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Review Article

Mathematical Models of Dissipative Systems in Quantum Engineering

Andreea Sterian and Paul Sterian

Academic Center of Optical Engineering and Photonics, Polytechnic University of Bucharest, 313 Spl. Independentei, 060042 Bucharest, Romania

Correspondence should be addressed to Paul Sterian, sterian@physics.pub.ro

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The paper shows the results of theoretical research concerning the modeling and characterization of the dissipative structures generally, the dissipation being an essential property of the system with self-organization which include the laser-type systems also. The most important results presented are new formulae which relate the coupling parameters a_{in} from Lindblad equation with environment operators Γ_i; microscopic quantitative expressions for the dissipative coefficients of the master equations; explicit expressions which describe the changes of the environment density operator during the system evolution for fermion systems coupled with free electromagnetic field; the generalized Bloch-Feynman equations for N-level systems with microscopic coefficients in agreement with generally accepted physical interpretations. Based on Maxwell-Bloch equations with consideration of the interactions between nearing atomic dipoles, for the dense optical media we have shown that in the presence of the short optical pulses, the population inversion oscillates between two extreme values, depending on the strength of the interaction and the optical pulse energy.

1. Introduction

An essential problem of the quantum information systems is the controllability and observability of the quantum systems. In this context, Fermi systems are essential for several important physical effects in quantum engineering as the dynamics of semiconductor nanostructures and high temperature superconductivity, nuclear resonances, fusion-fission reactions, and analysis of optical quantum systems. These effects are essentially determined by the dissipative coupling of the system.

Dissipation in quantum systems is a complex phenomenon which raises important theoretical investigations. A dissipative system is a system of interest, coupled with another system usually considered as being of much larger-environment. Fundamental and difficult problem of dissipative quantum theory is to design the total system (system of interest + environment) on the space system of interest. In this way obtain a quantum master equation describing the evolution of the system using two terms: (1) a hamiltonian term for processes with energy conservation and (2) a nonhamiltonian term with coefficients that depend on the dissipative coupling. A master equation is based on approximations that consist in mediating rapid oscillations of reduced density matrix describing the interaction.

Such an approximation is the assumption that the evolution operators of a dissipative system forms a semigroup, not a group like for isolated systems. In this framework was derived a quantum master equation with dissipative terms which is consistent with all principles of quantum mechanics. Considering two operators, coordinate q and momentum p, master equation was used to describe the harmonic oscillator. In this theoretical framework, dissipation is described by the friction and diffusion coefficients that satisfy certain conditions called basic restrictions and Heisenberg's uncertainty relations are observed during the whole evolution of the system.

A rigorous method for deducting the master equation with microscopic expressions of the dissipative coefficients is developed in the literature.

For a weak dissipative coupling one obtains a master equation of Lindblad form [1], but with the microscopic expressions of the dissipative coefficients.

In the development of quantum theory of dissipative systems an important step was the connection between Lindblad's generator and the previous phenomenological descriptions, realized by Săndulescu and Scutaru [2]. Besides, we must mention Isar et al.'s contributions [3]. This school developed by the above-mentioned researchers in the field are well recognized in the scientific world [4–8].

Firstly, in the paper general expressions which relate the coupling parameters a_{in} in Lindblad equation with environment operators Γ_i have been established [9–11]. In this way, became possible deeper causality understanding of processes of friction and diffusion and of related quantum effects: broadening and shift of spectral lines, tunneling rates, bifurcations and instability [12, 13].

Secondly, for a system of fermions, coupled with a dissipative environment quantitative microscopic expressions for the coefficients of the dissipative master equation depending on the potential matrix elements, the densities of states of the environment and the occupation probabilities of these states are presented [14–19].

The study continue with the systems of fermions coupled by electric dipole interactions of free electromagnetic field for which has established general explicit expressions which describe the changes of the environment density operator during the system evolution. This description is not restricted to the Born approximation, taking into account the environment time evolution as a function of the system evolution. The results of the dissipative dynamics of the system of fermions in the presence of laser field are applicable to the dissipative structures [14, 20–29].

Next, generalized Bloch-Feynman equations for N-level systems with microscopic coefficients in agreement with generally accepted physical interpretations are presented.

In the last part, we study the dynamics of dense media under the action of ultrafast optical pulses using Maxwell-Bloch formalism to include interaction between close atomic dipoles [30–34]. It is shown that, in a system initially without inversion, in the presence of optical pulses, the final population has two extreme values, results which contribute to understanding the specific mechanisms of switching for applications, with specific examples concerning the coherent radiation generation and amplification [35–43]. A computational specific software, to verify the experimental and numerical existing models and in the same time to discover new important situations for operative systems design and implementation, was developed [44–48].

2. Relationship between Coupling Coefficients in Lindblad Master Equation and Environment Observables

Research on dissipative processes has led to evidence for the first time concerning the relationship between coupling coefficients a_{in} in the Lindblad equation:

$$\dot{\rho} \equiv -\frac{i}{\hbar}[H,\rho] + \frac{1}{2\hbar}\sum_{n}\left\{ \left[X_n\rho, X_n^{+}\right] + \left[X_n, \rho X_n^{+}\right]\right\} \tag{2.1}$$

depending on the system Hamiltonian H and *the operators of opening* X_n:

$$X_n \equiv \sum_{i} a_{in}s_i, \tag{2.2}$$

where s_i are system operators, a_{in} are complex *coupling coefficients* or *amplitudes* and Γ_i operators of environment defined using the interaction Hamiltonian as

$$H^{SE} = \hbar \sum_{i} s_i\Gamma_i. \tag{2.3}$$

These relationships have been established under the form [10]

$$\sum_{n} a_{in}a_{jn}^{*} = 2\hbar\langle \Gamma_i\Gamma_j\rangle, \tag{2.4}$$

and allow an understanding of the physical causes of quantum processes of friction and diffusion, with their known effects: broadening and shift of spectral lines [11], increased rates of tunneling, nonlinear characteristics, leading to bifurcation, instability, and chaos.

3. Microscopic Quantitative Expressions of the Dissipative Coefficients in Master Equations

A general quantum master equation for a many-level many-particle system, with microscopic coefficients, that preserves the quantum-mechanical properties of the density matrix was obtained [12]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\frac{i}{\hbar}[H,\rho(t)] + \sum_{i,j}\lambda_{ij}\left\{\left[c_i^{+}c_j\rho(t), c_j^{+}c_i\right] + \left[c_i^{+}c_j, \rho(t)c_j^{+}c_i\right]\right\} \tag{3.1}$$

with dissipative coefficients:

$$\lambda_{ij} = \lambda_{ij}^{F} + \lambda_{ij}^{B} \tag{3.2}$$

including a component λ_{ij}^F for a dissipative environment of fermions and a component λ_{ij}^B for a dissipative environment of bosons.

Equation (3.1) is of Lindblad's form, with dissipative operators depending on the transition/population operators $c_i^+ c_j$. For a system with N levels, the total number of these operators is $N^2 - 1$ the number of the independent operators defined.

If we denote by V^F and V^B the interaction dissipative potentials of the environment containing Y^F fermions and Y^B bosons, respectively, it is possible to write the expressions of the coefficients λ_{ij}^F şi λ_{ij}^B for the resonant transition $|j\rangle \rightarrow |i\rangle$ of the system coupled with $|\beta\rangle \rightarrow |\alpha\rangle$ environmental transition, with fermionic state having densitiesaly g_α^F, g_β^F and populations $f_\alpha^F(\varepsilon_\alpha)$, $f_\beta^F(\varepsilon_\beta)$, and bosonic states with densities g_α^B, g_β^B and population $f_\alpha^B(\varepsilon_\alpha)$ şi $f_\beta^B(\varepsilon_\beta)$. The probability that the final state $|\alpha\rangle$ of the environment to be free is $1 - f(\varepsilon_\alpha)$ while the probability the initial state of the environment to be occupied is $f(\varepsilon_\beta)$.

General expressions of the dissipative coefficients are written for this type of interaction in the form:

$$\lambda_{ij}^F = \frac{\pi}{\hbar Y^F} \int \left| \langle \alpha_i | V^F | \beta_j \rangle \right|^2 \left[1 - f_\alpha^F(\varepsilon_\alpha) \right] f_\beta^F(\varepsilon_\beta) g_\alpha^F(\varepsilon_\alpha) g_\beta^F(\varepsilon_\beta) \mathrm{d}\varepsilon_\beta, \quad \varepsilon_\alpha - \varepsilon_\beta = \varepsilon_j - \varepsilon_i,$$
$$\lambda_{ij}^B = \frac{\pi}{\hbar Y^B} \int \left| \langle \alpha_i | V^B | \beta_j \rangle \right|^2 \left[1 + f_\alpha^B(\varepsilon_\alpha) \right] f_\beta^B(\varepsilon_\beta) g_\alpha^B(\varepsilon_\alpha) g_\beta^B(\varepsilon_\beta) \mathrm{d}\varepsilon_\beta, \quad \varepsilon_\alpha - \varepsilon_\beta = \varepsilon_j - \varepsilon_i. \tag{3.3}$$

4. The Environment Dynamics Correlated with that of a Fermion Systems Coupled with Free Electromagnetic Field

We consider a system of Z charged fermions with the coordinates $\overline{r}_n$ and momenta $\overline{p}_n$ ($n = 1, 2, \ldots, Z$) in a single-particle potential $U^{(1)}(\overline{r}_n)$, while $U^{(2)}(\overline{r}_n, \overline{r}_m)$ represents the two-particle residual potential. This system is coupled to the modes ν of the free electromagnetic field. In order to describe the dynamics of this system, for simplicity, we neglect the particle spin and its dimensions with respect to the electromagnetic field wavelength (the electric dipole approximation). In this case, the total hamiltonian is of the form [14]

$$H^T = \sum_{n=1}^{Z} \frac{\left(\overline{p}_n - e\overline{A}^B \right)^2}{2m} + \sum_{n=1}^{Z} U^{(1)}(\overline{r}_n) + \frac{1}{2} \sum_{n,m=1}^{Z} U^{(2)}(\overline{r}_n, \overline{r}_m) + H^B. \tag{4.1}$$

In the total hamiltonian (4.1),

$$V = -\frac{e}{m} \sum_{n=1}^{Z} \overline{p}_n \overline{A}^B \tag{4.2}$$

is the system-field interaction potential, while

$$H^S = \sum_{n=1}^{Z} \frac{\overline{p}_n^2}{2m} + \sum_{n=1}^{Z} U^{(1)}(\overline{r}_n) + \frac{1}{2} \sum_{n,m=1}^{Z} U^{(2)}(\overline{r}_n, \overline{r}_m) \tag{4.3}$$

is the fermion system hamiltonian, and

$$H^B = \sum_{\nu} H^{(\nu)} \tag{4.4}$$

is the field hamiltonian, where

$$H^{(\nu)} = \hbar\omega_\nu \left(a_\nu^+ a_\nu + \frac{1}{2} \right) \tag{4.5}$$

is the field mode ν hamiltonian.

Let us take the density operator $\chi(t)$ of the total system with hamitonian (4.1) and the reduced density matrix

$$\rho(t) = \mathrm{Tr}_B\{\chi(t)\} \tag{4.6}$$

over the environment states.

The total density operator $\chi(t)$ satisfies the equation of motion:

$$\frac{\mathrm{d}\tilde{\chi}}{\mathrm{d}t} = -\frac{i}{\hbar}\left[\varepsilon \tilde{V}^R(t) + \varepsilon \tilde{V}(t), \tilde{\chi}(t)\right], \tag{4.7}$$

where the sign above χ designs operators within the framework of interaction picture of the system and environment

$$\tilde{\chi}(t) = e^{(i/\hbar)(H^B + H_0^S)t} \chi(t) e^{-(i/\hbar)(H_0^S + H^B)t}, \tag{4.8}$$

while ε is an intensity parameter used to show the orders of the series expansion of this density. Considering the radiation field of the black body in the initial state R, the total density operator of the system can be taken under the form:

$$\tilde{\chi}(t) = R \otimes \tilde{\rho}(t) + \varepsilon \tilde{\chi}^{(1)}(t) + \varepsilon^2 \tilde{\chi}^{(2)}(t) + \cdots, \tag{4.9}$$

where $\tilde{\chi}^{(1)}(t)$, $\tilde{\chi}^{(2)}(t)$ represent modifications of the field during the system evolution. The first term of this expression corresponds to the Born approximation when the environment state is a constant state R, while the higher-order terms, which satisfy the normalization relations

$$\mathrm{Tr}_B\left\{\chi^{(1)}(t)\right\} = \mathrm{Tr}_B\left\{\chi^{(2)}(t)\right\} = \cdots = 0, \tag{4.10}$$

describe the environment dynamics that is correlated to the system dynamics. For an equation of motion of the form

$$\frac{\mathrm{d}\tilde{\rho}}{\mathrm{d}t} = \varepsilon \tilde{B}^{(1)}\left[\tilde{\rho}(t), t\right] + \varepsilon^2 \tilde{B}^{(2)}\left[\tilde{\rho}(t), t\right] \tag{4.11}$$

From (4.7), (4.9), and (4.11) we get a system of coupled equations:

$$R \otimes \widetilde{B}^{(1)}\left[\widetilde{\rho}(t), t\right] + \frac{\mathrm{d}\widetilde{\chi}^{(1)}}{\mathrm{d}t} = -\frac{i}{\hbar}\left[\widetilde{V}^{R}(t) + \widetilde{V}(t), R \otimes \widetilde{\rho}(t)\right],$$
$$R \otimes \widetilde{B}^{(2)}\left[\widetilde{\rho}(t), t\right] + \frac{\mathrm{d}\widetilde{\chi}^{(2)}}{\mathrm{d}t} = -\frac{i}{\hbar}\left[\widetilde{V}^{R}(t) + \widetilde{V}(t), \widetilde{\chi}^{(1)}(t)\right]. \tag{4.12}$$

By calculating the partial traces over the environment states and using the normalization conditions (4.10), from these equations we get successively the terms of the equation of motion (4.11):

$$\widetilde{B}^{(1)}\left[\widetilde{\rho}(t), t\right] = -\frac{i}{\hbar}\mathrm{Tr}_B\left[\widetilde{V}^{R}(t) + \widetilde{V}(t), R \otimes \widetilde{\rho}(t)\right],$$
$$\widetilde{B}^{(2)}\left[\widetilde{\rho}(t), t\right] = -\frac{i}{\hbar}\mathrm{Tr}_B\left[\widetilde{V}^{R}(t) + \widetilde{V}(t), \widetilde{\chi}^{(1)}(t)\right], \tag{4.13}$$

while, integrating by time, we get "excitation" terms of the total density operator (4.9):

$$\widetilde{\chi}^{(1)}(t) = \int_0^t \left\{ -\frac{i}{\hbar}\left[\widetilde{V}^{R}(t') + \widetilde{V}(t'), R \otimes \widetilde{\rho}(t')\right] - R \otimes \widetilde{B}^{(1)}\left[\widetilde{\rho}(t'), t'\right] \right\} \mathrm{d}t',$$
$$\widetilde{\chi}^{(2)}(t) = \int_0^t \left\{ -\frac{i}{\hbar}\left[\widetilde{V}^{R}(t') + \widetilde{V}(t'), \widetilde{\chi}^{(1)}(t')\right] - R \otimes \widetilde{B}^{(2)}\left[\widetilde{\rho}(t'), t'\right] \right\} \mathrm{d}t'. \tag{4.14}$$

The first-order equation (4.13) represents the system evolution when the environment is considered as being in a constant state R, while for the higher-order term (28), we take into consideration some changes of the environment matrix (4.14). Further on, we will show that the first-order terms (4.13) describe the hamiltonian dynamics of the system, while the second-order term (28) describes system one-particle transitions related to environment.

5. The Generalized Bloch-Feynman Equations

An alternative description of dissipative system dynamics is given by Bloch-Feynman equations for systems of fermions obtained by defining the pseudo-spin operators [14].

In particular, for a system with two-level known Bloch-Feynman, equations are obtained, where, Q_{12} is the field operator, P_{12} is the polarization operator, and N_2 is population operator:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle Q_{12}\rangle = -\gamma_{\perp}\langle Q_{12}\rangle + \omega_{21}\langle P_{12}\rangle,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle P_{12}\rangle = -\omega_{21}\langle Q_{12}\rangle - \gamma_{\perp}\langle P_{12}\rangle, \tag{5.1}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle N_{2}\rangle = -\gamma_{\|}\left[\langle N_{2}\rangle - N_2^{(0)}\right],$$

with microscopic coefficients $\gamma_\perp$ şi $\gamma_{||}$ expressed by dissipative coefficients λ_{ij} of the master equation:

$$\gamma_\perp = \lambda_{12} + \lambda_{21} + \lambda_{11} + \lambda_{22}, \tag{5.2}$$

$$\gamma_{||} = 2(\lambda_{12} + \lambda_{21}), \tag{5.3}$$

$$N_2^{(0)} = \frac{\lambda_{21}}{\lambda_{12} + \lambda_{21}}. \tag{5.4}$$

The condition $2\gamma_\perp \geq \gamma_{||}$ is a confirmation of master equation (4.7) which led to the establishment of Bloch equations-Feynman, because this condition is verified experimentally.

6. Dynamics of Dense Media under the Action of Short Optical Pulses

Maxwell-Bloch equations of a two-level atomic medium generalized to include interactions between the dipoles approach [15, 16, 32] have been used to describe the system dynamics under the action of ultrafast optical pulses. These equations, for systems with homogeneous broadening of spectral lines in about semiclassical treating, were established using the density matrix formalism as

$$\frac{dw}{dt} = -\gamma_L(w+1) + \frac{\mu}{\hbar}\left(\tilde{E}^* R_{ab} + \tilde{E} R_{ab}^*\right), \tag{6.1}$$

$$\frac{dR_{ab}}{dt} = -\left[\gamma_T + i(\Delta + \varepsilon w)\right] R_{ab} - \frac{\mu}{2\hbar}\tilde{E} w. \tag{6.2}$$

In the above equations, w is the inversion of population, R_{ab} nondiagonal elements of density matrix slow variable, indices a and b refer to lower and higher energy states, with the gap $\hbar\omega_0$, E_L is slowly varying local field $\Delta = \omega_0 - \omega$ is the frequency deviation in relation to the center frequency of the field resonance frequency, μ is the transition matrix element of the electric dipole, and $\gamma_{||}$, $\gamma_\perp$ are longitudinal and transverse relaxation rates.

Contributions of the dipole-dipole interactions occur in (6.2) by term $i\varepsilon w R_{ab}$, where $\varepsilon = n\mu^2/3\hbar\varepsilon_d \ll \omega_0$ is the strength parameter of dipole-dipole interactions having a dimension of a frequency. Equations (6.1) and (6.2) for the atomic variables and for field variables realise the description Maxwell-Bloch of optically dense environment. These equations were generalized and used to study intrinsic optical bistability, propagation effects in nonlinear media, and so forth.

For numerical simulation, we considered the case resonant ($\Delta = 0$), a characteristic distance between dipoles much smaller than the wavelength of the central field (propagation effects are negligible) and ultrafast pulses (pulses much shorter than $\gamma_{||}^{-1}$; this enables us to

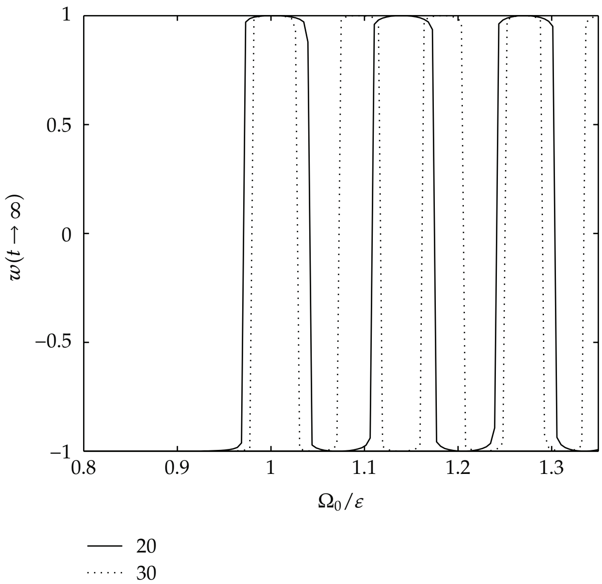

Figure 1: Final state of population inversion, depending on the hyperbolic secant pulse maximum value $\widetilde{E}(t) = \widetilde{E}_0 \mathrm{sech}(t/\tau_p)$ (solid line), $\varepsilon\tau_p = 20$ (continuous line) and $\varepsilon\tau_p = 30$.

neglect dissipation processes). In these conditions, the matrix element R_{ab} is decomposed into its real and imaginary parts $R_{ab} = 1/2(v + iu)$, resulting in system

$$\frac{\mathrm{d}u}{\mathrm{d}t'} = -(\varepsilon\tau_p)vw,$$
$$\frac{\mathrm{d}v}{\mathrm{d}t'} = (\varepsilon\tau_p)\left(u + \frac{\Omega}{\varepsilon}\right)w, \tag{6.3}$$
$$\frac{\mathrm{d}w}{\mathrm{d}t'} = -(\varepsilon\tau_p)\left(\frac{\Omega}{\varepsilon}\right)v$$

whose outcome is possible only numerically.

In the above equations $t' = t/\tau_p$ is the normalized time, τ_p is the measured width pulse, $\Omega(t) = \mu\widetilde{E}(t)/h$ is the instantaneous Rabi frequency, and $\widetilde{E}(t)$ is the intensty of electrical pulse.

In Figure 1, we present the final population inversion function of maximum Rabi frequency for hyperbolic secant pulses $\widetilde{E}(t) = \widetilde{E}_0 \mathrm{sech}(t/\tau_p)$. As long as the Rabi frequency has a value so that $\Omega_0/\varepsilon < 1$, the final population inversion is $w = -1$. In the region $\Omega_0/\varepsilon > 1$, the final population inversion has an oscillatory behavior, almost rectangular wave. As the parameter $\varepsilon\tau_p$ value is greater, the oscillation period decreases, the transitions become abrupt, and the first half cycle of the rectangular wave becomes more centered to $\Omega_0/\varepsilon = 1$.

In Figure 2, temporal evolution of the system is presented for a hyperbolic secant pulse with a peak higher than one (when $t \rightarrow \infty$, the population inversion performs a number of oscillations before reaching a value 1; under certain conditions when $t \rightarrow \infty$, after a number of oscillations, the system remains in the ground state).

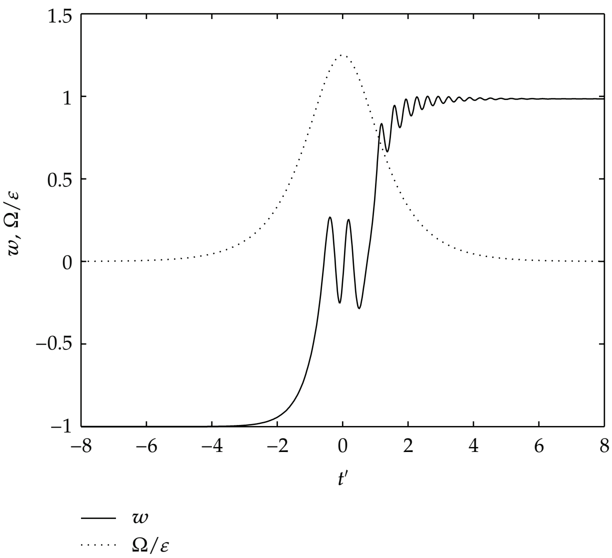

Figure 2: Temporal evolution of the system for a hyperbolic secant pulse, $\widetilde{E}(t) = \widetilde{E}_0 \text{sech}(t/\tau_p)$.

7. Conclusions

General expressions which relate the coupling parameters a_{in} in Lindblad equation with environment operators Γ_i have been established. These expressions allow deeper understanding of causal processes of friction- and diffusion-related quantum effects: broadening and shift of spectral lines, tunneling rates, bifurcations, and instability.

For a system of fermions coupled with a dissipative environment quantitative microscopic expressions for the coefficients of the dissipative master equation are presented.

These coefficients depend on the potential matrix elements, the densities of states of the environment, and the occupation probabilities of these states.

Expressions of the dependence of the particle distributions on temperature are taken into account. It can be shown that a system of fermions located in a dissipative environment of bosons tends to a Bose-Einstein distribution.

Studying the systems of fermions coupled by electric dipole interactions of free electromagnetic field, has established general explicit expressions which describe the changes of the environment density operator during the system evolution for fermion systems coupled with free electromagnetic field. This description is not restricted to the Born approximation, taking into account the environment time evolution as a function of the system evolution. The study can be continued with the calculation of the higher-order term of the reduced matrix equation in order to describe the correlated transition of the system particles. The results of the dissipative dynamics of the system of fermions in the presence of laser field are applicable to the dissipative structures.

Generalized Bloch-Feynman equations for N-level systems with microscopic coefficients in agreement with generally accepted physical interpretations are presented. On this basis, the problem of a quantum system control is explicitly formulated in terms of microscopic quantities: matrix elements of the dissipative two-body potential, densities of the environment states, and occupation probabilities of these states.

Studying the dynamics of dense media under the action of ultrafast optical pulses using Maxwell-Bloch formalism to include interaction between close atomic dipoles showed that, in a system initially without inversion, in the presence of optical pulses, the final population has two extreme values, the ratio of Rabi frequency and the parameter that describes the interactions between close dipoles, which contribute to understanding the specific mechanisms of switching.

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