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Research Article

On Series Solutions for MHD Plane and Axisymmetric Flow Near a Stagnation Point

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This investigation presents a mathematical model describing the momentum, heat and mass transfer characteristics of magnetohydrodynamic (MHD) flow and heat generating/absorbing fluid near a stagnation point of an isothermal two-dimensional body of an axisymmetric body. The fluid is electrically conducting in the presence of a uniform magnetic field. The series solution is obtained for the resulting coupled nonlinear differential equation. Homotopy analysis method (HAM) is utilized in obtaining the solution. Numerical values of the skin friction coefficient and the wall heat transfer coefficient are also computed.

1. Introduction

Stagnation point flows are classic problems in the theory of fluid dynamics. Pioneering works of Hiemenz [1] and Homann [2] for two-dimensional and axisymmetric three-dimensional stagnation point flows, respectively, have led to the extensive studies on such flows through various aspects. These flows subject to magnetic filed, and heat transfer characteristics have industrial applications, for instance, cooling of electronic devices by fans, heat exchangers design and MHD accelerators, and many others. In view of this motivation, Chamkha [3] studied the steady MHD flow and heat transfer of heat generating/absorbing viscous fluid at a stagnation point. Very recently, Abdelkhalek [4] discussed the steady forced convection MHD flow of heat generating/absorbing fluid by employing perturbation technique.

In the present paper, we developed the homotopy analysis solution for the problem considered in [3, 4]. The homotopy analysis method [5] is a powerful tool and has been already used for several nonlinear problems [6–18]. The governing partial differential equations are reduced into the ordinary differential equations. These ordinary differential equations are solved analytically. Some graphs depicting the variations of pertinent parameters are also shown and discussed.

2. Problem Statement

Here we consider the steady and MHD stagnation point flow impinging on a horizontal surface. The considered viscous fluid generates or absorbs heat at uniform rate. The X- and Y-axes are chosen along and normal to the plate. A uniform magnetic field is applied transversely to the flow. The induced magnetic field is negligible by choosing small magnetic Reynolds number. The governing equations are [3, 4, 19]

$$\frac{\partial}{\partial X}\left(X^{n-2}u\right)+\frac{\partial}{\partial Y}\left(X^{n-2}v\right)=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial X}+v\frac{\partial u}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\nu\left(\frac{\partial^2 u}{\partial X^2}+\frac{\partial}{\partial X}\left(\frac{u^{n-2}}{X^{n-2}}\right)+\frac{\partial^2 u}{\partial Y^2}\right)-\frac{\sigma B_0^2}{\rho}u, \tag{2.2}$$

$$u\frac{\partial v}{\partial X}+v\frac{\partial v}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\nu\left(X^{1-n}\frac{\partial}{\partial X}\left(X^{n-1}v\right)+\frac{\partial^2 v}{\partial Y^2}\right), \tag{2.3}$$

$$\rho C_P\left(u\frac{\partial T}{\partial X}+v\frac{\partial T}{\partial Y}\right)=K_e\left(X^{2-n}\frac{\partial}{\partial X}\left(X^{n-2}T\right)+\frac{\partial^2 T}{\partial Y^2}\right)+Q_0(T-T_w), \tag{2.4}$$

where u, v, P, and T are the velocity components, pressure, and temperature, respectively. ρ, ν, K_e, C_P and σ the fluid density, kinematic viscosity, thermal conductivity, specific heat at constant pressure and electrical conductivity, respectively. B_0, Q_0, T_w, and n are the respective magnetic induction, heat generation/absorption coefficient, wall temperature, and the dimensionality index such that $n = 2$ corresponding to plane flow and $n = 3$ corresponding to axisymmetric flow.

The boundary conditions for the problem under consideration are

$$u(X,0)=0,\qquad v(X,0)=-v_0,\qquad T(X,0)=T_w,\qquad u(X,\infty)=U_\infty,\qquad T(X,\infty)=T_\infty, \tag{2.5}$$

in which v_0 indicates the suction or injection velocity, and U_∞ and T_∞ are the free stream velocity and temperature, respectively.

Writing

$$\eta=Y\left(\frac{B}{\nu}\right)^{0.5},\qquad \psi=\frac{X^{n-1}}{n-1}(B\nu)^{0.5}f(\eta),\qquad \theta(\eta)=\frac{T-T_w}{T_\infty-T_w}, \tag{2.6}$$

$$u=\frac{BX}{n-1}f'(\eta),\qquad v=-(B\nu)^{0.5}f(\eta), \tag{2.7}$$

where the constant B is a sort of a velocity gradient parallel to the wall, and prime denotes ordinary differentiation with respect to η.

Invoking (2.6), equation (2.1) is identically satisfied, and (2.2)–(2.4) yield

$$f'''(\eta) + f(\eta)f''(\eta) + \frac{1}{n-1}\left(1 - f'^2(\eta)\right) - M^2 f'(\eta) = 0, \tag{2.8}$$

$$\theta''(\eta) + \Pr f(\eta)\theta'(\eta) + \Pr\, \alpha\, \theta(\eta) = 0, \tag{2.9}$$

where $M^2 = \sigma B_0^2(\rho B)^{-1}$, $\Pr = \mu C_P/K_e$ (μ is the dynamic viscosity of the fluid), and $\alpha = Q_0(\rho C_P B)^{-1}$ are the square of the Hartman number, the Prandtl number, and the dimensionless heat generation/absorption coefficient, respectively.

The boundary conditions (2.5) now become

$$f(0) = f_w, \qquad f'(0) = 0, \qquad f'(\infty) = 1, \qquad \theta(0) = 0, \qquad \theta(\infty) = 1, \tag{2.10}$$

in which f_w is the suction/injection parameter.

The expression of skin friction coefficient (C) and the wall heat transfer coefficient (H) are in the form

$$C = \frac{2\sqrt{n-1}\; f''(0)}{\sqrt{\mathrm{Re}_x}}, \tag{2.11}$$

$$H = \frac{\sqrt{n-1}\; \theta'(0)}{\Pr\sqrt{\mathrm{Re}_x}}. \tag{2.12}$$

In the above equations, $\mathrm{Re}_x = U_\infty X/\nu$ and $U_\infty = BX/(n-1)$ are the Reynolds number and the free stream velocity.

3. Solution by Homotopy Analysis Method (HAM)

According to equations (2.8) and (2.9) and the boundary conditions (2.10), solution can be expressed in the form

$$f(\eta) = a_0 + \eta + \sum_{m=1}^{+\infty}\sum_{q=0}^{\infty} a_{q,m}\eta^q e^{-m\eta}, \tag{3.1}$$

$$\theta(\eta) = 1 + \sum_{m=1}^{+\infty}\sum_{q=0}^{\infty} b_{q,m}\eta^q e^{-m\eta}, \tag{3.2}$$

where a_0, $a_{q,m}$, and $b_{q,m}$ are coefficients to be determined. According to the *rule of solution expression* denoted by (3.1) and (3.2) and the boundary conditions (2.10), it is natural to choose

$$\begin{aligned} f_0(\eta) &= f_w + \eta - (1 - e^{-\eta}), \\ \theta_0(\eta) &= (1 - e^{-\eta}) - \frac{1}{2}\eta e^{-\eta}, \end{aligned} \tag{3.3}$$

as the initial approximation to $f(\eta)$ and $\theta(\eta)$, respectively. We define an auxiliary linear operator $\mathcal{L}_1$ and $\mathcal{L}_2$ by

$$\mathcal{L}_1[\phi(\eta;p)] = \left(\frac{\partial^3}{\partial\eta^3} + \frac{\partial^2}{\partial\eta^2}\right)\phi(\eta;p),$$
$$\mathcal{L}_2[\psi(\eta;p)] = \left(\frac{\partial^2}{\partial\eta^2} - 1\right)\psi(\eta;p), \tag{3.4}$$

with the property

$$\mathcal{L}_1\left[C_1 + C_2\eta + C_3e^{-\eta}\right] = 0, \qquad \mathcal{L}_2\left[C_4e^{\eta} + C_5e^{-\eta}\right] = 0, \tag{3.5}$$

where C_i, $i = 1, 2, \ldots, 5$ are constants. This choice of $\mathcal{L}_1$ and $\mathcal{L}_2$ is motivated by (3.1) and (3.2), respectively, and from boundary conditions (2.10), we have $C_2 = C_4 = 0$.

From (2.8) and (2.9) we define nonlinear operators

$$\mathcal{N}_1[\phi(\eta;p), \psi(\eta;p)] = \frac{\partial^3\phi}{\partial\eta^3} + \phi\frac{\partial^2\phi}{\partial^2\eta} + \frac{1}{n-1}\left(1 - \left(\frac{\partial\phi}{\partial\eta}\right)^2\right) - M^2\frac{\partial\phi}{\partial\eta},$$
$$\mathcal{N}_2[\phi(\eta;p), \psi(\eta;p)] = \frac{\partial^2\psi}{\partial^2\eta} + \mathrm{Pr}\ \phi\ \frac{\partial\psi}{\partial\eta} + \mathrm{Pr}\ \alpha\ \psi, \tag{3.6}$$

and then construct the homotopy

$$\mathcal{H}_1[\phi(\eta;p), \psi(\eta;p)] = (1-p)\mathcal{L}_1[\phi(\eta;p) - f_0(\eta)] - \hbar_1 pH_1(\eta)\mathcal{N}_1[\phi(\eta;p), \psi(\eta;p)],$$
$$\mathcal{H}_2[\phi(\eta;p), \psi(\eta;p)] = (1-p)\mathcal{L}_2[\psi(\eta;p) - \theta_0(\eta)] - \hbar_2 pH_2(\eta)\mathcal{N}_2[\phi(\eta;p), \psi(\eta;p)], \tag{3.7}$$

where $\hbar_1 \neq 0$ and $\hbar_2 \neq 0$ are the convergence-control parameters [16], $H_1(\eta)$ and $H_2(\eta)$ are auxiliary functions. Setting $\mathcal{H}_i[\phi(\eta;p), \psi(\eta;p)] = 0$, for $i = 1, 2$, we have the zero-order deformation problems as follows:

$$(1-p)\mathcal{L}_1[\phi(\eta;p) - f_0(\eta)] = \hbar_1 pH_1(\eta)\mathcal{N}_1[\phi(\eta;p), \psi(\eta;p)], \tag{3.8}$$

$$(1-p)\mathcal{L}_2[\phi(\eta;p) - \theta_0(\eta)] = \hbar_2 pH_2(\eta)\mathcal{N}_2[\phi(\eta;p), \psi(\eta;p)], \tag{3.9}$$

subject to conditions

$$\phi(0;p) = f_w, \qquad \left.\frac{\partial\phi(\eta;p)}{\partial\eta}\right|_{\eta=0} = 0, \qquad \left.\frac{\partial\phi(\eta;p)}{\partial\eta}\right|_{\eta=\infty} = 1,$$
$$\psi(0;p) = 0, \qquad \psi(\infty;p) = 1, \tag{3.10}$$

where $p \in [0, 1]$ is an embedding parameter. When the parameter p increases from 0 to 1, the solution $\phi(\eta;p)$ varies from $f_0(\eta)$ to $f(\eta)$ and the solution $\psi(\eta;p)$ varies from $\theta_0(\eta)$ to $\theta(\eta)$.

If these continuous variation are smooth enough, the Maclaurin's series with respect to p can be constructed for $\phi(\eta;p)$ and $\psi(\eta;p)$, respectively, and further, if these series are convergent at $p=1$, we have

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{+\infty}f_m(\eta)=\sum_{m=0}^{+\infty}\phi_m(\eta,\hbar_1), \tag{3.11}$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{+\infty}\theta_m(\eta)=\sum_{m=0}^{+\infty}\psi_m(\eta,\hbar_2), \tag{3.12}$$

where

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m\phi(\eta;p)}{\partial p^m}\bigg|_{p=0}, \qquad \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\psi(\eta;p)}{\partial p^m}\bigg|_{p=0}. \tag{3.13}$$

Differentiating (3.8) and (3.9) and related conditions m times with respect to p, then setting $p=0$, and finally dividing by $m!$, we obtain the mth-order deformation problem:

$$\mathcal{L}_1\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=\hbar_1 H_1(\eta)R_{1,m}(\eta), \quad (m=1,2,3,\ldots), \tag{3.14}$$

$$\mathcal{L}_2\left[\theta_m(\eta)-\chi_m \theta_{m-1}(\eta)\right]=\hbar_2 H_2(\eta)R_{2,m}(\eta), \quad (m=1,2,3,\ldots), \tag{3.15}$$

subject to conditions

$$f_m(0)=0, \qquad f'_m(0)=0, \qquad f'_m(\infty)=0, \qquad \theta_m(0)=0, \qquad \theta_m(\infty)=0, \tag{3.16}$$

where $R_{1,m}(\eta)$ and $R_{2,m}(\eta)$ are defined as

$$\begin{gathered}R_{1,m}(\eta)=f'''_{m-1}+\sum_{i=0}^{m-1}f_i f''_{m-i-1}-\frac{1}{n-1}\sum_{i=0}^{m-1}f'_i f'_{m-i-1}-M^2 f'_{m-1}+\frac{1}{n-1}(1-\chi_m),\\ R_{2,m}(\eta)=\theta''_{m-1}+\Pr\sum_{i=0}^{m-1}f_i\theta'_{m-i-1}+\Pr\,\alpha\,\theta_{m-1},\end{gathered} \tag{3.17}$$

where prime denotes differentiation with respect to η and

$$\chi_m=\begin{cases}0, & m\le 1,\\ 1, & m>1.\end{cases} \tag{3.18}$$

The general solutions of (3.14) and (3.15) are

$$f_m(\eta) = \widehat{f}_m(\eta) + C_1 + C_2\eta + C_3 e^{-\eta},$$
$$\theta_m(\eta) = \widehat{\theta}_m(\eta) + C_4 e^{\eta} + C_5 e^{-\eta}, \tag{3.19}$$

where C_i for $i = 1, \ldots, 5$ are constants, $\widehat{f}_m(\eta)$ and $\widehat{\theta}_m(\eta)$ are particular solutions of (3.14) and (3.15), respectively.

According to the rule of solution expression denoted by (3.1) and (3.2), $C_2 = C_4 = 0$. The other unknowns are governed by

$$\widehat{f}_m(0) + C_1 + C_3 = 0, \qquad \widehat{f}'_m(0) - C_3 = 0, \qquad \widehat{\theta}_m(0) + C_5 = 0, \tag{3.20}$$

and according to our algorithm, the another boundary conditions are fulfilled. In this way, we derive $f_m(\eta)$ and $\theta_m(\eta)$ for $m = 1, 2, 3, \ldots$, successively.

For simplicity, here we take $H_1(\eta) = H_2(\eta) = H(\eta)$. According to the third rule of solution expression denoted by (3.11) and (3.12) and from (3.14) and (3.15), the auxiliary function $H(\tau)$ should be in the form

$$H(\eta) = e^{-\kappa\,\eta}, \tag{3.21}$$

where κ is an integer. To ensure that each coefficients $a_{q,m}$ in (3.1) and $b_{q,m}$ in (3.2) can be modified as the order of approximation tends to infinity, we set $\kappa = 1$.

At the Nth-order approximation, we have the analytic solution of (2.8) and (2.9), namely

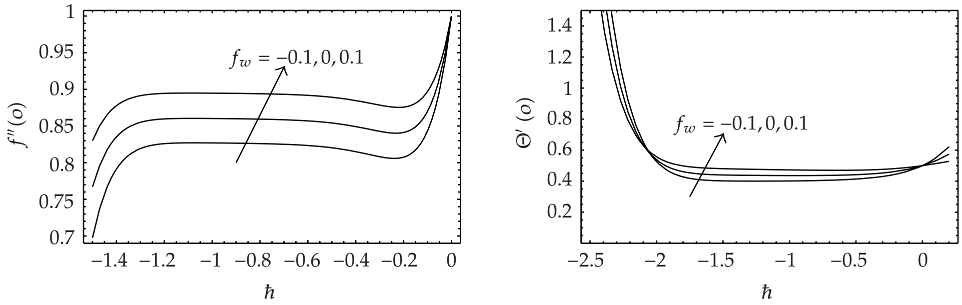
$$f(\eta) \approx F_N(\eta) = \sum_{i=0}^{N} f_i(\eta), \qquad \theta(\eta) \approx \Theta_N(\eta) = \sum_{i=0}^{N} \theta_i(\eta). \tag{3.22}$$

For simplicity, here we take $\hbar_1 = \hbar_2 = \hbar$. The auxiliary parameter $\hbar$ can be employed to adjust the convergence region of the series (3.22) in the homotopy analysis solution. By means of the so-called $\hbar$-curve, it is straightforward to choose an appropriate range for $\hbar$ which ensures the convergence of the solution series. As pointed out by Liao [5], the appropriate region for $\hbar$ is a horizontal line segment.

4. Numerical Results

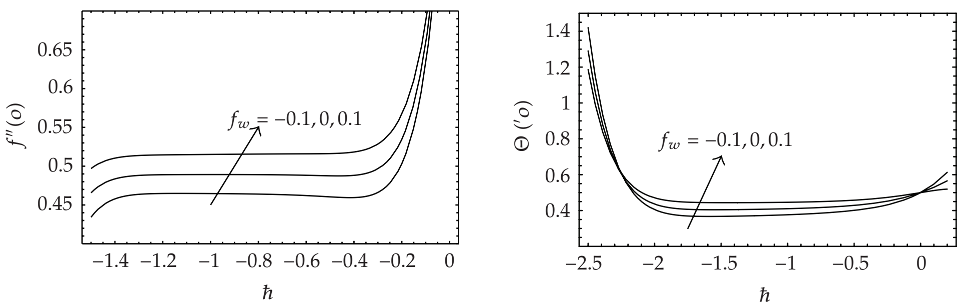
We use the widely applied symbolic computation software MATHEMATICA to solve (3.14) and (3.15) and find that $\phi_m(\eta, \hbar)$ and $\varphi_m(\eta, \hbar)$ have the following structure:

$$\phi_m(\eta, \hbar) = \sum_{i=0}^{\varphi(m)} \Phi_{m,i}(\eta, \hbar) \exp(-i\eta), \quad m \geq 0,$$
$$\varphi_m(\eta, \hbar) = \sum_{i=0}^{2m+1} \Psi_{m,i}(\eta, \hbar) \exp(-i\eta), \quad m \geq 0, \tag{4.1}$$



(a) (b)

Figure 1: The $\hbar$-curves for the 10th-order approximation and $n = 2$, $M = 1$, $\alpha = 0.1$, $\Pr = 0.7$.



(a) (b)

Figure 2: The $\hbar$-curves for the 10th-order approximation and $n = 3$, $M = 1$, $\alpha = 0.1$, $\Pr = 0.7$.

where

$$\varphi(m) = \begin{cases} 2m, & n = 2, \\ 2m + 1, & n = 3. \end{cases} \tag{4.2}$$

By means of the so-called $\hbar$-curve, it is straightforward to choose an appropriate range for $\hbar$ which ensures the convergence of the solution series. As pointed out by Liao [5], the appropriate region for $\hbar$ is a horizontal line segment. We can investigate the influence of $\hbar$ on the convergence of $f''(0)$ and $\theta'(0)$, by plotting the curve of it versus $\hbar$, as shown in Figure 1 for some examples in plane flow ($n = 2$), respectively. Also, Figure 2 shows the $\hbar$-curve in axisymmetric flow ($n = 3$). By considering the $\hbar$-curve we can obtain the reasonable interval for $\hbar$ in each case.

Also, by computing the error of norm 2 for two successive approximation of $F_N(\eta)$ or $\Theta_N(\eta)$, in each case, we can obtain the best value for $\hbar$ in each case. Figure 3 shows this error for $F_{10}(\eta)$ with $\alpha = 0.1$, $M = 1$ and $\Pr = 0.7$ in axisymmetric flow and $f_w = -0.1$ and 0.1 for $\eta \in [0, 10]$. One can compute easily that, in case $f_w = -0.1$, we have $\hbar = -1.056$, and for $f_w = 0.1$, $\hbar = -1.095$ and these values are match with $\hbar$-curve (in Figure 2).

Figure 4 presents representative profiles for the normal velocity f of both plane and axisymmetric flows for various values of Hartman number M and in each case the value of $\hbar$ computed by rule of minimizing the error of norm 2. Figures 5 and 6 show the respective

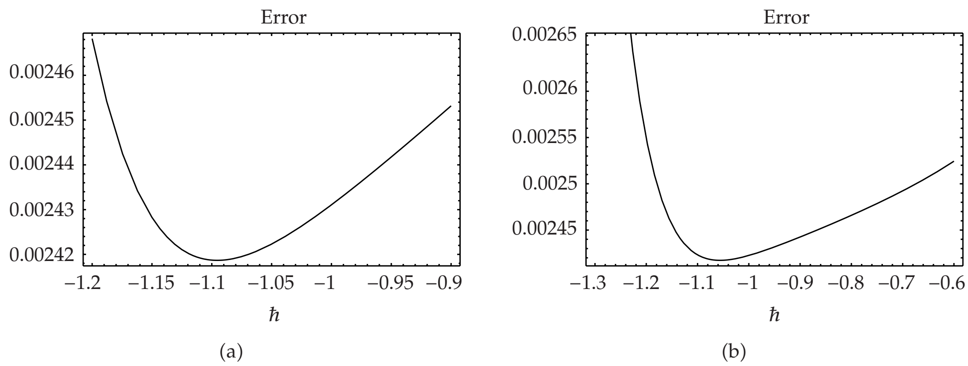


Figure 3: The norm 2 error of $F_{10}(\eta)$ versus $\hbar$; (a): $f_w = 0.1$, (b): $f_w = -0.1$.

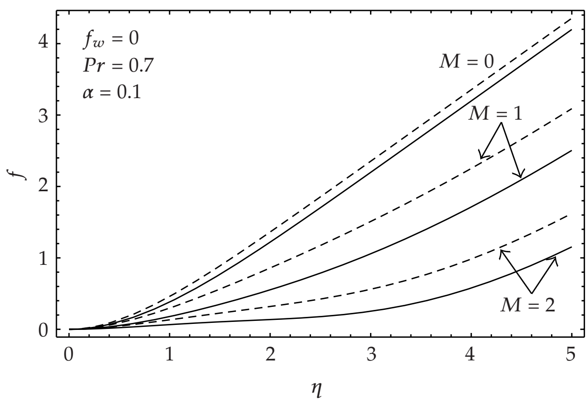


Figure 4: Effect of M on normal velocity profiles in 10th-order approximation.

effects of the Prandtl number Pr and the heat generation/absorption coefficient α on the temperature profiles for both plane and axisymmetric stagnation point flows. As pointed by Chamkha [3], for heat-generation case ($\alpha = 0.1$) in Figure 6, a sharp peak exists in the layer close the wall.

The so-called homotopy-Padé technique (see [5]) is employed, which greatly accelerates the convergence. The $[r, s]$ homotopy-Padé approximations of $f''(0)$, or C in (2.11), and $\theta'(0)$, or H in (2.12), according to (3.11) and (3.12) are formulated by

$$\frac{\sum_{k=0}^{r} \phi_k''(0, \hbar)}{1 + \sum_{k=1}^{s} \phi_{r+k+1}''(0, \hbar)}, \qquad \frac{\sum_{k=0}^{r} \varphi_k'(0, \hbar)}{1 + \sum_{k=1}^{s} \varphi_{r+k+1}'(0, \hbar)}, \tag{4.3}$$

respectively. In many cases, the $[r, r]$ homotopy-Padé approximation does not depend upon the auxiliary parameter $\hbar$. To verify the accuracy of HAM, a comparison of wall heat transfer coefficient $C_h = H\sqrt{\mathrm{Re}_x}$ with those reported by White [19], Chamkha [3], and Abdelkhalek [4] is given in Table 1 for $M = 0$ and $\alpha = 0$. The values of $C_f = C\sqrt{\mathrm{Re}_x}$

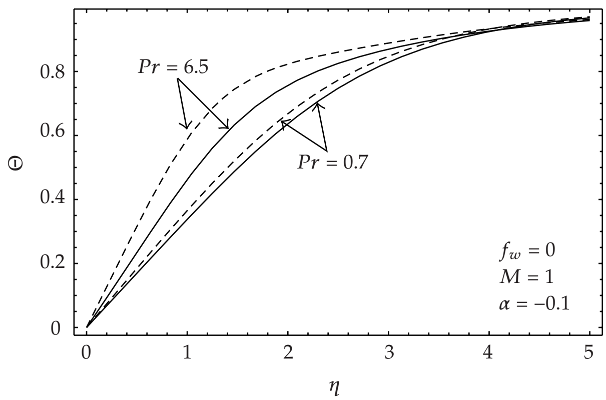


Figure 5: Effect of Pr on temperature profiles in 10th-order approximation.

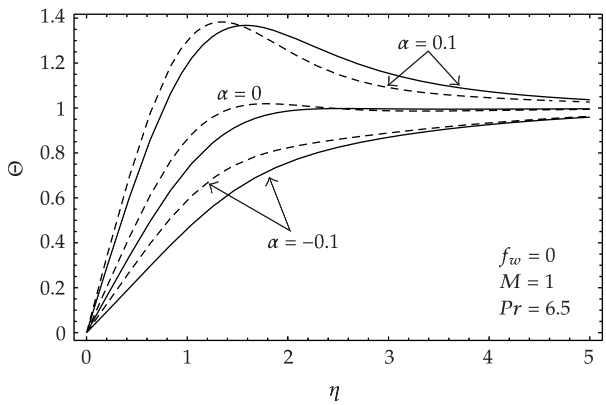


Figure 6: Effect of α on temperature profiles in 10th-order approximation.

also compare well since the obtained values for $n = 2$ and $n = 3$ by [15] Homotopy-Padé method are 2.4652 and 2.6239, while the values reported by Chamkha [3] are 2.4695 and 2.6240 and based on White's correlations $C_f \approx 2\mathrm{Pr}^{2/3} C_h$ are 2.4782 and 2.6275, respectively.

5. Final Remarks

Homotopy analysis method is employed to analyze the MHD flow near a stagnation point. The resulting nonlinear differential system is solved analytically. The effects of Hartman number, the Prandtl number and the heat generation/absorption coefficient are seen on the normal component of velocity and temperatures respectively in both plane and axisymmetric stagnation point cases. It is noticed that temperature profiles increase by increasing the heat generation/absorption coefficient. The behavior of Prandtl number on the temperature profile is similar to that of heat generation/absorption coefficient in a qualitative sense.

Table 1: Results for [15] Homotopy-Padé approach for C_h.

	$n = 2$				$n = 3$			
Pr	[19]	[3]	[4]	HAM	[19]	[3]	[4]	HAM
0.7	0.7060	0.7080	0.7054	0.70812	0.9438	0.9507	0.95421	0.95036
1.0	0.5700	0.5705	0.57235	0.56963	0.7620	0.7624	0.76421	0.76154
10.0	0.1432	0.1339	0.1446	0.13377	0.1914	0.1752	0.1925	0.13868
100.0	0.0360	0.0299	0.0381	0.02477	0.0481	0.0387	0.04923	0.02767

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