Research Article

Vehicle Vibration Analysis in Changeable Speeds Solved by Pseudoexcitation Method

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The vehicle driving comfort has become one of the important factors of vehicle quality and receives increasing attention. In this paper, the mechanical and mathematical models of the half-car, five degrees of freedom (DOF) of a vehicle were established, as well as the pseudoexcitation model of road conditions for the front wheel and the rear wheel. By the pseudoexcitation method, the equations of transient response and power spectrum density were established. After numerical simulation to vehicle vibration response of changeable driving, the results show that the pseudoexcitation method is more convenient than the traditional method and effectively solves the smoothness computation problems of vehicles while the pseudoexcitation method is used to analyze vehicle vibration under nonstationary random vibration environments.

1. Introduction

The mechanical and mathematical model of vehicle systems is usually simplified as a multiple-mass, complicated vibration system. Due to road excitation, vehicles may come into complicated vibration, which is disadvantageous to passenger health and goods protection [1, 2]. Therefore, it is important and necessary to control the vehicle's vibration within a limited and comfortable grade in order to ensure safety steering and physical health of drivers and passengers, as well as the operating stability of man-vehicle-road system. In the process of automobile moving, the random and changeable road surface is the main factor to induce vehicle vibration. Therefore, investigation of vehicle's stochastic vibration [3, 4] induced by road excitation has been a significant problem of vehicle design and its performance simulation.

At present, for this kind of problems, the Fourier transform analysis is used to investigate the dynamic characteristics of constant driving problems of automobiles based on stationary random vibration theory. After finishing vibration model of vehicles, it is important to derive the frequency characteristic of vehicle vibration responses and to

establish power spectrum density function of road excitation and vehicle vibration responses [5]. Then it can be used to analyze the influence of vehicle structural parameters and road excitation on vehicle random vibration [6]. Although this method was relatively simple, its derivation process is too complicated, that is, this method needs not only to derive the frequency response characteristics of vehicle vibration system, but also to derive the frequency response characteristics of vehicle vibration response values.

However, in some circumstances, vehicles are running in changeable speeds, such as in accelerating starting period and decelerating stopping period. In these cases, the road excitation and the vehicle dynamic response in time domain are nonstationary [7, 8]. The stochastic vibration analysis method based on Fourier transform and its inverse transform has been used to study the changeable speed response of vehicles in unevenness roads, but its computation work is enormous. The pseudoexcitation method was used to analyze the stochastic vibration of structural systems [9–11]. By pseudoexcitation method, stochastic vibration analysis was carried on a two-DOF system vibration of a quarter-vehicle model [12–14], in which the vibration response of a constant speed moving vehicle was investigated to a stationary random road excitation. The changeable speed vehicle vibration response was also conducted under one-point road excitation [15]. In addition, some last investigations [16, 17] have dealt with a quarter-car model with a harmonic excitation while the study [18] considers the additional stochastic component in the road surface roughness. In this study, the time-space frequency relationship of vehicle vibration under changeable speed moving was derived by pseudoexcitation method, and then the equation of transient power spectrum density of vehicle vibration response under nonstationary random road excitation input was obtained. At the end, we also conducted the numerical simulations to the vibration responses of a half-car, five-DOF vehicle system under changeable speed moving conditions.

2. Theory of Pseudoexcitation Method

When a linear system is randomly excited by self-spectrum density $S_{xx}(w)$, the self-power spectrum of its response y is
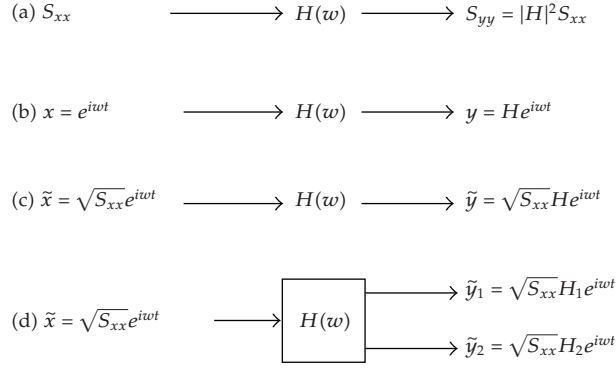
$$S_{yy}(w) = |H(w)|^2 S_{xx}(w), \tag{2.1}$$

where $H(w)$ is the frequency response function, and its meaning is shown in Figure 1 [19], that is, the corresponding harmonic response is $y = H(w)e^{iwt}$ when the random excitation is replaced by harmonic excitation e^{iwt}. From Figure 1, it can be seen that if it multiplies a constant $\sqrt{S_{xx}}$ before the excitation e^{iwt}, it can create a pseudoexcitation,

$$\tilde{x}(t) = \sqrt{S_{xx}}e^{iwt}. \tag{2.2}$$

After multiplying a same constant to its response value, it can give the following equations:

$$\tilde{y}^*\tilde{y} = |\tilde{y}|^2 S_{xx} = S_{yy},$$

$$\tilde{x}^*\tilde{y} = \sqrt{S_{xx}}e^{-iwt} \cdot \sqrt{S_{xx}}e^{iwt} = S_{xx}H = S_{xy}, \tag{2.3}$$

$$\tilde{y}^*\tilde{x} = \sqrt{S_{xx}}e^{-iwt} \cdot \sqrt{S_{xx}}He^{iwt} = S_{xx}H = S_{sy}.$$

(a) S_{xx} $\longrightarrow$ $H(w)$ $\longrightarrow$ $S_{yy} = |H|^2 S_{xx}$

(b) $x = e^{iwt}$ $\longrightarrow$ $H(w)$ $\longrightarrow$ $y = He^{iwt}$

(c) $\tilde{x} = \sqrt{S_{xx}}e^{iwt}$ $\longrightarrow$ $H(w)$ $\longrightarrow$ $\tilde{y} = \sqrt{S_{xx}}He^{iwt}$

(d) $\tilde{x} = \sqrt{S_{xx}}e^{iwt}$ $\longrightarrow$ $\boxed{H(w)}$ $\longrightarrow$ $\tilde{y}_1 = \sqrt{S_{xx}}H_1 e^{iwt}$
$\longrightarrow$ $\tilde{y}_2 = \sqrt{S_{xx}}H_2 e^{iwt}$

Figure 1: The principle of pseudoexcitation method.

The last formula in the above 3 equations is the conventional expressions of self-spectrum density or mutual spectrum density.

If the pseudoresponse values $\tilde{y}_1$ and $\tilde{y}_2$ are considered in the above system, the following equations can be validated:

$$\tilde{y}_1^* \tilde{y}_2 = H_1{}^* \sqrt{S_{xx}}e^{-iwt} \cdot H_2 \sqrt{S_{xx}}e^{iwt} = H_1{}^* S_{xx} H_2 = S_{y1y2},$$
$$\tilde{y}_2^* \tilde{y}_1 = H_2{}^* S_{xx} H_1 = S_{y2y1}. \tag{2.4}$$

Then the matrixes of power spectrum density are as follows:

$$[S_{yy}] = \{\tilde{y}\}^* \cdot \{\tilde{y}\}^T,$$
$$[S_{xy}] = \{\tilde{x}\}^* \cdot \{\tilde{y}\}^T, \tag{2.5}$$
$$[S_{yx}] = \{\tilde{y}\}^* \cdot \{\tilde{x}\}^T.$$

If the pseudoexcitation of a random process is $\tilde{x}(t) = \sqrt{S_{xx}}e^{iwt}$, then it can give

$$\dot{\tilde{x}} = iw\sqrt{S_{xx}}e^{iwt}, \qquad \ddot{\tilde{x}} = -w^2\sqrt{S_{xx}}e^{iwt}, \tag{2.6}$$
$$S_{\ddot{x}\ddot{x}} = w^4 S_{xx}. \tag{2.7}$$

That is the power spectrum density of accelerations, and in the same way, we can obtain the following equations:

$$S_{\ddot{y}\ddot{y}} = \{\ddot{\tilde{y}}\}^* \cdot \{\ddot{\tilde{y}}\}^T, \tag{2.8}$$
$$S_{\ddot{y}_1\ddot{y}_2} = \{\ddot{\tilde{y}}_1\}^* \cdot \{\ddot{\tilde{y}}_2\}^T, \tag{2.9}$$

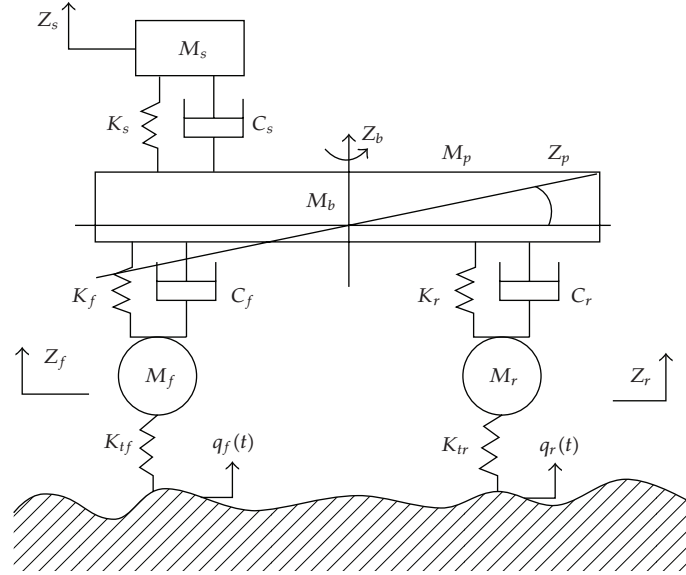where $*$ is a complex conjugate, and T is a matrix transfer.

Figure 2: The mechanical model of a half-car, five-DOF automobile system.

After computing the vibration response of the system in the case of the pseudo harmonic excitation, all the power spectrum densities of them can be solved according to (2.5)–(2.9). Then the self-spectrum density and mutual spectrum density of them can also be obtained.

3. Vibration Analysis of the Half-Car, Five-DOF Vehicle System in Changeable Speeds

3.1. Mechanical and Mathematical Models of the System

For analyzing automobile vibration, it is important to establish its mechanical and mathematical model of the automobile structural system, so that the vibration characteristic response value of the mathematical model of vehicle vibration can be solved and obtained. For the mechanical modeling of automobiles, a seven-DOF mechanical model [20] has been developed to investigate the influence of active and semiactive suspension to automobile dynamic performance. In addition, the moving smartness and operating stability of automobiles were also investigated by spatial mechanical models of automobiles [21]. In this study, a half-car, five-DOF linear mechanical model [22] of an automobile system was developed as shown in Figure 2.

In Figure 2, m_s is the mass of driver and chair and m_b is the mass of automobile structure. m_f, m_r are the nonspring supported mass of front and rear suspensions, respectively. k_s and c_s are the rigidity coefficient and damping coefficient of the chair, respectively. k_f, k_r are the rigidity coefficient of front and rear suspensions, respectively. k_{tf}, k_{tr} are the rigidity coefficient of front and rear wheels, respectively. q_f, q_r are the road excitation forces at front and rear wheels, respectively. l_1 is the distance between chair and vehicle mass center. l_2, l_3 are the distances from the vehicle mass center to front and rear wheel axles, respectively.

By Lagrange equations, the mathematical model of the vehicle mechanical model in Figure 2 is as follows:

$$[M]\{\ddot{Z}\} + [C]\{\dot{Z}\} + [K]\{Z\} = [F]\{Q\}, \tag{3.1}$$

where

$$M = \begin{bmatrix} m_s & 0 & 0 & 0 & 0 \\ 0 & m_b & 0 & 0 & 0 \\ 0 & 0 & m_p & 0 & 0 \\ 0 & 0 & 0 & m_f & 0 \\ 0 & 0 & 0 & 0 & m_r \end{bmatrix},$$

$$C = \begin{bmatrix} c_s & -c_s & c_s l_1 & 0 & 0 \\ -c_s & c_s + c_f + c_r & -c_s l_1 - c_f l_2 + c_r l_3 & -c_f & -c_r \\ c_s l_1 & -c_s l_1 - c_f l_2 + Cr l_3 & c_s l_1^2 + c_f l_2^2 + c_r l_3^2 & c_f l_2 & -Cr l_3 \\ 0 & -c_f & c_f l_1 & c_f & 0 \\ 0 & -c_r & -c_r l_3 & 0 & c_r \end{bmatrix},$$

$$K = \begin{bmatrix} k_s & -k_s & k_s l_1 & 0 & 0 \\ -k_s & k_s + k_f + k_r & -k_s l_1 - k_f l_2 + k_r l_3 & -k_f & -k_r \\ k_s & -k_s l_1 - k_f l_2 + k_r l_3 & k_s l_1^2 + k_f l_2^2 + k_r l_3^2 & k_f l_2 & -k_r l_3 \\ 0 & -k_f & k_f l_2 & k_f + k_{tf} & 0 \\ 0 & -k_r & -k_r l_3 & 0 & k_r + k_{tr} \end{bmatrix},$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_{tf} & 0 \\ 0 & k_{tr} \end{bmatrix}, \quad \{Z\} = \{z_s \ z_b \ z_p \ z_f \ z_r\}^T, \quad \{Q\} = \{q_f \ q_r\}^T. \tag{3.2}$$

3.2. Road Excitation

The unevenness degree of road profile can be generally described by power spectrum density. The international GB7031 recommends that the power spectrum density of road profile is described by

$$S_q(n) = S_q(n_0)\left(\frac{n}{n_0}\right)^{-w}, \tag{3.3}$$

where $S_q(n_0)$ is the unevenness coefficient of road profile. n_0 is the referenced spatial frequency, $n_0 = 0.1\,(m^{-1})$. n is the spatial frequency (m^{-1}). w is the frequency exponent of the graded road spectrum and generally chosen as 2. In this study, $S_q(n_0)$ is set as $S_q(n_0) = 64 \times 10^{-6}\,(m^2/m^{-1})$, that is, Grade B road condition.

When automobiles move in changeable speeds, the excitations of automobile systems are different in time domain and space domain. It is not stationary in space domain but in time domain. However, the automobile's mechanical responses are all nonstationary. By the inherent characteristics of frequency response function $H(w)$ of vehicle system in time domain and the relation of time frequency w and space frequency n, the transient frequency response function $H(s, n)$ can be obtained, then we can solve the stochastic vibration of the vehicle system in changeable speed moving [11, 23].

The unevenness degree of roads in time domain is shown as follows [24]:

$$q(t) = h_0 e^{jwt}, \tag{3.4}$$

where h_0 is the amplitude of unevenness degree of roads. The expression of unevenness degree of roads in space domain is as follows:

$$q = h_0 e^{j\Omega s}, \tag{3.5}$$

$$wt = \Omega v, \tag{3.6}$$

where Ω is the spatial angular frequency.

When the automobile is moving in a constant speed, $s = vt$, it has the following relation, $w = \Omega s$ or $f = nv$.

When the automobile is moving in a changeable speed, it has

$$s = v_0 + \frac{at^2}{2}, \tag{3.7}$$

where v_0 is the initial velocity of the automobile and a is its acceleration. Then (3.6) can be rewritten as

$$wdt = \Omega ds,$$

$$w = \Omega \frac{ds}{dt} = 2n\pi(v_0 + at) = 2n\pi\left(2as + v_0^2\right)^{1/2}. \tag{3.8}$$

Equation (3.8) reflects the time-space frequency relation of automobiles in an accelerated moving.

3.3. Pseudoexcitation of Random Road at Front and Rear Wheels

By the time-frequency expression of unevenness degree of roads, a pseudoexcitation of road $\tilde{q}(t)$ can be built which is corresponding with the road excitation $q(t)$, as follows:

$$\tilde{q}(t) = \sqrt{S_q(n)}e^{iwt}. \tag{3.9}$$

Hypothesize that the road excitations on four wheels of the automobile are the same and the delay relation [14] between front wheel excitation $q_f(t)$ and rear wheel excitation $q_r(t)$ is as follows:

$$q_r(t) = q_f(t - \tau), \tag{3.10}$$

where $\tau = l/v$, l is the distance between two wheel axles, then it has

$$\tilde{q}_f(t) = \sqrt{S_q(n)}e^{iwt},$$
$$\tilde{q}_r(t) = \tilde{q}_f(t - \tau) = \sqrt{S_q(n)}e^{iw(t-\tau)} = e^{-iwt\tau}\tilde{q}(t). \tag{3.11}$$

Therefore, the excitation input is written as follows:

$$\{\tilde{q}(t)\} = \begin{bmatrix} \tilde{q}_f(t) \\ \tilde{q}_r(t) \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-iw\tau} \end{bmatrix}\tilde{q}(t) = \{H_q(w)\}\tilde{q}(t). \tag{3.12}$$

The road excitations from front wheel and rear wheel can be simplified as an excitation input $\{\tilde{q}(t)\}$, and its frequency response characteristic is $\{H_q(w)\}$. Thus, the two-point excitations are simplified as one-point excitation.

3.4. Formulation of System Response

For a multiple degrees-of-freedom system, its frequency response characteristic is the complex number ratio of response vector and excitation vector. For the half-car, five-DOF vehicle system in this study, if we supposed that its frequency response is $[H(w)]$, then the relation of pseudoresponse and pseudoexcitation is

$$\{\tilde{z}(t)\} = [H(w)]\{\tilde{q}(t)\}. \tag{3.13}$$

Substituting (3.12) into (3.13) gives

$$\{\tilde{z}(t)\} = [H(w)]\{H_q(w)\}\tilde{q}(t) = \{h_g(w)\}\tilde{q}(t). \tag{3.14}$$

Since $\{h_g(w)\} = [H(w)]\{H_q(w)\}$, then

$$\{\dot{\tilde{z}}(t)\} = \{h_g(w)\}\dot{\tilde{q}}(t) = iw\{h_g(w)\}\tilde{q}(t), \tag{3.15}$$

$$\{\ddot{\tilde{z}}(t)\} = -w^2\{h_g(w)\}\tilde{q}(t). \tag{3.16}$$

Substitute (3.15) and (3.16) into the system equation, then the system frequency response function can be obtained as follows:

$$[H(w)] = \left[[K] - w^2[M] + iw[C]\right]^{-1}F, \tag{3.17}$$

where

$$[H(w)]_{5 \times 2} = \begin{bmatrix} H(w)_s \\ H(w)_b \\ H(w)_p \\ H(w)_f \\ H(w)_r \end{bmatrix}. \tag{3.18}$$

The frequency response functions of the relative displacement of suspension and the dynamic loads of tires are as follows, respectively:

$$H_{rd}(w) = \frac{z_b(w) - l_2 z_p(w) - z_f(w)}{q_f(w)},$$

$$H_{tf}(w) = \frac{z_f(w) - z_r(w)}{q_f(w)} k_{tf}. \tag{3.19}$$

By substituting (3.13) into (3.19), it gives the frequency response functions of the relative displacement of suspension and the dynamic loads of tires as follows:

$$H_{rd}(w) = H_b(w) - l_2 H_p(w) - H_f(w),$$

$$H_{tf}(w) = [H_f(w) - H_r(w)] k_{tf}. \tag{3.20}$$

It can be found that the system response result by pseudoexcitation method is the same as the result obtained by Fourier transform analysis. After obtaining the automobile structural parameters and the road excitation parameters, the pseudoexcitation responses $\{\tilde{z}(t)\}$ and $\{\ddot{\tilde{z}}(t)\}$ can be solved in accordance with (3.14) and (3.16). The response power spectrum can be achieved according to (2.5) and (2.6).

The power spectrum matrix of vertical acceleration of the system is

$$\{S_{\ddot{z}\ddot{z}}(w)\} = \{\ddot{\tilde{z}}(t)\}^* \cdot \{\ddot{\tilde{z}}(t)\}^T = w^4 \{h_g(w)\} \sqrt{S_q(n)} e^{-iwt} \{h_g(w)\}^T \sqrt{S_q(n)} e^{iwt}$$

$$= w^4 \{h_g(w)\} \{h_g(w)\}^T S_q(n) \tag{3.21}$$

$$= w^4 [H(w)] \{H_q(w)\} \{H_q(w)\}^T [H(w)]^T S_q(n).$$

By substituting $w = 2n\pi(2as + v_0{}^2)^{1/2}$ into (3.21), the spatial acceleration power spectrum density of system responses can be obtained as follows:

$$\{S_{\ddot{z}\ddot{z}}(s,n)\} = (2\pi n)^4 \left(2as + v_0{}^2\right)^2 [H(s,n)] \{H_q(s,n)\} \{H_q(s,n)\}^T [H(s,n)]^T S_q(n), \tag{3.22}$$
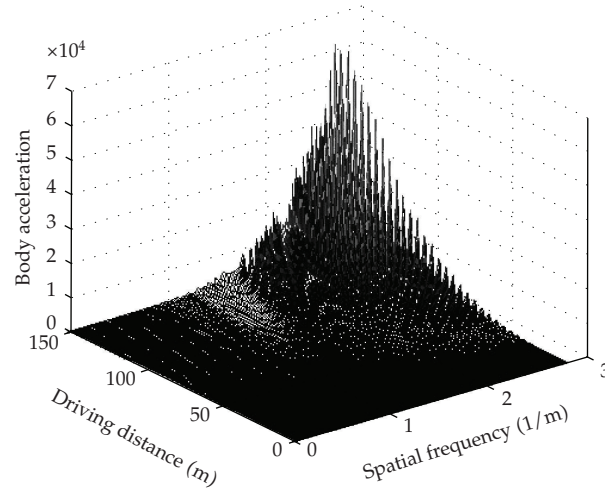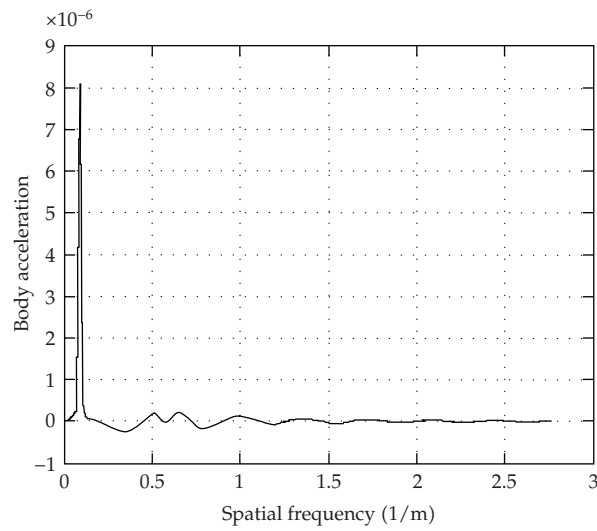
Figure 3: 3D spectrum of body acceleration.

Figure 4: 2D spectrum of body acceleration.

where

$$H(s,n) = \left[[K] - 4n^2\pi^2\left(2as + v_0{}^2\right)[M] + i\left(2n\pi\sqrt{2as + v_0{}^2}[C]\right)\right]^{-1} F,$$

$$\{H_q(s,n)\} = \begin{bmatrix} 1 \\ e^{-i\left(2n\pi\sqrt{2as+v_0{}^2}\tau\right)} \end{bmatrix} \sqrt{S_q(n)}\, e^{-i\left(2n\pi\sqrt{2as+v_0{}^2}t\right)}.$$

$$(3.23)$$

In the same way, according to the frequency response function of relative displacement of suspensions and dynamic loads of tires, we can obtain the transient spatial power spectrum
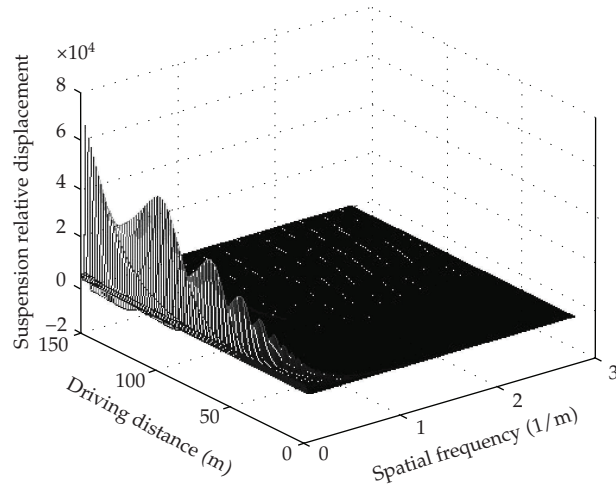
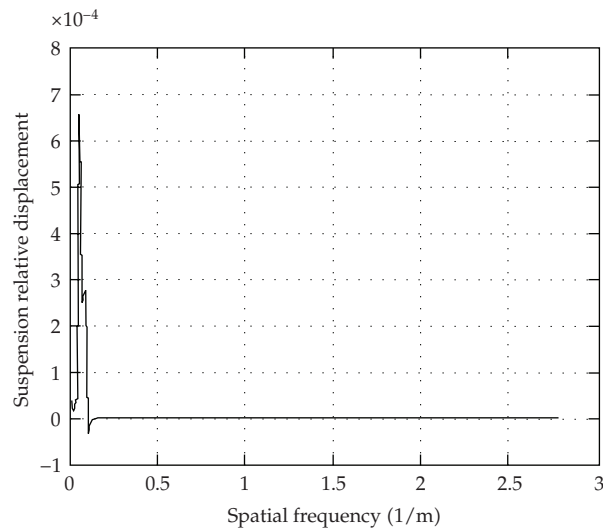**Figure 5:** 3D spectrum of relative displacement of vehicle suspension.

Figure 6: 2D spectrum of relative displacement of vehicle suspension.

density functions. It is important to substitute $w = 2n\pi(2as + v_0{}^2)^{1/2}$ into them to get the corresponding power spectrum density as follows:

$$
\begin{aligned}
S_{rd}(s,n) &= H_{rd}(s,n)S_q(n)[H_{rd}(s,n)]^T, \\
S_{tf}(s,n) &= H_{tf}(s,n)S_q(n)[H_{tf}(s,n)]^T.
\end{aligned}
\tag{3.24}
$$

By means of computing $\{S_{\ddot{z}\ddot{z}}(s,n)\}$, $S_{rd}(s,n)$, and $S_{tf}(s,n)$, we can obtain the dynamic characteristics of vehicles and driving comfort of vehicles.
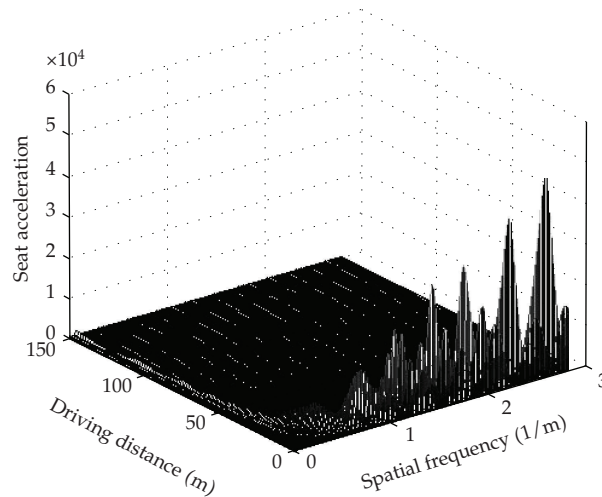
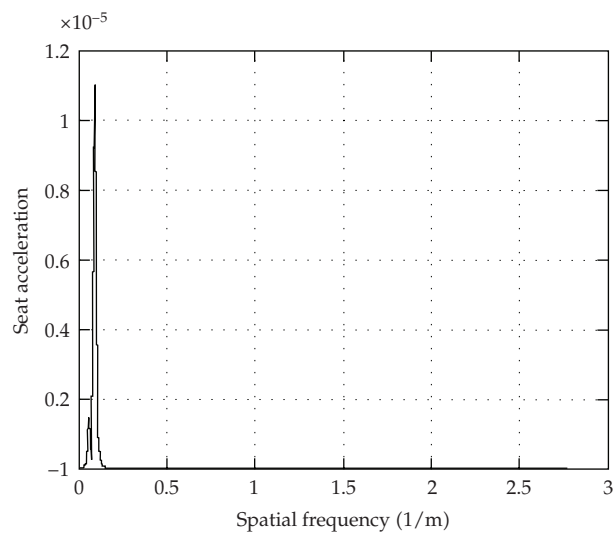**Figure 7:** 3D spectrum of seat acceleration.

Figure 8: 2D spectrum of seat acceleration.

4. Computing Case

In order to check the validation of the above mathematical models of transient response analysis of the half-car, five-DOF automobile system based on pseudoexcitation, we carried out the simulations as follows. To mimic an accelerating process of the automobile, set the acceleration as $a = 1.0$ ms and the running distance as $s = 150$ m, respectively. The other mechanical model parameters of the five-DOF automobile system are shown in Table 1.

Figures 3 and 4 show the 3D (three-dimensional) and 2D (two-dimensional) spectrums of body acceleration, respectively. Figures 5 and 6 show the 3D and 2D spectrums of relative displacement of vehicle suspension, respectively. Figures 7 and 8 show the 3D and 2D spectrums of seat acceleration, respectively. Figures 9 and 10 show the 3D and
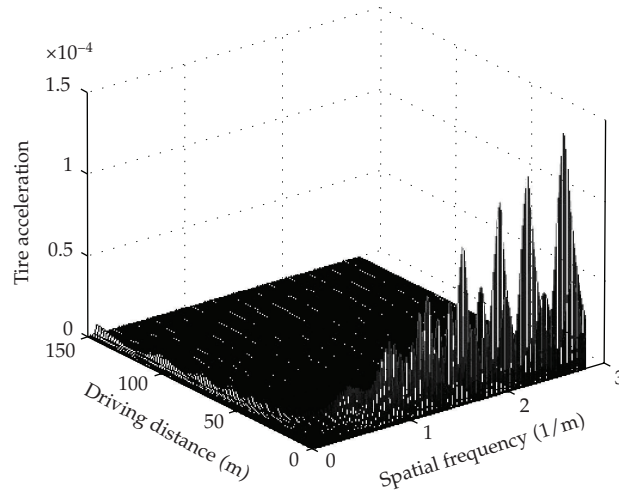
Figure 9: 3D spectrum of tire acceleration.

Table 1: Mechanical model parameters of the five-DOF automobile system.

Items	m_s(kg)	m_b(kg)	m_p(kg.m^2)	m_f(kg)	m_r(kg)
Values	70	2100	3500	140	210
Items	k_s	k_f	k_r	k_{tf}	k_{tr}
Values	12200	74000	120000	520000	520000
Items	c_s	c_f	c_r		
Values	550	1800	1200		

2D spectrums of tire acceleration, respectively. Figure 11 shows the 2D spectrum of tire dynamic load. The nonstationary response spectrum analysis of the vehicle system shows that the low-spatial frequency ingredient has a main role with vehicle speed increasing when the vehicle moves in a constant acceleration. With the speed increasing, the peak values of power spectrum of seat and body acceleration, as well as the low frequency values of power spectrum of dynamic tire loads, are not monotonously increasing, and some local values are decreasing. The changing of relative displacement of vehicle suspensions is not large. The nonlinearity of the vehicle suspension is not included in the current study, this is the limitation of this vehicle dynamic model, and it will be considered in the coming study.

5. Conclusion

By pseudoexcitation method, the vibration response characteristics of the half-car, five-DOF automobile system were obtained. The results show that the pseudoexcitation method is more convenient than the traditional method and effectively solves the smoothness computation of vehicles while the pseudoexcitation method is used to analyze vehicle vibration under nonstationary random vibration.
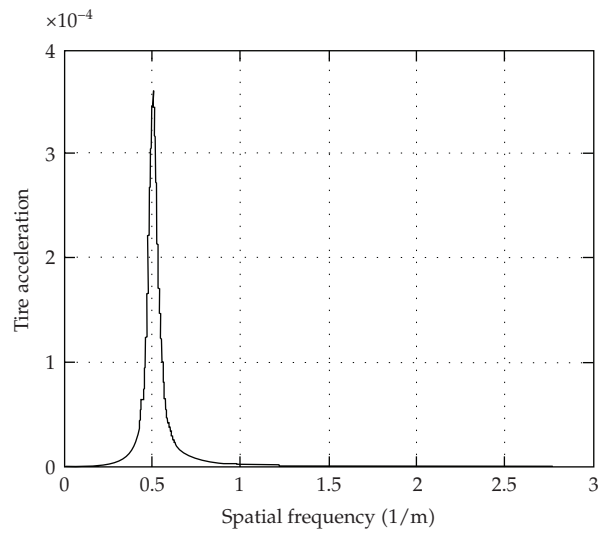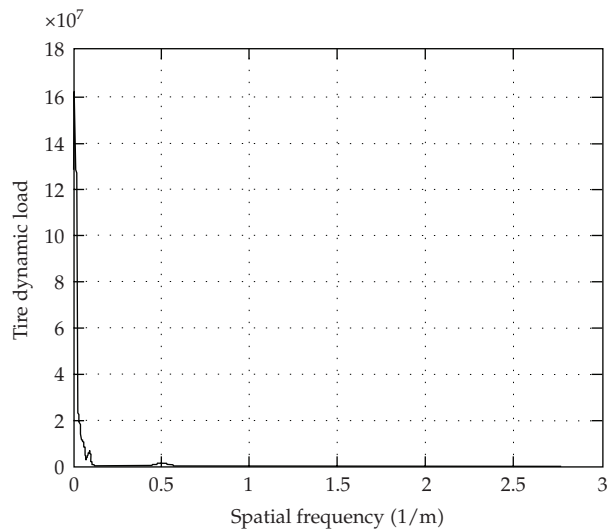
Figure 10: 2D spectrum of tire acceleration.

Figure 11: 2D spectrum of tire dynamic load.

Acknowledgments

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