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Research Article

Vibration Analysis of an Optical Fiber Coupler Using the Differential Quadrature Method

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This study elucidates the dynamic characteristics of an optical fiber coupler via the differential quadrature simulations. A novel modeling scheme is suitable for developing an optical fiber coupler application. Exactly how the locations of bonding points, string tension, and spring stiffness of the rubber pad affect the dynamic behavior of the optical fiber coupler is investigated.

1. Introduction

An optical fiber coupler is an optical device with several input fibers and several output fibers. The optical fibers in couplers may be under shock and impact. Cheng and Zu [1] and Sun [2] studied vibration of an optical fiber coupler subjected to a half-sine shock. Malomed and Tasgal [3] analyzed the dynamics of small internal vibrations in a two-component gap soliton. They found three oscillation modes, which are composed of dilation-contraction of each component's width, and a relative translation of the two components. Brown et al. [4] performed vibration tests on commercial grade fiber optic connectors and splices. Huang et al. [5] presented optical coupling loss and vibration characterization for packaging of 2×2 MEMS vertical torsion mirror switches. Thomes et al. [6] presented vibration performance of current fiber optic connector.

In this study the idea of differential quadrature formulation is extended to an optical fiber coupler. During the last decade, the differential quadrature approach applied to engineering and science problems has attracted considerable attention [7–31]. Liew et al. [7–16] applied the differential quadrature method to Mindlin plates on Winkler foundations and developed an application of the differential quadrature method to thick symmetric cross-ply laminates with first-order shear flexibility. Liew et al. [7–16] also employed the generalized differential quadrature method for buckling analysis and examined static and free vibration of beams and rectangular and annular plates. Sherbourne and Pandey [17] investigated the buckling behavior of beams and composite plates using the differential quadrature method. Mirfakhraei and Redekop [18] evaluated the buckling of circular cylindrical shells using the differential quadrature method. Tomasiello [19] applied the differential quadrature method to evaluate initial-boundary-value problems. Moradi and Taheri [20] conducted buckling analysis of general laminated composite beams using the differential quadrature method. De Rosa and Franciosi [21] solved the dynamic problem of circular arches using the differential quadrature method. Sun and Zhu [22] investigated incompressible viscous flow problems using the differential quadrature method. Via the differential quadrature method, Tanaka and Chen [23] solved transient elastodynamic problems. Chen and Zhong [24] observed that the differential quadrature and differential cubature methods, due to their global domain characteristics, are more efficient in solving nonlinear problems than conventional numerical schemes, such as the finite element and finite difference methods. Civan [25] solved multivariable mathematical models using the differential quadrature and differential cubature methods. Hua and Lam [26] identified the frequency characteristics of a thin rotating cylindrical shell using the differential quadrature method. Wang et al. [27–30] employed new versions of the differential quadrature and differential quadrature element methods to analyze anisotropic rectangular plates, frame structures, nonuniform beams, circular annular plates, and isotropic skew plates. The dynamic behavior of an optical fiber coupler is elucidated using the differential quadrature method in this work. Few studies have conducted vibration analysis of an optical fiber coupler using the differential quadrature method.

2. Differential Quadrature Method

Solutions to numerous complex beam problems have been efficiently acquired using fast computers and various numerical schemes, including the Galerkin technique, finite element method, boundary element method, and Rayleigh-Ritz method. In this study, the differential quadrature scheme is employed to generate discrete eigenvalue problems for an optical fiber coupler. The basic concept of the differential quadrature method is that the derivative of a function at a given point can be approximated as a weighted linear sum of functional values at all sample points in the domain of that variable. The partial differential equation is then reduced to a set of algebraic equations. For a function, $f(x)$, the differential quadrature approximation for the mth-order derivative at the ith sample point is given by

$$\begin{bmatrix} \dfrac{d^m f(x_1)}{dx^m} \\ \dfrac{d^m f(x_2)}{dx^m} \\ \vdots \\ \dfrac{d^m f(x_N)}{dx^m} \end{bmatrix} \cong \left[D_{ij}^{(m)}\right] \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix} \quad \text{for } i,j = 1,2,\ldots,N, \tag{2.1}$$

where $f(x_i)$ is the value of the function at sample point x_i, $D_{ij}^{(m)}$ is the weighted coefficient of the mth-order differentiation attached to these functional values, N is the number of sample points, and x_i is the location of the ith sample point in the domain. The most convenient technique is to distribute sample points uniformly [31]. A Lagrangian interpolation polynomial is utilized to eliminate possible adverse conditions when determining the weighted coefficients $D_{ij}^{(m)}$ [31], which are as follows:

$$f(x) = \frac{M(x)}{(x - x_i) M_1(x_i)} \quad \text{for } i = 1, 2, \ldots, N, \tag{2.2}$$

where

$$\begin{aligned} M(x) &= \prod_{j=1}^{N} (x - x_j), \\ M_1(x_i) &= \prod_{j=1, j \neq i}^{N} (x_i - x_j) \quad \text{for } i = 1, 2, \ldots, N. \end{aligned} \tag{2.3}$$

Inputting (2.2) into (2.1) yields

$$\begin{aligned} D_{ij}^{(1)} &= \frac{M_1(x_i)}{(x_i - x_j) M_1(x_j)} \quad \text{for } i, j = 1, 2, \ldots, N,\ i \neq j, \\ D_{ii}^{(1)} &= -\sum_{j=1, j \neq i}^{N} D_{ij}^{(1)} \quad \text{for } i = 1, 2, \ldots, N. \end{aligned} \tag{2.4}$$

The coefficients of the weighted matrix can be acquired using (2.4). For the mth-order derivative, the weighted coefficients can be obtained using the following recurrence relation equations:

$$D_{ij}^{(m)} = \sum_{k=1}^{N} D_{ik}^{(1)} D_{kj}^{(m-1)} \quad \text{for } i, j = 1, 2, \ldots, N. \tag{2.5}$$

The selection of sample points always is very important to solution accuracy when using the differential quadrature approach. For a beam problem, the most convenient technique is to choose equally spaced sample points [31]. Unequally spaced sample points, such as Chebyshev-Gauss-Lobatto sample points, have been utilized by a number of studies. With the Chebyshev-Gauss-Lobatto distribution, the sample points of an optical fiber coupler are distributed as

$$x_i = \frac{1}{2}\left(1 - \cos\frac{(i-1)\pi}{N-1}\right) \quad \text{for } i = 1, 2, \ldots, N. \tag{2.6}$$

3. Vibration of an Optical Fiber Coupler with a Continuous Elastic Support

Figure 1 shows a sectional view of the optical fiber coupler with a continuous elastic support. The fibers, the substrate, and the rubber pad are on the inside of the steel tube. The optical

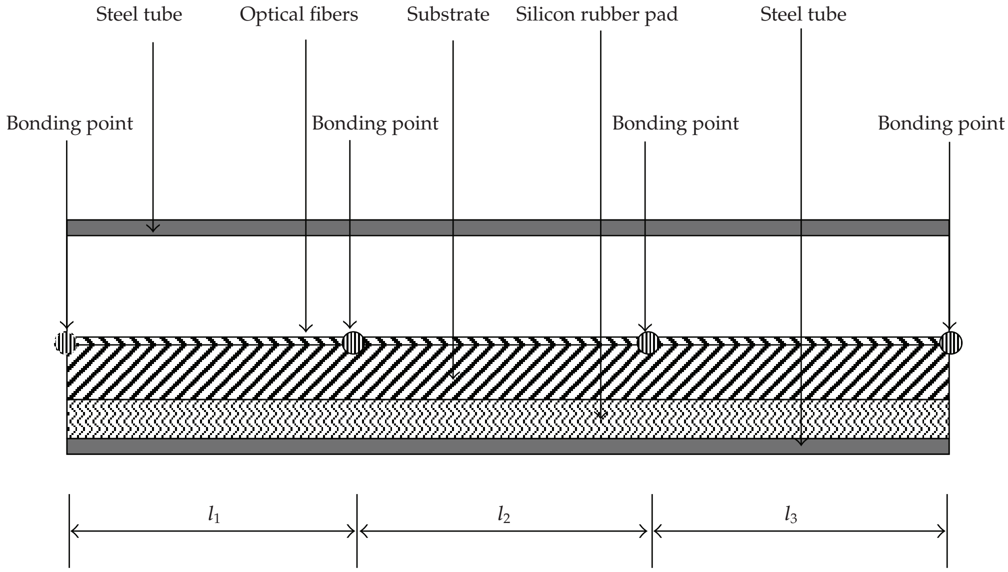

Figure 1: Sectional view of the optical fiber coupler with an elastic foundation [1, 2].

fibers are placed onto the substrate. A beam represents the substrate. The optical fibers model as a string. The rubber pad is placed between the substrate and steel tube. The linear model assumes that fiber tension is constant. The equation of motion for the optical fiber coupler is derived as [1, 2]

$$\begin{gathered}
P\frac{\partial^2 u_1}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_1}{\partial t^2} = 0 \quad \text{for } x : 0 \text{ to } l_1, \\
P\frac{\partial^2 u_2}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_2}{\partial t^2} = 0 \quad \text{for } x : l_1 \text{ to } l_1 + l_2, \\
P\frac{\partial^2 u_3}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_3}{\partial t^2} = 0 \quad \text{for } x : l_1 + l_2 \text{ to } l_1 + l_2 + l_3, \\
E_2 I_2 \frac{\partial^4 v_1}{\partial x^4} + k_f v_1 + \rho_2 A_2 \frac{\partial^2 v_1}{\partial t^2} = 0 \quad \text{for } x : 0 \text{ to } l_1, \\
E_2 I_2 \frac{\partial^4 v_2}{\partial x^4} + k_f v_2 + \rho_2 A_2 \frac{\partial^2 v_2}{\partial t^2} = 0 \quad \text{for } x : l_1 \text{ to } l_1 + l_2, \\
E_2 I_2 \frac{\partial^4 v_3}{\partial x^4} + k_f v_3 + \rho_2 A_2 \frac{\partial^2 v_3}{\partial t^2} = 0 \quad \text{for } x : l_1 + l_2 \text{ to } l_1 + l_2 + l_3,
\end{gathered} \tag{3.1}$$

where u_1, u_2, and u_3 are displacements of the fibers; v_1, v_2, and v_3 are displacements of the substrate; P is string tension; t is time; k_f is the constant determined by the material constants of the silicon rubber pad; A_1 is cross-section area of fibers; A_2 is cross-section area of substrate; ρ_1 is density of the fiber material; ρ_2 is density of the substrate material; I_2 is the second moment of the cross-sectional area A_2; E_2 is Young's modulus of the substrate. The optical fibers and substrate are bonded at four points. The boundary conditions of the optical fiber coupler are

$$
\begin{aligned}
&u_1(0,t) - v_1(0,t) = 0,\\
&u_1(l_1,t) - v_1(l_1,t) = 0,\\
&u_2(l_1,t) - v_2(l_1,t) = 0,\\
&u_2(l_1+l_2,t) - v_2(l_1+l_2,t) = 0,\\
&u_3(l_1+l_2,t) - v_3(l_1+l_2,t) = 0,\\
&u_3(l_1+l_2+l_3,t) - v_3(l_1+l_2+l_3,t) = 0,\\
&E_2I_2\frac{\partial^2 v_1(0,t)}{\partial x^2} = 0,\\
&E_2I_2\frac{\partial^3 v_1(0,t)}{\partial x^3} = 0,\\
&v_1(l_1,t) - v_2(l_1,t) = 0,\\
&\frac{\partial v_1(l_1,t)}{\partial x} - \frac{\partial v_2(l_1,t)}{\partial x} = 0,\\
&E_2I_2\frac{\partial^2 v_1(l_1,t)}{\partial x^2} - E_2I_2\frac{\partial^2 v_2(l_1,t)}{\partial x^2} = 0,\\
&E_2I_2\frac{\partial^3 v_1(l_1,t)}{\partial x^3} - E_2I_2\frac{\partial^3 v_2(l_1,t)}{\partial x^3} = 0,\\
&v_2(l_1+l_2,t) - v_3(l_1+l_2,t) = 0,\\
&\frac{\partial v_2(l_1+l_2,t)}{\partial x} - \frac{\partial v_3(l_1+l_2,t)}{\partial x} = 0,\\
&E_2I_2\frac{\partial^2 v_2(l_1+l_2,t)}{\partial x^2} - E_2I_2\frac{\partial^2 v_3(l_1+l_2,t)}{\partial x^2} = 0,\\
&E_2I_2\frac{\partial^3 v_2(l_1+l_2,t)}{\partial x^3} - E_2I_2\frac{\partial^3 v_3(l_1+l_2,t)}{\partial x^3} = 0,\\
&E_2I_2\frac{\partial^2 v_3(l_1+l_2+l_3,t)}{\partial x^2} = 0,\\
&E_2I_2\frac{\partial^3 v_3(l_1+l_2+l_3,t)}{\partial x^3} = 0.
\end{aligned}
\tag{3.2}
$$

To obtain frequencies, a harmonic movement of the optical fiber coupler is assumed as

$$
\begin{aligned}
u_s(x,t) &= \overline{u}_s(x)\cos(\omega t) \quad \text{for } s = 1,2,3,\\
v_s(x,t) &= \overline{v}_s(x)\cos(\omega t) \quad \text{for } s = 1,2,3,
\end{aligned}
\tag{3.3}
$$

where $\overline{u}_1(x)$, $\overline{u}_2(x)$, $\overline{u}_3(x)$, $\overline{v}_1(x)$, $\overline{v}_2(x)$, and $\overline{v}_3(x)$ are vibrational modes, and ω is the natural frequency of the optical fiber coupler. Substituting (3.3) into (3.1) yields

$$\begin{gathered} P\frac{d^2\overline{u}_s}{dx^2} = -\omega^2\rho_1 A_1\overline{u}_s \quad \text{for } s = 1,2,3, \\ E_2 I_2\frac{d^4\overline{v}_s}{dx^4} + k_f\overline{v}_s = \omega^2\rho_2 A_2\overline{v}_s \quad \text{for } s = 1,2,3. \end{gathered} \tag{3.4}$$

The boundary conditions of the optical fiber coupler are rewritten as

$$\begin{gathered} \overline{u}_1(0) - \overline{v}_1(0) = 0, \\ \overline{u}_1(l_1) - \overline{v}_1(l_1) = 0, \\ \overline{u}_2(l_1) - \overline{v}_2(l_1) = 0, \\ \overline{u}_2(l_1+l_2) - \overline{v}_2(l_1+l_2) = 0, \\ \overline{u}_3(l_1+l_2) - \overline{v}_3(l_1+l_2) = 0, \\ \overline{u}_3(l_1+l_2+l_3) - \overline{v}_3(l_1+l_2+l_3) = 0, \\ E_2 I_2\frac{d^2\overline{v}_1(0)}{dx^2} = 0, \\ E_2 I_2\frac{d^3\overline{v}_1(0)}{dx^3} = 0, \\ \overline{v}_1(l_1) - \overline{v}_2(l_1) = 0, \\ \frac{dv_1(l_1)}{dx} - \frac{dv_2(l_1)}{dx} = 0, \\ E_2 I_2\frac{d^2\overline{v}_1(l_1)}{dx^2} - E_2 I_2\frac{d^2\overline{v}_2(l_1)}{dx^2} = 0, \\ E_2 I_2\frac{d^3\overline{v}_1(l_1)}{dx^3} - E_2 I_2\frac{d^3\overline{v}_2(l_1)}{dx^3} = 0, \\ \overline{v}_2(l_1+l_2) - \overline{v}_3(l_1+l_2) = 0, \\ \frac{d\overline{v}_2(l_1+l_2)}{dx} - \frac{d\overline{v}_3(l_1+l_2)}{dx} = 0, \\ E_2 I_2\frac{d^2\overline{v}_2(l_1+l_2)}{dx^2} - E_2 I_2\frac{d^2\overline{v}_3(l_1+l_2)}{dx^2} = 0, \\ E_2 I_2\frac{d^3\overline{v}_2(l_1+l_2)}{dx^3} - E_2 I_2\frac{d^3\overline{v}_3(l_1+l_2)}{dx^3} = 0, \\ E_2 I_2\frac{d^2\overline{v}_3(l_1+l_2+l_3)}{dx^2} = 0, \\ E_2 I_2\frac{d^3\overline{v}_3(l_1+l_2+l_3)}{dx^3} = 0. \end{gathered} \tag{3.5}$$

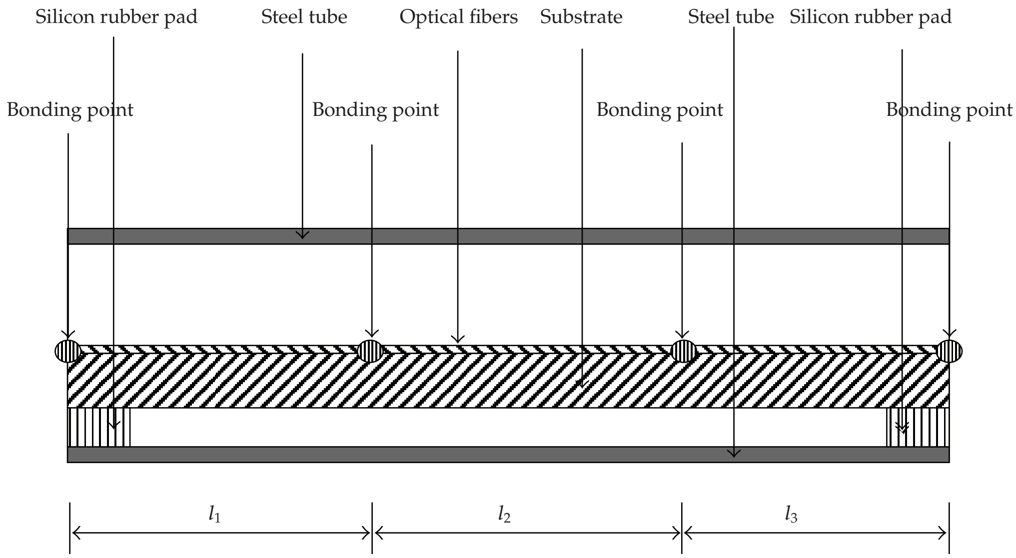

Figure 2: Sectional view of the optical fiber coupler with two spring supports [1, 2].

The equations of motion of the optical fiber coupler can be rearranged in the differential quadrature method formula by substituting (2.1) into (3.4) and (3.5). The equations of motion of the optical fiber coupler are derived as

$$\begin{gathered}
\sum_{j=1}^{N} \frac{PD_{i,j}^{(2)}}{l_s^2} \overline{u}_{s,j} = -\omega^2 \rho_1 A_1 \overline{u}_{s,i} \quad \text{for } i = 1, 2, \ldots, N, \; s = 1, 2, 3, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{i,j}^{(4)}}{l_s^4} \overline{v}_{s,j} + k_f \overline{v}_{s,i} = \omega^2 \rho_2 A_2 \overline{v}_{s,i} \quad \text{for } i = 1, 2, \ldots, N, \; s = 1, 2, 3.
\end{gathered} \tag{3.6}$$

Using the differential quadrature method, the boundary conditions of the optical fiber coupler can be rearranged into the matrix form as

$$\begin{gathered}
\overline{u}_{1,1} - \overline{v}_{1,1} = 0, \\
\overline{u}_{1,N} - \overline{v}_{1,N} = 0, \\
\overline{u}_{2,1} - \overline{v}_{2,1} = 0, \\
\overline{u}_{2,N} - \overline{v}_{2,N} = 0, \\
\overline{u}_{3,1} - \overline{v}_{3,1} = 0, \\
\overline{u}_{3,N} - \overline{v}_{3,N} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(2)}}{l_1^2} \overline{v}_{1,j} = 0,
\end{gathered}$$

$$\begin{gathered}
\sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(3)}}{l_1^3} \overline{v}_{1,j} = 0, \\
\overline{v}_{1,N} - \overline{v}_{2,1} = 0, \\
\sum_{j=1}^{N} \frac{D_{N,j}^{(1)}}{l_1} \overline{v}_{1,j} - \sum_{j=1}^{N} \frac{D_{1,j}^{(1)}}{l_2} \overline{v}_{2,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(2)}}{l_1^2} \overline{v}_{1,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(2)}}{l_2^2} \overline{v}_{2,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(3)}}{l_1^3} \overline{v}_{1,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(3)}}{l_2^3} \overline{v}_{2,j} = 0, \\
\overline{v}_{2,N} - \overline{v}_{3,1} = 0, \\
\sum_{j=1}^{N} \frac{D_{N,j}^{(1)}}{l_2} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{D_{1,j}^{(1)}}{l_3} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(2)}}{l_2^2} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(2)}}{l_3^2} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(3)}}{l_2^3} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(3)}}{l_3^3} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(2)}}{l_3^2} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(3)}}{l_3^3} \overline{v}_{3,j} = 0.
\end{gathered} \tag{3.7}$$

4. The Optical Fiber Coupler with Two Spring Supports

Figure 2 shows a sectional view of the optical fiber coupler with two rubber pads at each end of the coupler. The rubber pads are placed between the substrate and steel tube. The rubber pads model as the two spring supports. The equations of motion for the optical fiber coupler are [1, 2]

$$\begin{gathered}
P \frac{\partial^2 u_1}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_1}{\partial t^2} = 0 \quad \text{for } x : 0 \text{ to } l_1, \\
P \frac{\partial^2 u_2}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_2}{\partial t^2} = 0 \quad \text{for } x : l_1 \text{ to } l_1 + l_2,
\end{gathered}$$

$$
\begin{gathered}
P\frac{\partial^2 u_3}{\partial x^2} - \rho_1 A_1 \frac{\partial^2 u_3}{\partial t^2} = 0 \quad \text{for } x: l_1 + l_2 \text{ to } l_1 + l_2 + l_3, \\
E_2 I_2 \frac{\partial^4 v_1}{\partial x^4} + \rho_2 A_2 \frac{\partial^2 v_1}{\partial t^2} = 0 \quad \text{for } x: 0 \text{ to } l_1, \\
E_2 I_2 \frac{\partial^4 v_2}{\partial x^4} + \rho_2 A_2 \frac{\partial^2 v_2}{\partial t^2} = 0 \quad \text{for } x: l_1 \text{ to } l_1 + l_2, \\
E_2 I_2 \frac{\partial^4 v_3}{\partial x^4} + \rho_2 A_2 \frac{\partial^2 v_3}{\partial t^2} = 0 \quad \text{for } x: l_1 + l_2 \text{ to } l_1 + l_2 + l_3.
\end{gathered} \tag{4.1}
$$

The boundary conditions of the optical fiber coupler are

$$
\begin{gathered}
u_1(0,t) - v_1(0,t) = 0, \\
u_1(l_1,t) - v_1(l_1,t) = 0, \\
u_2(l_1,t) - v_2(l_1,t) = 0, \\
u_2(l_1+l_2,t) - v_2(l_1+l_2,t) = 0, \\
u_3(l_1+l_2,t) - v_3(l_1+l_2,t) = 0, \\
u_3(l_1+l_2+l_3,t) - v_3(l_1+l_2+l_3,t) = 0, \\
E_2 I_2 \frac{\partial^2 v_1(0,t)}{\partial x^2} = 0, \\
E_2 I_2 \frac{\partial^3 v_1(0,t)}{\partial x^3} = -k_{\text{spring}} v_1(0,t), \\
v_1(l_1,t) - v_2(l_1,t) = 0, \\
\frac{\partial v_1(l_1,t)}{\partial x} - \frac{\partial v_2(l_1,t)}{\partial x} = 0, \\
E_2 I_2 \frac{\partial^2 v_1(l_1,t)}{\partial x^2} - E_2 I_2 \frac{\partial^2 v_2(l_1,t)}{\partial x^2} = 0, \\
E_2 I_2 \frac{\partial^3 v_1(l_1,t)}{\partial x^3} - E_2 I_2 \frac{\partial^3 v_2(l_1,t)}{\partial x^3} = 0, \\
v_2(l_1+l_2,t) - v_3(l_1+l_2,t) = 0, \\
\frac{\partial v_2(l_1+l_2,t)}{\partial x} - \frac{\partial v_3(l_1+l_2,t)}{\partial x} = 0, \\
E_2 I_2 \frac{\partial^2 v_2(l_1+l_2,t)}{\partial x^2} - E_2 I_2 \frac{\partial^2 v_3(l_1+l_2,t)}{\partial x^2} = 0, \\
E_2 I_2 \frac{\partial^3 v_2(l_1+l_2,t)}{\partial x^3} - E_2 I_2 \frac{\partial^3 v_3(l_1+l_2,t)}{\partial x^3} = 0, \\
E_2 I_2 \frac{\partial^2 v_3(l_1+l_2+l_3,t)}{\partial x^2} = 0, \\
E_2 I_2 \frac{\partial^3 v_3(l_1+l_2+l_3,t)}{\partial x^3} = k_{\text{spring}} v_3(l_1+l_2+l_3,t),
\end{gathered} \tag{4.2}
$$

where k_{spring} is the constant determined by material constants of the spring support. Substituting (3.3) into (4) yields

$$
\begin{aligned}
P\frac{d^2\overline{u}_s}{dx^2} &= -\omega^2\rho_1 A_1\overline{u}_s \quad \text{for } s = 1,2,3,\\
E_2I_2\frac{d^4\overline{v}_s}{dx^4} &= \omega^2\rho_2 A_2\overline{v}_s \quad \text{for } s = 1,2,3.
\end{aligned}
\tag{4.3}
$$

The boundary conditions of the optical fiber coupler are rewritten as

$$
\begin{gathered}
\overline{u}_1(0) - \overline{v}_1(0) = 0,\\
\overline{u}_1(l_1) - \overline{v}_1(l_1) = 0,\\
\overline{u}_2(l_1) - \overline{v}_2(l_1) = 0,\\
\overline{u}_2(l_1 + l_2) - \overline{v}_2(l_1 + l_2) = 0,\\
\overline{u}_3(l_1 + l_2) - \overline{v}_3(l_1 + l_2) = 0,\\
\overline{u}_3(l_1 + l_2 + l_3) - \overline{v}_3(l_1 + l_2 + l_3) = 0,\\
E_2I_2\frac{d^2\overline{v}_1(0)}{dx^2} = 0,\\
E_2I_2\frac{d^3\overline{v}_1(0)}{dx^3} = -k_{\text{spring}}\overline{v}_1(0),\\
\overline{v}_1(l_1) - \overline{v}_2(l_1) = 0,\\
\frac{dv_1(l_1)}{dx} - \frac{dv_2(l_1)}{dx} = 0,\\
E_2I_2\frac{d^2\overline{v}_1(l_1)}{dx^2} - E_2I_2\frac{d^2\overline{v}_2(l_1)}{dx^2} = 0,\\
E_2I_2\frac{d^3\overline{v}_1(l_1)}{dx^3} - E_2I_2\frac{d^3\overline{v}_2(l_1)}{dx^3} = 0,\\
\overline{v}_2(l_1 + l_2) - \overline{v}_3(l_1 + l_2) = 0,\\
\frac{d\overline{v}_2(l_1 + l_2)}{dx} - \frac{d\overline{v}_3(l_1 + l_2)}{dx} = 0,\\
E_2I_2\frac{d^2\overline{v}_2(l_1 + l_2)}{dx^2} - E_2I_2\frac{d^2\overline{v}_3(l_1 + l_2)}{dx^2} = 0,\\
E_2I_2\frac{d^3\overline{v}_2(l_1 + l_2)}{dx^3} - E_2I_2\frac{d^3\overline{v}_3(l_1 + l_2)}{dx^3} = 0,\\
E_2I_2\frac{d^2\overline{v}_3(l_1 + l_2 + l_3)}{dx^2} = 0,\\
E_2I_2\frac{d^3\overline{v}_3(l_1 + l_2 + l_3)}{dx^3} = k_{\text{spring}}\overline{v}_3(l_1 + l_2 + l_3).
\end{gathered}
\tag{4.4}
$$

The equations of motion of the optical fiber coupler can be rearranged in the differential quadrature method formula by substituting (2.1) into (4.3). The equations of motion of the optical fiber coupler then become

$$\begin{aligned}
&\sum_{j=1}^{N}\frac{PD_{i,j}^{(2)}}{l_s^2}\overline{u}_{s,j} = -\omega^2\rho_1 A_1\overline{u}_{s,i} \quad \text{for } i = 1,2,\ldots,N,\ s = 1,2,3,\\
&\sum_{j=1}^{N}\frac{E_2I_2D_{i,j}^{(4)}}{l_s^4}\overline{v}_{s,j} = \omega^2\rho_2 A_2\overline{v}_{s,i} \quad \text{for } i = 1,2,\ldots,N,\ s = 1,2,3.
\end{aligned}\tag{4.5}$$

Using the differential quadrature method, the boundary conditions of the optical fiber coupler can be rearranged into the matrix form as

$$\begin{gathered}
\overline{u}_{1,1} - \overline{v}_{1,1} = 0,\\
\overline{u}_{1,N} - \overline{v}_{1,N} = 0,\\
\overline{u}_{2,1} - \overline{v}_{2,1} = 0,\\
\overline{u}_{2,N} - \overline{v}_{2,N} = 0,\\
\overline{u}_{3,1} - \overline{v}_{3,1} = 0,\\
\overline{u}_{3,N} - \overline{v}_{3,N} = 0,\\
\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(2)}}{l_1^2}\overline{v}_{1,j} = 0,\\
\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(3)}}{l_1^3}\overline{v}_{1,j} = -k_{\text{spring}}\overline{v}_{1,1},\\
\overline{v}_{1,N} - \overline{v}_{2,1} = 0,\\
\sum_{j=1}^{N}\frac{D_{N,j}^{(1)}}{l_1}\overline{v}_{1,j} - \sum_{j=1}^{N}\frac{D_{1,j}^{(1)}}{l_2}\overline{v}_{2,j} = 0,\\
\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(2)}}{l_1^2}\overline{v}_{1,j} - \sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(2)}}{l_2^2}\overline{v}_{2,j} = 0,\\
\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(3)}}{l_1^3}\overline{v}_{1,j} - \sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(3)}}{l_2^3}\overline{v}_{2,j} = 0,
\end{gathered}$$

$$
\begin{gathered}
\overline{v}_{2,N} - \overline{v}_{3,1} = 0, \\
\sum_{j=1}^{N} \frac{D_{N,j}^{(1)}}{l_2} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{D_{1,j}^{(1)}}{l_3} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(2)}}{l_2^2} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(2)}}{l_3^2} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(3)}}{l_2^3} \overline{v}_{2,j} - \sum_{j=1}^{N} \frac{E_2 I_2 D_{1,j}^{(3)}}{l_3^3} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(2)}}{l_3^2} \overline{v}_{3,j} = 0, \\
\sum_{j=1}^{N} \frac{E_2 I_2 D_{N,j}^{(3)}}{l_3^3} \overline{v}_{3,j} = k_{\text{spring}} \overline{v}_{3,N}.
\end{gathered}
\tag{4.6}
$$

5. Nonlinear Dynamic Analysis of an Optical Fiber Coupler with a Continuous Elastic Support

The nonlinear model assumes that fiber tension varies. The coupler is subjected to a sine shock motion. The equations of motion for the optical fiber coupler with a continuous elastic support are [1, 2]

$$
\begin{aligned}
&P \frac{\partial^2 u_1}{\partial x^2} + k_{\text{string}} \left(\int_0^{l_1} \sqrt{1 + \left(\frac{\partial u_1}{\partial x} \right)^2} dx + \int_{l_1}^{l_1+l_2} \sqrt{1 + \left(\frac{\partial u_2}{\partial x} \right)^2} dx \right. \\
&\qquad\qquad \left. + \int_{l_1+l_2}^{l_1+l_2+l_3} \sqrt{1 + \left(\frac{\partial u_3}{\partial x} \right)^2} dx - l_1 - l_2 - l_3 \right) \\
&- \rho_1 A_1 \frac{\partial^2 u_1}{\partial t^2} = \rho_1 A_1 a \ \sin(\Omega t) \quad \text{for } x : 0 \text{ to } l_1, \\
&P \frac{\partial^2 u_2}{\partial x^2} + k_{\text{string}} \left(\int_0^{l_1} \sqrt{1 + \left(\frac{\partial u_1}{\partial x} \right)^2} dx + \int_{l_1}^{l_1+l_2} \sqrt{1 + \left(\frac{\partial u_2}{\partial x} \right)^2} dx \right. \\
&\qquad\qquad \left. + \int_{l_1+l_2}^{l_1+l_2+l_3} \sqrt{1 + \left(\frac{\partial u_3}{\partial x} \right)^2} dx - l_1 - l_2 - l_3 \right) \\
&- \rho_1 A_1 \frac{\partial^2 u_2}{\partial t^2} = \rho_1 A_1 a \ \sin(\Omega t) \quad \text{for } x : l_1 \text{ to } l_1 + l_2,
\end{aligned}
$$

$$P\frac{\partial^2 u_3}{\partial x^2}+k_{\text{string}}\left(\int_0^{l_1}\sqrt{1+\left(\frac{\partial u_1}{\partial x}\right)^2}dx+\int_{l_1}^{l_1+l_2}\sqrt{1+\left(\frac{\partial u_2}{\partial x}\right)^2}dx\right.$$
$$\left.+\int_{l_1+l_2}^{l_1+l_2+l_3}\sqrt{1+\left(\frac{\partial u_3}{\partial x}\right)^2}dx-l_1-l_2-l_3\right)$$
$$-\rho_1 A_1\frac{\partial^2 u_3}{\partial t^2}=\rho_1 A_1 a\ \sin(\Omega t)\quad \text{for } x: l_1+l_2 \text{ to } l_1+l_2+l_3, \tag{5.1}$$

$$\begin{aligned}
&E_2 I_2\frac{\partial^4 v_1}{\partial x^4}+k_f v_1+\rho_2 A_2\frac{\partial^2 v_1}{\partial t^2}=-\rho_2 A_2 a\ \sin(\Omega t)\quad \text{for } x: 0 \text{ to } l_1,\\
&E_2 I_2\frac{\partial^4 v_2}{\partial x^4}+k_f v_2+\rho_2 A_2\frac{\partial^2 v_2}{\partial t^2}=-\rho_2 A_2 a\ \sin(\Omega t)\quad \text{for } x: l_1 \text{ to } l_1+l_2,\\
&E_2 I_2\frac{\partial^4 v_3}{\partial x^4}+k_f v_3+\rho_2 A_2\frac{\partial^2 v_3}{\partial t^2}=-\rho_2 A_2 a\ \sin(\Omega t)\quad \text{for } x: l_1+l_2 \text{ to } l_1+l_2+l_3,
\end{aligned} \tag{5.2}$$

where k_{string} is the elastic coefficient of the string, a is the acceleration of shock motion, and Ω is the circular frequency of shock motion. With the following approximation equation

$$\sqrt{1+\left(\frac{\partial u_s}{\partial x}\right)^2}\approx 1+\frac{1}{2}\left(\frac{\partial u_s}{\partial x}\right)^2\quad \text{for } s=1,2,3 \tag{5.3}$$

(5.1) can be rewritten as [1, 2]

$$\begin{aligned}
&P\frac{\partial^2 u_1}{\partial x^2}+\frac{k_{\text{string}}}{2}\left(\int_0^{l_1}\left(\frac{\partial u_1}{\partial x}\right)^2dx+\int_{l_1}^{l_1+l_2}\left(\frac{\partial u_2}{\partial x}\right)^2dx+\int_{l_1+l_2}^{l_1+l_2+l_3}\left(\frac{\partial u_3}{\partial x}\right)^2dx\right)-\rho_1 A_1\frac{\partial^2 u_1}{\partial t^2}\\
&\quad=\rho_1 A_1 a\ \sin(\Omega t)\quad \text{for } x: 0 \text{ to } l_1,\\
&P\frac{\partial^2 u_2}{\partial x^2}+\frac{k_{\text{string}}}{2}\left(\int_0^{l_1}\left(\frac{\partial u_1}{\partial x}\right)^2dx+\int_{l_1}^{l_1+l_2}\left(\frac{\partial u_2}{\partial x}\right)^2dx+\int_{l_1+l_2}^{l_1+l_2+l_3}\left(\frac{\partial u_3}{\partial x}\right)^2dx\right)-\rho_1 A_1\frac{\partial^2 u_2}{\partial t^2}\\
&\quad=\rho_1 A_1 a\ \sin(\Omega t)\quad \text{for } x: l_1 \text{ to } l_1+l_2,\\
&P\frac{\partial^2 u_3}{\partial x^2}+\frac{k_{\text{string}}}{2}\left(\int_0^{l_1}\left(\frac{\partial u_1}{\partial x}\right)^2dx+\int_{l_1}^{l_1+l_2}\left(\frac{\partial u_2}{\partial x}\right)^2dx+\int_{l_1+l_2}^{l_1+l_2+l_3}\left(\frac{\partial u_3}{\partial x}\right)^2dx\right)-\rho_1 A_1\frac{\partial^2 u_3}{\partial t^2}\\
&\quad=\rho_1 A_1 a\ \sin(\Omega t)\quad \text{for } x: l_1+l_2 \text{ to } l_1+l_2+l_3.
\end{aligned} \tag{5.4}$$

The boundary conditions of the optical fiber coupler are

$$
\begin{gathered}
u_1(0,t)-v_1(0,t)=0,\\
u_1(l_1,t)-v_1(l_1,t)=0,\\
u_2(l_1,t)-v_2(l_1,t)=0,\\
u_2(l_1+l_2,t)-v_2(l_1+l_2,t)=0,\\
u_3(l_1+l_2,t)-v_3(l_1+l_2,t)=0,\\
u_3(l_1+l_2+l_3,t)-v_3(l_1+l_2+l_3,t)=0,\\
E_2I_2\frac{\partial^2 v_1(0,t)}{\partial x^2}=0,\\
E_2I_2\frac{\partial^3 v_1(0,t)}{\partial x^3}=0,\\
v_1(l_1,t)-v_2(l_1,t)=0,\\
\frac{\partial v_1(l_1,t)}{\partial x}-\frac{\partial v_2(l_1,t)}{\partial x}=0,\\
E_2I_2\frac{\partial^2 v_1(l_1,t)}{\partial x^2}-E_2I_2\frac{\partial^2 v_2(l_1,t)}{\partial x^2}=0,\\
E_2I_2\frac{\partial^3 v_1(l_1,t)}{\partial x^3}-E_2I_2\frac{\partial^3 v_2(l_1,t)}{\partial x^3}=0,\\
v_2(l_1+l_2,t)-v_3(l_1+l_2,t)=0,\\
\frac{\partial v_2(l_1+l_2,t)}{\partial x}-\frac{\partial v_3(l_1+l_2,t)}{\partial x}=0,\\
E_2I_2\frac{\partial^2 v_2(l_1+l_2,t)}{\partial x^2}-E_2I_2\frac{\partial^2 v_3(l_1+l_2,t)}{\partial x^2}=0,\\
E_2I_2\frac{\partial^3 v_2(l_1+l_2,t)}{\partial x^3}-E_2I_2\frac{\partial^3 v_3(l_1+l_2,t)}{\partial x^3}=0,\\
E_2I_2\frac{\partial^2 v_3(l_1+l_2+l_3,t)}{\partial x^2}=0,\\
E_2I_2\frac{\partial^3 v_3(l_1+l_2+l_3,t)}{\partial x^3}=0.
\end{gathered}
\tag{5.5}
$$

The equation of motion of the optical fiber coupler can be rearranged in the differential quadrature method formula by substituting (2.1) into (5.2) and (5.4). The equations of motion of the optical fiber coupler are

$$\sum_{j=1}^{N}\frac{PD_{i,j}^{(2)}}{l_s^2}u_{s,j}+\frac{k_{\text{string}}}{2}\left(\int_0^{l_1}\left(\frac{\partial u_1}{\partial x}\right)^2dx+\int_{l_1}^{l_1+l_2}\left(\frac{\partial u_2}{\partial x}\right)^2dx+\int_{l_1+l_2}^{l_1+l_2+l_3}\left(\frac{\partial u_3}{\partial x}\right)^2dx\right)-\rho_1A_1\frac{\partial^2u_{s,i}}{\partial t^2}$$
$$=\rho_1A_1a\ \sin(\Omega t)\quad \text{for } i=1,2,\ldots,N,\ s=1,2,3,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{i,j}^{(4)}}{l_s^4}v_{s,j}+k_fv_{s,i}+\rho_2A_2\frac{\partial^2v_{s,i}}{\partial t^2}=-\rho_2A_2a\ \sin(\Omega t)\quad \text{for } i=1,2,\ldots,N,\ s=1,2,3. \tag{5.6}$$

Using the differential quadrature method, the boundary conditions of the optical fiber coupler can be rearranged into the matrix form as

$$u_{1,1}-v_{1,1}=0,$$
$$u_{1,N}-v_{1,N}=0,$$
$$u_{2,1}-v_{2,1}=0,$$
$$u_{2,N}-v_{2,N}=0,$$
$$u_{3,1}-v_{3,1}=0,$$
$$u_{3,N}-v_{3,N}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(2)}}{l_1^2}v_{1,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(3)}}{l_1^3}v_{1,j}=0,$$
$$v_{1,N}-v_{2,1}=0,$$
$$\sum_{j=1}^{N}\frac{D_{N,j}^{(1)}}{l_1}v_{1,j}-\sum_{j=1}^{N}\frac{D_{1,j}^{(1)}}{l_2}v_{2,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(2)}}{l_1^2}v_{1,j}-\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(2)}}{l_2^2}v_{2,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(3)}}{l_1^3}v_{1,j}-\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(3)}}{l_2^3}v_{2,j}=0,$$
$$v_{2,N}-v_{3,1}=0,$$

$$\sum_{j=1}^{N}\frac{D_{N,j}^{(1)}}{l_2}v_{2,j}-\sum_{j=1}^{N}\frac{D_{1,j}^{(1)}}{l_3}v_{3,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(2)}}{l_2^2}v_{2,j}-\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(2)}}{l_3^2}\overline{v}_{3,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(3)}}{l_2^3}v_{2,j}-\sum_{j=1}^{N}\frac{E_2I_2D_{1,j}^{(3)}}{l_3^3}v_{3,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(2)}}{l_3^2}v_{3,j}=0,$$
$$\sum_{j=1}^{N}\frac{E_2I_2D_{N,j}^{(3)}}{l_3^3}v_{3,j}=0. \tag{5.7}$$

6. Numerical Results and Discussion

Figure 3 presents the natural frequencies of the optical fiber coupler for various values of P. The material and geometric parameters of the optical fiber coupler are $k_f = 50000\,\mathrm{N/m^2}$, $A_1 = 3.1\times10^{-8}\,\mathrm{m^2}$, $A_2 = 6.61\times10^{-6}\,\mathrm{m^2}$, $\rho_1 = 2.2\times10^3\,\mathrm{kg/m^3}$, $\rho_2 = 2.2\times10^3\,\mathrm{kg/m^3}$, $I_2 = 4.34\times10^{-12}\,\mathrm{m^4}$, $l_1 = 0.1333\,\mathrm{m}$, $l_2 = 0.1333\,\mathrm{m}$, $l_3 = 0.1333\,\mathrm{m}$, and $E_2 = 7.24\times10^{10}\,\mathrm{N/m^2}$ [1, 2]. The first and second natural frequencies of the optical fiber coupler are robust to the string tension, P. The third and fourth natural frequencies of the optical fiber coupler increase as the string tension, P, increases. Figure 4 shows the natural frequencies of the optical fiber coupler for various values of k_f. The material and geometric parameters of the optical fiber coupler are $P = 0.01\,\mathrm{N}$, $A_1 = 3.1\times10^{-8}\,\mathrm{m^2}$, $A_2 = 6.61\times10^{-6}\,\mathrm{m^2}$, $\rho_1 = 2.2\times10^3\,\mathrm{kg/m^3}$, $\rho_2 = 2.2\times10^3\,\mathrm{kg/m^3}$, $I_2 = 4.34\times10^{-12}\,\mathrm{m^4}$, $l_1 = 0.1333\,\mathrm{m}$, $l_2 = 0.1333\,\mathrm{m}$, $l_3 = 0.1333\,\mathrm{m}$, and $E_2 = 7.24\times10^{10}\,\mathrm{N/m^2}$ [1, 2]. The first and third natural frequencies of the optical fiber coupler increase as the rubber pad stiffness increases. The rubber pad stiffness does not significantly affect the second and fourth natural frequencies of the optical fiber coupler. Figure 5 lists the natural frequencies of the optical fiber coupler with bonding points at various locations. The material and geometric parameters of the optical fiber coupler are $P = 0.01\,\mathrm{N}$, $k_f = 50000\,\mathrm{N/m^2}$, $A_1 = 3.1\times10^{-8}\,\mathrm{m^2}$, $A_2 = 6.61\times10^{-6}\,\mathrm{m^2}$, $\rho_1 = 2.2\times10^3\,\mathrm{kg/m^3}$, $\rho_2 = 2.2\times10^3\,\mathrm{kg/m^3}$, $I_2 = 4.34\times10^{-12}\,\mathrm{m^4}$, $l_1+l_2+l_3 = 0.4\,\mathrm{m}$, and $E_2 = 7.24\times10^{10}\,\mathrm{N/m^2}$ [1, 2]. The fourth natural frequency of the optical fiber coupler generally increases rapidly as lengths l_1 and l_3 increase. The locations of bonding points markedly impact the second and third natural frequencies of the optical fiber coupler. Figure 6 plots the natural frequencies of the optical fiber coupler for various values of k_{spring}. The material and geometric parameters of the optical fiber coupler are $P = 0.01\,\mathrm{N}$, $A_1 = 3.1\times10^{-8}\,\mathrm{m^2}$, $A_2 = 6.61\times10^{-6}\,\mathrm{m^2}$, $\rho_1 = 2.2\times10^3\,\mathrm{kg/m^3}$, $\rho_2 = 2.2\times10^3\,\mathrm{kg/m^3}$, $I_2 = 4.34\times10^{-12}\,\mathrm{m^4}$, $l_1 = 0.1333\,\mathrm{m}$, $l_2 = 0.1333\,\mathrm{m}$, $l_3 = 0.1333\,\mathrm{m}$, and $E_2 = 7.24\times10^{10}\,\mathrm{N/m^2}$ [1, 2]. The spring constant, k_{spring}, does not affect the first, third and fourth natural frequencies of the optical fiber coupler. Notably, the spring constant, k_{spring}, increases the second natural frequencies of the optical fiber coupler. Figures 7 and 8 show the displacements of the center of the fibers

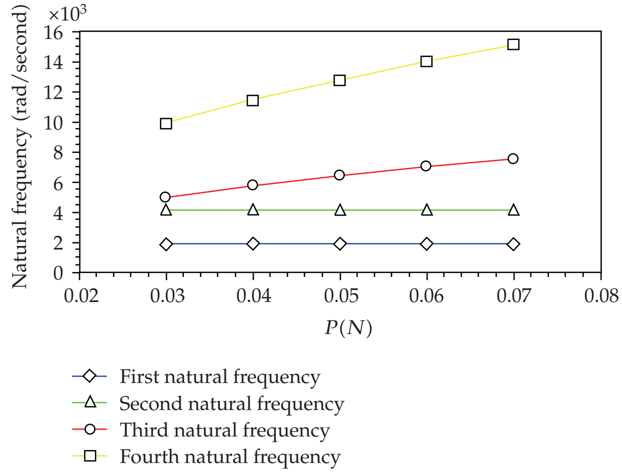

Figure 3: Natural frequencies of the optical fiber coupler for various values of P.

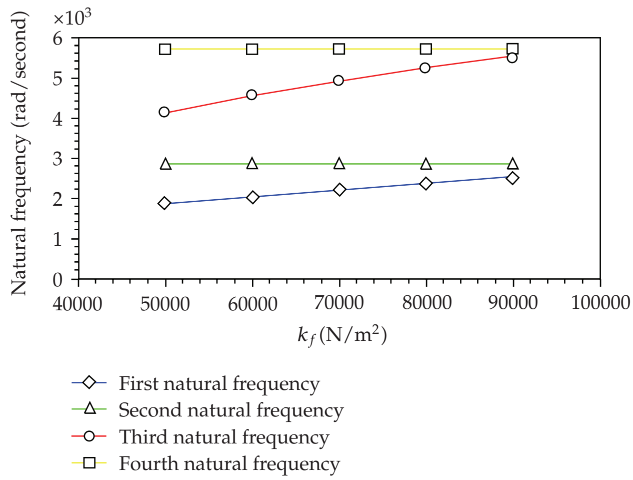

Figure 4: Natural frequencies of the optical fiber coupler for various values of k_f.

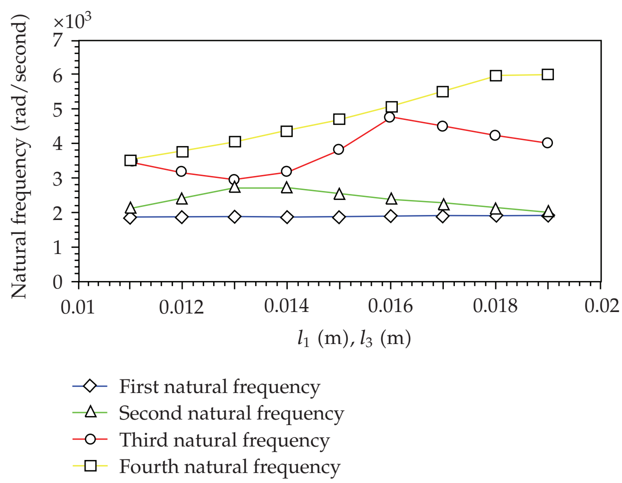

Figure 5: Natural frequencies of the optical fiber coupler with bonding points at various locations.

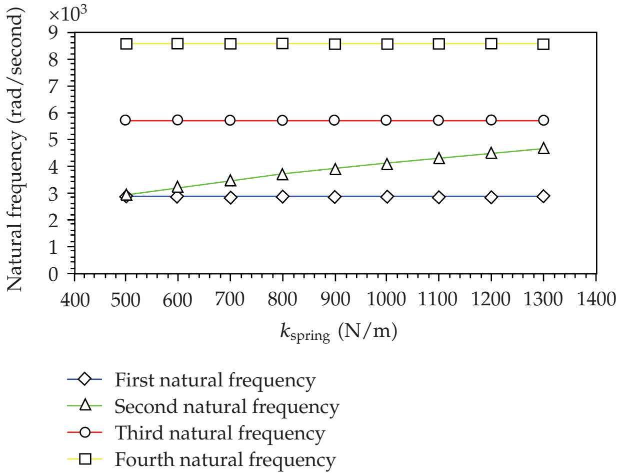

Figure 6: Natural frequencies of the optical fiber coupler for various values of k_{spring}.

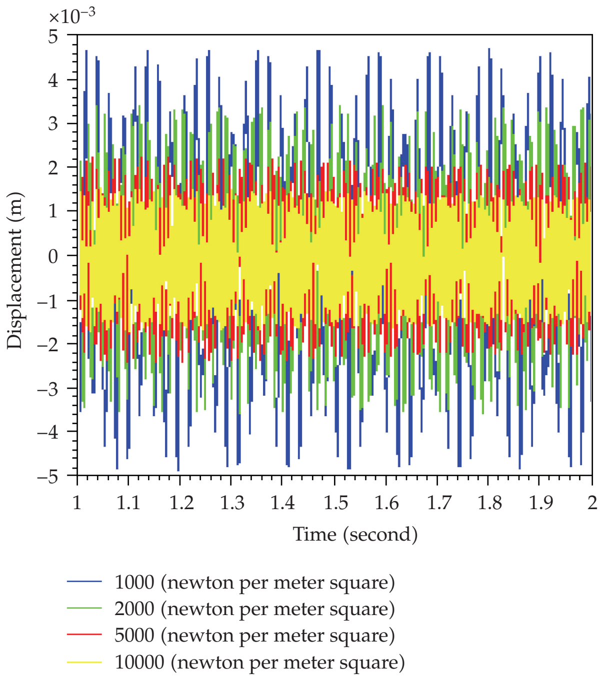

Figure 7: Displacements at the fiber center for various values of k_f.

and the substrate under a shock, respectively. The material and geometric parameters of the optical fiber coupler are $P = 0.01\,\text{N}$, $A_1 = 3.1\times10^{-8}\,\text{m}^2$, $A_2 = 6.61\times10^{-6}\,\text{m}^2$, $\rho_1 = 2.2\times10^3\,\text{kg/m}^3$, $\rho_2 = 2.2\times10^3\,\text{kg/m}^3$, $I_2 = 4.34\times10^{-12}\,\text{m}^4$, $l_1 = 0.1333\,\text{m}$, $l_2 = 0.1333\,\text{m}$, $l_3 = 0.1333\,\text{m}$, $E_2 = 7.24\times10^{10}\,\text{N/m}^2$, and $k_{string} = 5000\,\text{N/m}$ [1, 2]. The fibers and substrate stiffen when the foundation stiffness, k_f, is large. The differential quadrature method is effective in treating this problem.

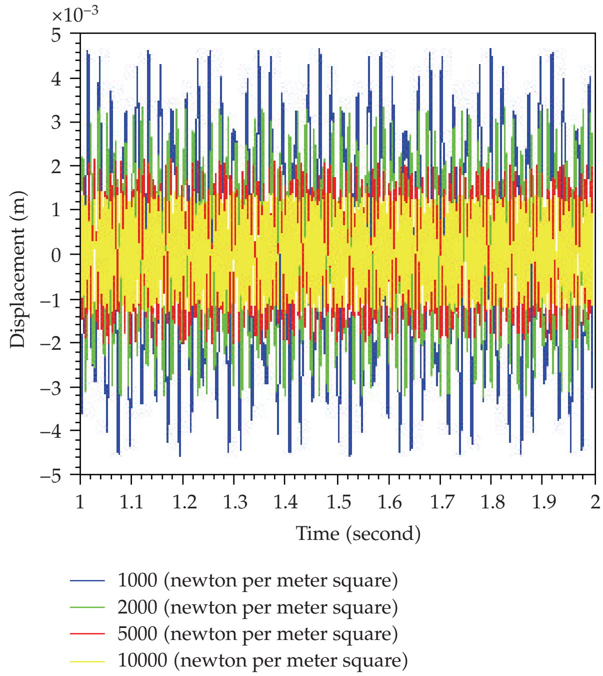

Figure 8: Displacements at the substrate center for various values of k_f.

7. Conclusions

This study demonstrates the value of the differential quadrature method for vibration analysis of an optical fiber coupler. The effects of string tension P, bonding locations, surrounding medium, spring constant k_{spring}, and rubber pad stiffness k_f on the natural frequencies of the optical fiber coupler are discussed. The effect of stiffness of the silicon rubber pad during vibrations is significant and should be incorporated into the designs of the optical fiber couplers.

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