks," *Journal of Guidance, Control, and Dynamics*, vol. 20, no. 1, pp. 26–33, 1997.

[33] Y. G. Leu, W. Y. Wang, and T. T. Lee, "Observer-based direct adaptive fuzzy-neural control for nonaffine nonlinear systems," *IEEE Transactions on Neural Networks*, vol. 16, no. 4, pp. 853–861, 2005.

[34] T. Zhang, S. S. Ge, and C. C. Hang, "Design and performance analysis of a direct adaptive controller for nonlinear systems," *Automatica*, vol. 35, no. 11, pp. 1809–1817, 1999.

[35] J. Q. Han and W. Wang, "Nonlinear tracking differentiator," *System Science and Mathematics*, vol. 14, no. 2, pp. 177–183, 1994.

[36] S. S. Ge, C. C. Hang, T. H. Lee, and T. Zhang, *Stable Adaptive Neural Network Control*, Kluwer Academic Publishers, Norwell, Mass, USA, 2001.