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Research Article

Switching Signal Design for Global Exponential Stability of Uncertain Switched Neutral Systems

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The switching signal design for global exponential stability of switched neutral systems is investigated in this paper. LMI-based delay-dependent and delay-independent criteria are proposed to guarantee the global stability via the constructed switching signal. Razumikhin-like approach is used to find the stability results. Finally, some numerical examples are illustrated to show the main results.

1. Introduction

It is well known that the existence of delay in a system may cause instability or bad system performance in control systems. Time-delay phenomenon appears in many practical systems, such as AIDS epidemic, aircraft stabilization, chemical engineering systems, inferred grinding model, manual control, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines. Hence stability analysis for time-delay systems has been considered in the recent years [1–3]. Neutral systems are described by functional differential equations which depend on the delays of state and state derivative. Some practical examples of neutral systems include distributed networks, heat exchanges, and processes including steam [4].

Switched system is a class of hybrid systems which is consisting of several subsystems and uses the switching signal to specify which subsystem is activated to the system trajectories at each instant of time. Some examples for switched systems are automated highway systems, constrained robotics, power systems and power electronics, transmission and stepper motors [5]. Stability analysis of switched time-delay systems has been an attractive research topic [6–13]. It is interesting to note that the stability for each subsystem cannot imply that of the overall system under arbitrary switching signal [9]. Another interesting fact is that the stability of a switched system can be achieved by choosing the switching signal even when each subsystem is unstable [6, 7, 10]. In this paper, the switching signal design will be considered for uncertain switched neutral systems with mixed delays. The switching signal will be proposed to guarantee the stability of switched system even when each subsystem is unstable. Based on Razumikhin-like approach [11], delay-dependent and delay-independent results are provided. New and flexible LMI conditions are proposed to design the switching signal which guarantees the global exponential and asymptotic stability of uncertain switched neutral systems. Some numerical examples are provided to demonstrate the use of our results.

The notation used throughout this paper is as follows. For a matrix A, we denote the transpose by A^T, spectral norm by $\|A\|$, symmetric positive (negative) definite by $A > 0$ ($A < 0$), maximal eigenvalue by $\lambda_{\max}(A)$, and minimal eigenvalue by $\lambda_{\min}(A)$. $A \le B$ means that matrix $B - A$ is symmetric positive semidefinite. For two sets X and Y, $X - Y$ means that the set of all points in X that are not in Y. For a vector x, we denote the Euclidean norm by $\|x\|$ and $\|x_t\|_s = \sup_{-H\le\theta\le 0}\sqrt{\|x(t+\theta)\|^2 + \|\dot{x}(t+\theta)\|^2}$. I denotes the identity matrix. $\Re^n$ denotes n-dimensional real space.

2. Problem Formulations and Main Results

Consider the following switched neutral system with mixed time delays:

$$\begin{aligned}\dot{x}(t) - D\dot{x}(t-\tau) &= A_{0\sigma}x(t) + A_{1\sigma}x(t-h(t)), \quad t \ge 0,\\ x(t) &= \phi(t), \quad t \in [-H, 0],\end{aligned} \tag{2.1}$$

where $x \in \Re^n$, x_t is state at time t defined by $x_t(\theta) := x(t+\theta)$, $\forall\theta \in [-H, 0]$, σ is a switching signal which is a piecewise constant function and may depend on t or x, σ, taking its values in the finite set $\{1, 2, \ldots, N\}$, and time-varying delay satisfies $0 \le h(t) \le h_M$, $\dot{h}(t) \le h_D$, $h_M > 0$, $\tau > 0$, $H = \max\{h_M, \tau\}$. Matrices D, A_{0i}, and $A_{1i} \in \Re^{n\times n}$, $i = 1, 2, \ldots, N$, are constant, and the initial vector $\phi \in C_1$, where C_1 is the set of differentiable functions from $[-H, 0]$ to $\Re^n$.

Now we define some functions $\lambda_i(t)$, $i = 1, 2, \ldots, N$, that will be used to represent our system:

$$\lambda_i(t) = \begin{cases} 1, & \sigma = i,\\ 0, & \text{otherwise,}\end{cases} \quad i = 1, 2, \ldots, N. \tag{2.2}$$

The switched system in (2.1) can be rewritten as follows:

$$\begin{aligned}\dot{x}(t) - D\dot{x}(t-\tau) &= \sum_{i=1}^{N}\lambda_i(t)\{A_{0i}x(t) + A_{1i}x(t-h(t))\}, \quad t \ge 0,\\ x(t) &= \phi(t), \quad t \in [-H, 0],\end{aligned} \tag{2.3}$$

where $\lambda_i(t)$ is defined in (2.2) and $\sum_{i=1}^{N}\lambda_i(t) = 1$, $\forall t \ge 0$.

Lemma 2.1 (see [14]). *Let U, V, W, and M be real matrices of appropriate dimensions with M satisfying $M = M^T$, then*

$$M + UVW + W^T V^T U^T < 0 \quad \forall V^T V \leq I, \tag{2.4}$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} \cdot UU^T + \varepsilon \cdot W^T W = M + \varepsilon^{-1} \cdot UU^T + \varepsilon^{-1} \cdot (\varepsilon W)^T (\varepsilon W) < 0. \tag{2.5}$$

Lemma 2.2 (Schur complement of [15]). *For a given matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ with $S_{11} = S_{11}^T$, $S_{22} = S_{22}^T$, the following conditions are equivalent:*

(1) $S < 0$,

(2) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Assumption 2.3. Assume that there exists a convex combination $F = \sum_{i=1}^{N} \alpha_i A_{0i}$ such that F is Hurwitz, where $0 \leq \alpha_i \leq 1$ and $\sum_{i=1}^{N} \alpha_i = 1$.

Since F is Hurwitz, there exist positive definite matrices P and Q satisfying

$$F^T P = PF \leq -Q. \tag{2.6}$$

Define some domains

$$\Omega_i = \left\{ x \in \mathfrak{R}^n : x^T \left({A_{0i}}^T P + P A_{0i} \right) x \leq -x^T Q x \right\}, \quad i \in \{1, 2, \ldots, N\}. \tag{2.7}$$

From the similar proof of [7], it is easy to show $\bigcup_{i=1}^{N} \Omega_i = \mathfrak{R}^n$. Construct some domains

$$\overline{\Omega}_1 = \Omega_1, \overline{\Omega}_2 = \Omega_2 - \overline{\Omega}_1, \overline{\Omega}_3 = \Omega_3 - \overline{\Omega}_1 - \overline{\Omega}_2, \ldots, \overline{\Omega}_N = \Omega_N - \overline{\Omega}_1 - \cdots - \overline{\Omega}_{N-1}. \tag{2.8}$$

We can obtain $\bigcup_{i=1}^{N} \overline{\Omega}_i = \mathfrak{R}^n$ and $\overline{\Omega}_i \cap \overline{\Omega}_j = \Phi$, $i \neq j$, where Φ is an empty set. If Assumption 2.3 is satisfied, then the following results can be derived:

$$x^T \left({A_{0i}}^T P + P A_{0i} \right) x \leq -x^T Q x, \quad x(t) \in \overline{\Omega}_i. \tag{2.9}$$

Define the following switching function:

$$\sigma = i, \quad \forall x \in \overline{\Omega}_i. \tag{2.10}$$

Definition 2.4 (see [14]). The system (2.1) with the designed switching signal is said to be the globally exponentially stabilizable with convergence rate $\alpha > 0$ by the designed switching signal, if there are two positive constants α and Ψ such that

$$\|x(t)\| \leq \Psi \cdot e^{-\alpha t}, \quad t \geq 0. \tag{2.11}$$

Now we present a result to design the switching signal that guarantees global exponential stability of system (2.1).

Theorem 2.5. *Assume that for* $\|D\| < 1$, $0 < \alpha < -(\ln\|D\|)/\tau$, $0 \leq \alpha_i \leq 1$, $i = 1, 2, \ldots, N$, *and* $\sum_{i=1}^{N} \alpha_i = 1$, *there exist some* $n \times n$ *matrices* $P, Q, R_1, R_2 > 0$, *such that the following LMI conditions hold for all* $i = 1, 2, \ldots, N$*:*

$$\Xi_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & \Xi_{13i} \\ * & \Xi_{22i} & \Xi_{23i} \\ * & * & \Xi_{33i} \end{bmatrix} < 0,$$
$$F^T P + PF + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \tag{2.12}$$

where

$$\Xi_{11i} = 2\alpha \cdot P + R_1 + R_2 - Q, \qquad \Xi_{12i} = -2\alpha \cdot PD - A_{0i}{}^T PD, \qquad \Xi_{13i} = PA_{1i},$$
$$\Xi_{22i} = 2\alpha \cdot D^T PD - e^{-2\alpha\tau} \cdot R_1, \qquad \Xi_{23i} = -D^T PA_{1i}, \qquad \Xi_{33i} = -(1 - h_D) \cdot e^{-2\alpha h_M} \cdot R_2. \tag{2.13}$$

Then the system (2.1) *is globally exponentially stabilizable with convergence rate* α *by the switching signal given in* (2.10).

Proof. Define the Lyapunov functional

$$V(x_t) = e^{2\alpha t} \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau))$$
$$+ \int_{t-\tau}^{t} e^{2\alpha s} \cdot x^T(s) R_1 x(s) ds + \int_{t-h(t)}^{t} e^{2\alpha s} \cdot x^T(s) R_2 x(s) ds, \tag{2.14}$$

where $P, R_1, R_2 > 0$. The time derivatives of $V(x_t)$ along the trajectories of system (2.3) under the switching function (2.10) satisfy

$$\dot{V}(x_t) = e^{2\alpha t} \cdot \left[2\alpha \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau))\right]$$
$$+ e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \left[(A_{0i}x(t) + A_{1i}x(t-h(t)))^T P(x(t) - Dx(t-\tau))\right]$$
$$+ e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \left[(x(t) - Dx(t-\tau))^T P(A_{0i}x(t) + A_{1i}x(t-h(t)))\right] \tag{2.15}$$
$$+ e^{2\alpha t} \cdot \left[x^T(t) R_1 x(t) - e^{-2\alpha\tau} \cdot x^T(t-\tau) R_1 x(t-\tau)\right]$$
$$+ e^{2\alpha t} \cdot \left[x^T(t) R_2 x(t) - \left(1 - \dot{h}(t)\right) \cdot e^{-2\alpha h(t)} \cdot x^T(t-h(t)) R_2 x(t-h(t))\right].$$

By the condition (2.9) and switching function (2.10), we obtain

$$\dot{V}(x_t) \le e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \cdot \left[X^T \cdot \Xi_i \cdot X\right], \tag{2.16}$$

where Ξ_i, $i = 1, 2, \ldots, N$, are defined in (2.12), $X^T = [x^T(t) \quad x^T(t-\tau) \quad x^T(t-h(t))]$. From (2.16) with $\Xi_i < 0$, we have

$$V(x_t) \le V(x_0), \quad t \ge 0, \tag{2.17}$$

where

$$V(x_0) \le \delta_1 \cdot \|x_0\|_s^2, \quad \delta_1 = \lambda_{\max}(P)(1 + \|D\|)^2 + \tau \cdot \lambda_{\max}(R_1) + h_M \cdot \lambda_{\max}(R_2). \tag{2.18}$$

From (2.14), we have

$$\lambda_{\min}(P) \cdot e^{2\alpha t} \cdot \|\wp(t)\|^2 \le e^{2\alpha t} \cdot \wp^T(t) P \wp(t) \le V(x_t) \le \delta_1 \cdot \|x_0\|_s^2, \tag{2.19}$$

where $\wp(t) = x(t) - Dx(t-\tau)$. From (2.19), we can obtain

$$\|x(t)\| = \|\wp(t) + Dx(t-\tau)\| \le \|D\| \cdot \|x(t-\tau)\| + \|\wp(t)\| \le \|D\| \cdot \|x(t-\tau)\| + \delta_2 \cdot e^{-\alpha t}, \quad t \ge 0, \tag{2.20}$$

where $\delta_2 = \sqrt{\delta_1/\lambda_{\min}(P)} \cdot \|x_0\|_s$. Since $\|D\| < 1$ and $\tau > 0$, we can choose a sufficiently small positive constant $\xi = \alpha < -(\ln \|D\|)/\tau$ satisfying $\|D\| \cdot e^{\xi\tau} < 1$. By the Razumikhin-like approach of [14], we have

$$\|x(t)\| \le \left[\|x_0\|_s + \frac{\delta_2}{1 - \|D\| e^{\xi h}}\right] \cdot e^{-\xi t}, \quad t \ge 0. \tag{2.21}$$

This completes the proof. □

Consider the following uncertain switched neutral system with mixed time delays:

$$\dot{x}(t) - D\dot{x}(t-\tau) = [A_{0\sigma} + \Delta A_{0\sigma}(t)]x(t) + [A_{1\sigma} + \Delta A_{1\sigma}(t)]x(t-h(t)), \quad t \ge 0, \tag{2.22a}$$

$$x(t) = \phi(t), \quad t \in [-H, 0], \tag{2.22b}$$

where $\Delta A_{0i}(t)$ and $\Delta A_{1i}(t)$ are some perturbed matrices and satisfy the following condition:

$$\left[\Delta A_{0i}(t) \quad \Delta A_{1i}(t)\right] = M_i F_i(t) \left[N_{0i} \quad N_{1i}\right], \quad \forall i \in \{1, 2, \ldots, N\}, \ t \ge 0, \tag{2.22c}$$

where M_i, N_{0i}, and N_{1i}, $i = 1,2,\ldots,N$, are some given constant matrices with appropriate dimensions, and $F_i(t)$, $i = 1,2,\ldots,N$, are unknown matrices representing the parameter perturbation which satisfy

$$F_i^T(t)F_i(t) \leq I, \quad \forall i \in \{1,2,\ldots,N\},\ t \geq 0. \tag{2.23}$$

The uncertain switched system in (2.22a)–(2.22c) can be rewritten as follows:

$$\begin{gathered}\dot{x}(t) - D\dot{x}(t-\tau) = \sum_{i=1}^{N} \lambda_i(t)\{[A_{0i} + \Delta A_{0i}(t)]x(t) + [A_{1i} + \Delta A_{1i}(t)]x(t-h(t))\}, \quad t \geq 0, \\ x(t) = \phi(t), \quad t \in [-H, 0],\end{gathered} \tag{2.24}$$

where $\lambda_i(t)$ is defined in (2.2) and $\sum_{i=1}^{N} \lambda_i(t) = 1, \forall t \geq 0$.

Now we consider the exponential stability for uncertain switched system (2.22a)–(2.22c).

Theorem 2.6. *Assume that for* $\|D\| < 1$, $0 < \alpha < -(\ln\|D\|)/\tau$, $0 \leq \alpha_i \leq 1$, $i = 1,2,\ldots,N$, *and* $\sum_{i=1}^{N} \alpha_i = 1$, *there exist constants* $\varepsilon_i > 0$, $i = 1,2,\ldots,N$, *and some* $n \times n$ *matrices* $P, Q, R_1, R_2 > 0$, *such that the following LMI conditions hold for all* $i = 1,2,\ldots,N$:

$$\begin{gathered}\overline{\Xi}_i = \begin{bmatrix} \Xi_{11i} & \Xi_{12i} & \Xi_{13i} & \overline{\Xi}_{14i} & \overline{\Xi}_{15i} \\ * & \Xi_{22i} & \Xi_{23i} & \overline{\Xi}_{24i} & 0 \\ * & * & \Xi_{33i} & 0 & \overline{\Xi}_{35i} \\ * & * & * & \overline{\Xi}_{44i} & 0 \\ * & * & * & * & \overline{\Xi}_{55i} \end{bmatrix} < 0, \\ F^T P + PF + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i},\end{gathered} \tag{2.25}$$

where Ξ_{jki}, $j,k = 1,2,3$, *are defined in* (2.12)

$$\overline{\Xi}_{14i} = PM_i, \quad \overline{\Xi}_{15i} = \varepsilon_i \cdot N_{0i}^T, \quad \overline{\Xi}_{24i} = -D^T PM_i, \quad \overline{\Xi}_{35i} = \varepsilon_i \cdot N_{1i}^T, \quad \overline{\Xi}_{44i} = \overline{\Xi}_{55i} = -\varepsilon_i \cdot I. \tag{2.26}$$

Then the system (2.22a)–(2.22c) *is globally exponentially stabilizable with convergence rate* α *by the switching signal given in* (2.10).

Proof. The time derivatives of $V(x_t)$ in (2.14) along the trajectories of system (2.22a)–(2.22c) under the switching function (2.9) satisfy

$$\dot{V}(x_t) \leq e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \cdot \left[X^T \cdot \widehat{\Xi}_i \cdot X\right], \tag{2.27}$$

where

$$\widehat{\Xi}_i = \Xi_i + \begin{bmatrix} PM_i \\ -D^T PM_i \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} N_{0i} & 0 & N_{1i} \end{bmatrix} + \begin{bmatrix} N_{0i}^T \\ 0 \\ N_{1i}^T \end{bmatrix} F_i^T(t) \begin{bmatrix} M_i^T P & -M_i^T PD & 0 \end{bmatrix}. \tag{2.28}$$

By Lemmas 2.1 and 2.2, the condition $\overline{\Xi}_i < 0$ in (2.25) is equivalent to $\widehat{\Xi}_i < 0$. By the same derivation of Theorem 2.5, this proof can be completed. □

If we choose the convergence rate $\alpha = 0$, we can obtain the following delay-independent condition for the global asymptotic stability of system (2.22a)–(2.22c).

Corollary 2.7. *Assume that for $\|D\| < 1$, $0 \le \alpha_i \le 1$, $i = 1, 2, \ldots, N$, and $\sum_{i=1}^{N} \alpha_i = 1$, there exist constants $\varepsilon_i > 0$, $i = 1, 2, \ldots, N$, some $n \times n$ matrices $P, Q, R_1, R_2 > 0$, such that the following LMI conditions hold for all $i = 1, 2, \ldots, N$:*

$$\widetilde{\Xi}_i = \begin{bmatrix} \widetilde{\Xi}_{11i} & \widetilde{\Xi}_{12i} & \widetilde{\Xi}_{13i} & \widetilde{\Xi}_{14i} & \widetilde{\Xi}_{15i} \\ * & \widetilde{\Xi}_{22i} & \widetilde{\Xi}_{23i} & \widetilde{\Xi}_{24i} & 0 \\ * & * & \widetilde{\Xi}_{33i} & 0 & \widetilde{\Xi}_{35i} \\ * & * & * & \widetilde{\Xi}_{44i} & 0 \\ * & * & * & * & \widetilde{\Xi}_{55i} \end{bmatrix} < 0,$$
$$F^T P + PF + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \tag{2.29}$$

where

$$\widetilde{\Xi}_{11i} = R_1 + R_2 - Q, \quad \widetilde{\Xi}_{12i} = -A_{0i}{}^T PD, \quad \widetilde{\Xi}_{13i} = PA_{1i}, \quad \widetilde{\Xi}_{14i} = PM_i, \quad \widetilde{\Xi}_{15i} = \varepsilon_i \cdot N_{0i}^T,$$
$$\widetilde{\Xi}_{22i} = -R_1, \quad \widetilde{\Xi}_{23i} = -D^T PA_{1i}, \quad \widetilde{\Xi}_{24i} = -D^T PM_i, \quad \widetilde{\Xi}_{33i} = -(1 - h_D) \cdot R_2,$$
$$\widetilde{\Xi}_{35i} = \varepsilon_i \cdot N_{1i}^T, \quad \widetilde{\Xi}_{44i} = \widetilde{\Xi}_{55i} = -\varepsilon_i \cdot I. \tag{2.30}$$

Then the system (2.22a)–(2.22c) *is globally asymptotically stabilizable by the switching signal given in* (2.10).

If $D = 0$, Corollary 2.7 can be reduced to the following corollary.

Corollary 2.8. *Assume that for some constants α_i, $i = 1,2,\ldots,N$, there exist constants $\varepsilon_i > 0$, $i = 1,2,\ldots,N$, some $n \times n$ matrices $P, Q, R_2 > 0$, such that the following LMI conditions hold for all $i = 1,2,\ldots,N$:*

$$\tilde{\tilde{\tilde{\Xi}}}_i = \begin{bmatrix} \tilde{\tilde{\tilde{\Xi}}}_{11i} & \tilde{\tilde{\tilde{\Xi}}}_{12i} & \tilde{\tilde{\tilde{\Xi}}}_{13i} & \tilde{\tilde{\tilde{\Xi}}}_{14i} \\ * & \tilde{\tilde{\tilde{\Xi}}}_{22i} & 0 & \tilde{\tilde{\tilde{\Xi}}}_{24i} \\ * & * & \tilde{\tilde{\tilde{\Xi}}}_{33i} & 0 \\ * & * & * & \tilde{\tilde{\tilde{\Xi}}}_{44i} \end{bmatrix} < 0,$$
$$F^T P + PF + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \tag{2.31}$$

where

$$\tilde{\tilde{\tilde{\Xi}}}_{11i} = R_2 - Q, \quad \tilde{\tilde{\tilde{\Xi}}}_{12i} = PA_{1i}, \quad \tilde{\tilde{\tilde{\Xi}}}_{13i} = PM_i, \quad \tilde{\tilde{\tilde{\Xi}}}_{14i} = \varepsilon_i \cdot N_{0i}^T, \quad \tilde{\tilde{\tilde{\Xi}}}_{22i} = -(1 - h_D) \cdot R_2,$$
$$\tilde{\tilde{\tilde{\Xi}}}_{24i} = \varepsilon_i \cdot N_{1i}^T, \quad \tilde{\tilde{\tilde{\Xi}}}_{33i} = \tilde{\tilde{\tilde{\Xi}}}_{44i} = -\varepsilon_i \cdot I. \tag{2.32}$$

Then the system (2.22a)–(2.22c) *is globally asymptotically stabilizable by the switching signal given in* (2.10).

Assumption 2.9. Assume that there exists a convex combination $F = \sum_{i=1}^{N} \alpha_i A_{0i}$, some positive definite matrices P and Q, some matrices $\widehat{S}_i$, $i = 1,2,\ldots,N$, such that

$$F^T P + PF + \sum_{i=1}^{N} \alpha_i \cdot \left(\widehat{S}_i + \widehat{S}_i^T\right) < -Q, \tag{2.33}$$

where $0 \le \alpha_i \le 1$ and $\sum_{i=1}^{N} \alpha_i = 1$.

Define some domains

$$\widehat{\Omega}_i = \left\{x \in \Re^n : x^T \left(A_{0i}{}^T P + PA_{0i} + \widehat{S}_i + \widehat{S}_i^T\right) x \le -x^T Q x\right\}, \quad i \in \{1,2,\ldots,N\}. \tag{2.34}$$

From the similar proof of [7], it is easy to show $\bigcup_{i=1}^{N} \widehat{\Omega}_i = \Re^n$. Construct some domains

$$\tilde{\Omega}_1 = \widehat{\Omega}_1, \tilde{\Omega}_2 = \widehat{\Omega}_2 - \tilde{\Omega}_1, \tilde{\Omega}_3 = \widehat{\Omega}_3 - \tilde{\Omega}_1 - \tilde{\Omega}_2, \ldots, \tilde{\Omega}_N = \widehat{\Omega}_N - \tilde{\Omega}_1 - \cdots - \tilde{\Omega}_{N-1}. \tag{2.35}$$

We can obtain $\bigcup_{i=1}^{N}\widetilde{\Omega}_i = \Re^n$ and $\widetilde{\Omega}_i \cap \widetilde{\Omega}_j = \Phi$, $i \neq j$, where Φ is an empty set. If Assumption 2.9 is satisfied, then the following results can be derived:

$$x^T\left(A_{0i}{}^T P + PA_{0i} + \widehat{S}_i + \widehat{S}_i^T\right)x \leq -x^T Qx, \quad x(t) \in \widetilde{\Omega}_i. \tag{2.36}$$

Define the following switching function:

$$\sigma = i, \quad \forall x \in \widetilde{\Omega}_i. \tag{2.37}$$

Remark 2.10. In [6, 7, 10], their assumption is given by

$$F_1^T P + PF_1 = -Q, \tag{2.38}$$

where $F_1 = \sum_{i=1}^{N} \alpha_i \cdot (A_{0i} + A_{1i})$. We can see that our Assumption 2.9 is more flexible with $\widehat{S}_i = PA_{1i}$, $\forall i = 1, 2, \ldots, N$. The main difference of Assumptions 2.3 and 2.9 is that some matrices $\widehat{S}_i$ are introduced in Assumption 2.9. These matrices play a key role to derive the delay-dependent results.

Theorem 2.11. *Assume that for $\|D\| < 1$, $0 < \alpha < -(\ln\|D\|)/\tau$, $0 \leq \alpha_i \leq 1$, $i = 1, 2, \ldots, N$, and $\sum_{i=1}^{N} \alpha_i = 1$, there exist constants $\varepsilon_i > 0$, $i = 1, 2, \ldots, N$, some $n \times n$ matrices $P, Q, R_1, R_2, R_3, R_4 > 0$, and some $n \times n$ matrices R_{12}, U, $\widehat{S}_i$, V_{1i}, V_{2i}, and V_{3i}, $i = 1, 2, \ldots, N$, such that the following LMI conditions hold for all $i = 1, 2, \ldots, N$:*

$$\overline{\Sigma}_i = \begin{bmatrix} \Sigma_{11i} & \Sigma_{12i} & \Sigma_{13i} & \Sigma_{14i} & \Sigma_{15i} & \Sigma_{16i} & \Sigma_{17i} & \Sigma_{18i} \\ * & \Sigma_{22i} & 0 & \Sigma_{24i} & \Sigma_{25i} & 0 & \Sigma_{27i} & 0 \\ * & * & \Sigma_{33i} & \Sigma_{34i} & \Sigma_{35i} & \Sigma_{36i} & \Sigma_{37i} & 0 \\ * & * & * & \Sigma_{44i} & \Sigma_{45i} & 0 & \Sigma_{47i} & 0 \\ * & * & * & * & \Sigma_{55i} & \Sigma_{56i} & 0 & \Sigma_{58i} \\ * & * & * & * & * & \Sigma_{66i} & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77i} & 0 \\ * & * & * & * & * & * & * & \Sigma_{88i} \end{bmatrix} < 0,$$

$$F^T P + PF + \sum_{i=1}^{N} \alpha_i \cdot \left(\widehat{S}_i + \widehat{S}_i^T\right) + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \tag{2.39}$$

$$\begin{bmatrix} R_1 & R_{12} \\ * & R_2 \end{bmatrix} > 0,$$

where

$$\Sigma_{11i} = 2\alpha \cdot P - Q + R_1 + R_3 - V_{1i} - V_{1i}^T, \quad \Sigma_{12i} = R_{12} + A_{0i}^T U^T,$$
$$\Sigma_{13i} = -2\alpha PD - A_{0i}{}^T PD - \widehat{S}_i^T D, \quad \Sigma_{14i} = -A_{0i}{}^T U^T D,$$
$$\Sigma_{15i} = PA_{1i} - \widehat{S}_i - V_{2i} + V_{1i}^T, \quad \Sigma_{16i} = -\widehat{S}_i - V_{3i} + V_{1i}^T, \quad \Sigma_{17i} = PM_i, \quad \Sigma_{18i} = \varepsilon_i \cdot N_{0i}^T,$$
$$\Sigma_{22i} = R_2 + h_M^2 \cdot R_4 - U - U^T, \quad \Sigma_{24i} = \left(U + U^T\right)D, \quad \Sigma_{25i} = UA_{1i}, \quad \Sigma_{27i} = UM_i,$$
$$\Sigma_{33i} = 2\alpha D^T PD - e^{-2\alpha\tau} \cdot R_1, \quad \Sigma_{34i} = -e^{-2\alpha\tau} \cdot R_{12}, \quad \Sigma_{35i} = -D^T PA_{1i} + D^T \widehat{S}_i, \quad \Sigma_{36i} = D^T \widehat{S}_i,$$
$$\Sigma_{37i} = -D^T PM_i, \quad \Sigma_{44i} = -e^{-2\alpha\tau} \cdot R_2 - D^T\left(U + U^T\right)D, \quad \Sigma_{45i} = -D^T UA_{1i}, \quad \Sigma_{47i} = -D^T UM_i,$$
$$\Sigma_{55i} = -(1 - h_D) \cdot e^{-2\alpha h_M} \cdot R_3 + V_{2i} + V_{2i}^T, \quad \Sigma_{56i} = V_{3i} + V_{2i}^T, \quad \Sigma_{58i} = \varepsilon_i \cdot N_{1i}^T,$$
$$\Sigma_{66i} = -e^{-2\alpha h_M} \cdot R_4 + V_{3i} + V_{3i}^T, \quad \Sigma_{77i} = \Sigma_{88i} = -\varepsilon_i \cdot I. \tag{2.40}$$

Then the system (2.22a)–(2.22c) *is globally exponentially stabilizable with convergence rate* α *by the switching signal given in* (2.37)*.*

Proof. Define the Lyapunov functional

$$\begin{aligned} V(x_t) &= e^{2\alpha t} \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau)) \\ &\quad + \int_{t-\tau}^{t} e^{2\alpha s} \cdot \begin{bmatrix} x^T(s) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} R_1 & R_{12} \\ * & R_2 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ &\quad + \int_{t-h(t)}^{t} e^{2\alpha s} \cdot x^T(s) R_3 x(s) ds + h_M \cdot \int_{t-h_M}^{t} e^{2\alpha s} \cdot (s - (t - h_M)) \dot{x}^T(s) R_4 \dot{x}(s) ds, \end{aligned} \tag{2.41}$$

where $P, R_3, R_4 > 0$, $\left[\begin{smallmatrix} R_1 & R_{12} \\ * & R_2 \end{smallmatrix}\right] > 0$. The time derivatives of $V(x_t)$ along the trajectories of system (2.24) satisfy

$$\begin{aligned} \dot{V}(x_t) &= e^{2\alpha t} \cdot \left[2\alpha \cdot (x(t) - Dx(t-\tau))^T P(x(t) - Dx(t-\tau))\right] \\ &\quad + e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \Bigg[\Bigg((A_{0i} + S_i + \Delta A_{0i}(t))x(t) - S_i \cdot \int_{t-h(t)}^{t} \dot{x}(s) ds \\ &\qquad + (A_{1i} + \Delta A_{1i}(t) - S_i)x(t - h(t))\Bigg)^T P(x(t) - Dx(t-\tau))\Bigg] \\ &\quad + e^{2\alpha t} \cdot \sum_{i=1}^{N} \lambda_i(t) \Bigg[(x(t) - Dx(t-\tau))^T P\Bigg((A_{0i} + S_i + \Delta A_{0i}(t))x(t) - S_i \cdot \int_{t-h(t)}^{t} \dot{x}(s) ds \\ &\qquad + (A_{1i} + \Delta A_{1i}(t) - S_i)x(t - h(t))\Bigg)\Bigg] \end{aligned}$$

$$+e^{2\alpha t}\cdot\left[\left[x^T(t)\ \ \dot{x}^T(t)\right]\begin{bmatrix}R_1 & R_{12}\\ * & R_2\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}\right.$$
$$\left.-e^{-2\alpha\tau}\cdot\left[x^T(t-\tau)\ \ \dot{x}^T(t-\tau)\right]\begin{bmatrix}R_1 & R_{12}\\ * & R_2\end{bmatrix}\begin{bmatrix}x(t-\tau)\\ \dot{x}(t-\tau)\end{bmatrix}\right]$$
$$+e^{2\alpha t}\cdot\left[x^T(t)R_3x(t)-\left(1-\dot{h}(t)\right)\cdot e^{-2\alpha h(t)}\cdot x^T(t-h(t))R_3x(t-h(t))\right]$$
$$+e^{2\alpha t}\cdot\left[h_M^2\cdot\dot{x}^T(t)R_4\dot{x}(t)-h_M\cdot\int_{t-h_M}^{t}e^{2\alpha(s-t)}\cdot\dot{x}^T(s)R_4\dot{x}(s)ds\right],\tag{2.42}$$

where $\widehat{S}_i=PS_i$. By the inequality in [1, page 322], we have

$$\begin{aligned}-h_M\cdot\int_{t-h_M}^{t}e^{2\alpha s}\cdot\dot{x}^T(s)R_4\dot{x}(s)ds &\le -h(t)\cdot e^{2\alpha t}\cdot e^{-2\alpha h_M}\cdot\int_{t-h(t)}^{t}\dot{x}^T(s)R_4\dot{x}(s)ds\\ &\le -e^{2\alpha t}\cdot e^{-2\alpha h_M}\cdot\left[\int_{t-h(t)}^{t}\dot{x}(s)ds\right]^T R_4\left[\int_{t-h(t)}^{t}\dot{x}(s)ds\right].\end{aligned}\tag{2.43}$$

By system (2.24) and Leibniz-Newton formula, we have

$$\sum_{i=1}^{N}\lambda_i(t)\cdot e^{2\alpha t}\cdot\left[\int_{t-h(t)}^{t}\dot{x}(s)ds-x(t)+x(t-h(t))\right]^T\left[V_{1i}x(t)+V_{2i}x(t-h(t))+V_{3i}\int_{t-h(t)}^{t}\dot{x}(s)ds\right]$$
$$+\sum_{i=1}^{N}\lambda_i(t)\cdot e^{2\alpha t}\cdot\left[V_{1i}x(t)+V_{2i}x(t-h(t))+V_{3i}\int_{t-h(t)}^{t}\dot{x}(s)ds\right]^T$$
$$\times\left[\int_{t-h(t)}^{t}\dot{x}(s)ds-x(t)+x(t-h(t))\right]=0,$$
$$-e^{2\alpha t}\cdot(\dot{x}(t)-D\dot{x}(t-\tau))^T\left(U+U^T\right)(\dot{x}(t)-D\dot{x}(t-\tau))$$
$$+\sum_{i=1}^{N}\lambda_i(t)\cdot e^{2\alpha t}\cdot(\dot{x}(t)-D\dot{x}(t-\tau))^TU\{[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}$$
$$+\sum_{i=1}^{N}\lambda_i(t)\cdot e^{2\alpha t}\cdot\{[A_{0i}+\Delta A_{0i}(t)]x(t)+[A_{1i}+\Delta A_{1i}(t)]x(t-h(t))\}^TU^T(\dot{x}(t)-D\dot{x}(t-\tau))=0.\tag{2.44}$$

By the conditions (2.42)–(2.44), we obtain the following result:

$$\dot{V}(x_t)\le e^{2\alpha t}\cdot\sum_{i=1}^{N}\lambda_i(t)\cdot\left[X^T\cdot\Sigma_i\cdot X\right],\tag{2.45}$$

where

$$X^T = \left[x^T(t) \;\; \dot{x}^T(t) \;\; x^T(t-\tau) \;\; \dot{x}^T(t-\tau) \;\; x^T(t-h(t)) \;\; \int_{t-h(t)}^{t} \dot{x}^T(s)ds \right],$$

$$\Sigma_i = \begin{bmatrix} \Sigma_{11i} & \Sigma_{12i} & \Sigma_{13i} & \Sigma_{14i} & \Sigma_{15i} & \Sigma_{16i} \\ * & \Sigma_{22i} & 0 & \Sigma_{24i} & \Sigma_{25i} & 0 \\ * & * & \Sigma_{33i} & \Sigma_{34i} & \Sigma_{35i} & \Sigma_{36i} \\ * & * & * & \Sigma_{44i} & \Sigma_{45i} & 0 \\ * & * & * & * & \Sigma_{55i} & \Sigma_{56i} \\ * & * & * & * & * & \Sigma_{66i} \end{bmatrix} + \begin{bmatrix} PM_i \\ UM_i \\ -D^T PM_i \\ -D^T UM_i \\ 0 \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} N_{0i}^T \\ 0 \\ 0 \\ 0 \\ N_{1i}^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} N_{0i}^T \\ 0 \\ 0 \\ 0 \\ N_{1i}^T \\ 0 \end{bmatrix} F_i^T(t) \begin{bmatrix} PM_i \\ UM_i \\ -D^T PM_i \\ -D^T UM_i \\ 0 \\ 0 \end{bmatrix}^T. \quad (2.46)$$

By Lemmas 2.1 and 2.2, the condition $\overline{\Sigma}_i < 0$ in (2.39) is equivalent to $\Sigma_i < 0$ in (2.45). From $\Sigma_i < 0$ and by the similar derivation of Theorem 2.5, the proof can be completed. □

If $D = 0$, Theorem 2.11 can be reduced to the following corollary.

Corollary 2.12. *Assume that for $\alpha > 0$, $0 \le \alpha_i \le 1$, $i = 1, 2, \ldots, N$, there exist constants $\varepsilon_i > 0$, $i = 1, 2, \ldots, N$, some $n \times n$ matrices $P, Q, R_3, R_4 > 0$, some $n \times n$ matrices U, $\widehat{S}_i$, V_{1i}, V_{2i}, and V_{3i}, $i = 1, 2, \ldots, N$, such that the following LMI conditions hold for all $i = 1, 2, \ldots, N$:*

$$\widehat{\Sigma}_i = \begin{bmatrix} \widehat{\Sigma}_{11i} & \widehat{\Sigma}_{12i} & \widehat{\Sigma}_{13i} & \widehat{\Sigma}_{14i} & \widehat{\Sigma}_{15i} & \widehat{\Sigma}_{16i} \\ * & \widehat{\Sigma}_{22i} & \widehat{\Sigma}_{23i} & 0 & \widehat{\Sigma}_{25i} & 0 \\ * & * & \widehat{\Sigma}_{33i} & \widehat{\Sigma}_{34i} & 0 & \widehat{\Sigma}_{36i} \\ * & * & * & \widehat{\Sigma}_{44i} & 0 & 0 \\ * & * & * & * & \widehat{\Sigma}_{55i} & 0 \\ * & * & * & * & * & \widehat{\Sigma}_{66i} \end{bmatrix} < 0,$$

$$F^T P + PF + \sum_{i=1}^{N} \alpha_i \cdot \left(\widehat{S}_i + \widehat{S}_i^T \right) + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \quad (2.47)$$

where

$$\widehat{\Sigma}_{11i} = 2\alpha \cdot P - Q + R_3 - V_{1i} - V_{1i}^T, \quad \widehat{\Sigma}_{12i} = A_{0i}^T U^T, \quad \widehat{\Sigma}_{13i} = PA_{1i} - \widehat{S}_i - V_{2i} + V_{1i}^T,$$
$$\widehat{\Sigma}_{14i} = -\widehat{S}_i - V_{3i} + V_{1i}^T, \quad \widehat{\Sigma}_{15i} = PM_i, \quad \widehat{\Sigma}_{16i} = \varepsilon_i \cdot N_{0i}^T, \quad \widehat{\Sigma}_{22i} = h_M^2 \cdot R_4 - U - U^T,$$
$$\widehat{\Sigma}_{23i} = UA_{1i}, \quad \widehat{\Sigma}_{25i} = UM_i, \quad \widehat{\Sigma}_{33i} = -(1 - h_D) \cdot e^{-2\alpha h_M} \cdot R_3 + V_{2i} + V_{2i}^T, \quad \widehat{\Sigma}_{34i} = V_{3i} + V_{2i}^T,$$
$$\widehat{\Sigma}_{36i} = \varepsilon_i \cdot N_{1i}^T, \quad \widehat{\Sigma}_{44i} = -e^{-2\alpha h_M} \cdot R_4 + V_{3i} + V_{3i}^T, \quad \widehat{\Sigma}_{55i} = \widehat{\Sigma}_{66i} = -\varepsilon_i \cdot I. \tag{2.48}$$

Then the system (2.22a)–(2.22c) *with* $D = 0$ *is globally exponentially stabilizable with convergence rate* α *by the switching signal given in* (2.37).

If $D = M_i = N_{0i} = N_{1i} = 0$, $i = 1, 2, \ldots, N$, Corollary 2.12 can be reduced to the following corollary.

Corollary 2.13. *Assume that for* $\alpha > 0$, $0 \le \alpha_i \le 1$, $i = 1, 2, \ldots, N$, *there exist some* $n \times n$ *matrices* $P, Q, R_3, R_4 > 0$, *some* $n \times n$ *matrices* U, $\widehat{S}_i$, V_{1i}, V_{2i}, *and* V_{3i}, $i = 1, 2, \ldots, N$, *such that the following LMI conditions hold for all* $i = 1, 2, \ldots, N$*:*

$$\widehat{\Sigma}_i = \begin{bmatrix} \widehat{\Sigma}_{11i} & \widehat{\Sigma}_{12i} & \widehat{\Sigma}_{13i} & \widehat{\Sigma}_{14i} \\ * & \widehat{\Sigma}_{22i} & \widehat{\Sigma}_{23i} & 0 \\ * & * & \widehat{\Sigma}_{33i} & \widehat{\Sigma}_{34i} \\ * & * & * & \widehat{\Sigma}_{44i} \end{bmatrix} < 0,$$
$$F^T P + PF + \sum_{i=1}^{N} \alpha_i \cdot \left(\widehat{S}_i + \widehat{S}_i^T\right) + Q < 0, \quad F = \sum_{i=1}^{N} \alpha_i A_{0i}, \tag{2.49}$$

where $\widehat{\Sigma}_{jki}$, $j, k = 1, 2, 3, 4$, $i = 1, 2, \ldots, N$, *are defined in* Corollary 2.12. *Then the system* (2.22a)–(2.22c) *with* $D = M_i = N_{0i} = N_{1i} = 0$ *is globally exponentially stabilizable with convergence rate* α *by the switching signal given in* (2.37).

Remark 2.14. By setting $\alpha = 0$ in Theorems 2.5–2.11 and Corollaries 2.7–2.13, the global asymptotic stability for system (2.22a)–(2.22c) can be guaranteed.

3. Numerical Examples

Example 3.1. Consider the system (2.22a)–(2.22c) and the following parameters:

$$N = 2, \quad D = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix}, \quad A_{01} = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} 0 & -1 \\ -1 & -4 \end{bmatrix}, \tag{3.1}$$

Table 1: Comparison with other previous results.

Allowable time-varying delay bounds retaining global asymptotic stability ($\alpha = 0$) of the system (2.22a)–(2.22c) with (3.1) under switching signal (3.4)			
Results	$h_D = 0$	$h_D = 0.2$	$h_D = 0.3$
Reference [10] (without perturbations and $D = 0$)	$h_M = 0.9999998$	$h_M = 0.9999997$	$h_M = 0.9999996$
Our results (with perturbations)	$h_M \geq 0$	$h_M \geq 0$	$h_M \geq 0$

$$A_{12} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 1 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad N_{01} = N_{02} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad N_{11} = N_{12} = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}. \tag{3.2}$$

By Corollary 2.7, a feasible solution of LMI (2.29) with (3.1), $\alpha = 0$, $h_D = 0.2$, and $\alpha_1 = \alpha_2 = 0.5$ is given by

$$P = \begin{bmatrix} 169.0808 & 6.9691 \\ 6.9691 & 147.4925 \end{bmatrix}, \quad Q = \begin{bmatrix} 618.4486 & -21.7634 \\ -21.7634 & 538.3595 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 120.4512 & -58.0284 \\ -58.0284 & 90.4631 \end{bmatrix},$$
$$R_2 = \begin{bmatrix} 275.0262 & 25.6702 \\ 25.6702 & 180.9190 \end{bmatrix}, \quad \varepsilon_1 = 145.7753, \quad \varepsilon_2 = 180.2272. \tag{3.3}$$

Select the switching signal by

$$\sigma = \begin{cases} 1, & x \in \overline{\Omega}_1, \\ 2, & x \in \overline{\Omega}_2, \end{cases} \tag{3.4}$$

where $\overline{\Omega}_1 = \Omega_1$, $\overline{\Omega}_2 = \Omega_2 - \overline{\Omega}_1$,

$$\begin{aligned} \Omega_1 &= \left\{ x \in \Re^n : -720.2596x_1^2 + 872.0285x_1x_2 + 566.2359x_2^2 \leq 0 \right\}, \\ \Omega_2 &= \left\{ x \in \Re^n : 604.5104x_1^2 - 732.4262x_1x_2 - 655.5187x_2^2 \leq 0 \right\}. \end{aligned} \tag{3.5}$$

The switching regions Ω_1 and Ω_2 are sketched in Figure 1. The system (2.22a)–(2.22c) with $h_D = 0.2$ and (3.1) is globally asymptotically stabilizable by the switching signal (3.4). Some comparisons are made in Table 1. The result of this paper provides a major improvement to guarantee the global asymptotic stability of system (2.22a)–(2.22c) with (3.1).

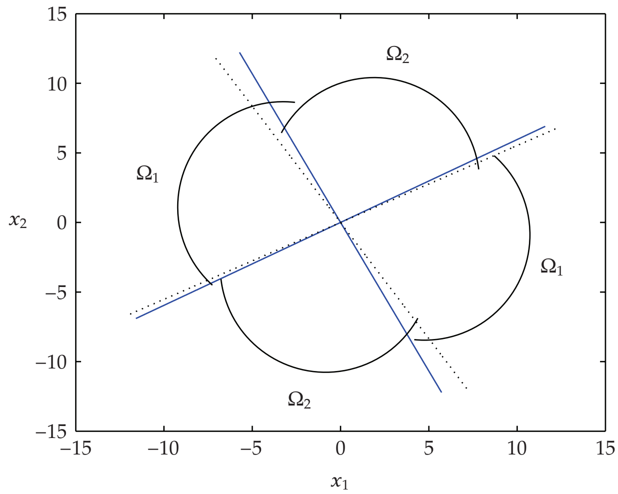

Figure 1: Switching regions.

Table 2: Comparison with other previous results.

Allowable time-varying delay bounds retaining global asymptotic stability ($\alpha = 0$) of the system (2.22a)–(2.22c) with (3.6)					
Results	$h_D = 0$	$h_D = 0.1$	$h_D = 0.5$	$h_D = 0.9$	$h_D = 1$
Reference [7]	$h_M = 0.001573$	No results			
Reference [6]	$h_M = 0.00392$	No results			
Reference [10]	$h_M = 0.0202$	$h_M = 0.0179$	$h_M = 0.0176$	$h_M = 0.0176$	$h_M = 0.0176$
Our results	$h_M = 0.0309$	$h_M = 0.0271$	$h_M = 0.0193$	$h_M = 0.0181$	$h_M = 0.0180$

Example 3.2. Consider the system (2.22a)–(2.22c) and the following parameters [7]:

$$N = 2, \qquad D = M_i = N_{0i} = N_{1i} = 0, \qquad A_{01} = \begin{bmatrix} -2 & 2 \\ -20 & -2 \end{bmatrix},$$
$$A_{11} = \begin{bmatrix} -1 & -7 \\ 23 & 6 \end{bmatrix}, \qquad A_{02} = \begin{bmatrix} -2 & 10 \\ -4 & -2 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 4 & -5 \\ 1 & -8 \end{bmatrix}. \tag{3.6}$$

By Corollary 2.13, some comparisons with the obtained results for switched system (2.22a)–(2.22c) with (3.6) are made in Table 2. The results of this paper provide a larger allowable upper bound for time delay to guarantee the global asymptotic stability of system (2.22a)–(2.22c) with (3.6) by the switching signal (2.37).

Example 3.3. Consider the following switched system with input time delay [7]:

$$\dot{x}(t) = A_{0\sigma}x(t) + B_\sigma u(t - h(t)), \quad t \geq 0, \tag{3.7}$$

Table 3: Comparison with other previous results.

Allowable time-varying delay bounds retaining global asymptotic and exponential stability of the system (3.7) with (3.9)				
Results	$h_D = 0$, $\alpha = 0$	$h_D = 0.1$, $\alpha = 0$	$h_D = 0$, $\alpha = 0.1$	$h_D = 0$, $\alpha = 0.2$
Reference [7]	$h_M = 0.008723$	No results		
Reference [10]	$h_M = 0.0189$	$h_M = 0.0182$	No results	
Our results	$h_M = 0.03112$	$h_M = 0.0294$	$h_M = 0.0296$	$h_M = 0.0281$

where

$$N = 2, \quad A_{01} = \begin{bmatrix} 3 & 2 \\ -5 & -1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -1 & 20 \\ -2 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}. \tag{3.8}$$

The feedback control is given by $u(t) = K_{\sigma(t)}x(t)$ with

$$K_1 = \begin{bmatrix} 5 & 0 \\ 20 & 1 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -3 & -14 \\ 2 & 4 \end{bmatrix}. \tag{3.9}$$

For the given feedback control (3.9), system (3.7) can be rewritten as

$$\dot{x}(t) = A_{0\sigma}x(t) + A_{1\sigma}x(t - h(t)), \quad t \geq 0, \tag{3.10}$$

where $A_{1\sigma} = B_\sigma K_\sigma$. As shown in Table 3, the results obtained in this paper provide larger allowable time delay bounds guaranteeing the global stability of system (3.7) with (3.9) by switching signal (2.37). In [7, 10], the convex combination parameters are chosen by $\alpha_1 = 1/3$ and $\alpha_2 = 2/3$. The convex combination parameters of our results are chosen by $\alpha_1 = 0.1$ and $\alpha_2 = 0.9$.

4. Conclusions

In this paper, the switching signal design for global exponential stability of uncertain switched neutral systems with mixed time delays has been considered. LMI and Razumikhin-like approaches are used to derive delay-dependent and delay-independent stability criteria. The results obtained in this paper are less conservative than the previous ones for the numerical examples investigated in this paper.

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