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Research Article

On the Stability of Quadratic Double Centralizers and Quadratic Multipliers: A Fixed Point Approach

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We prove the superstability of quadratic double centralizers and of quadratic multipliers on Banach algebras by fixed point methods. These results show that we can remove the conditions of being weakly commutative and weakly without order which are used in the work of M. E. Gordji et al. (2011) for Banach algebras.

1. Introduction

In 1940, Ulam [1] raised the following question concerning stability of group homomorphisms: *under what condition does there exist an additive mapping near an approximately additive mapping?* Hyers [2] answered the problem of Ulam for Banach spaces. He showed that for two Banach spaces $\mathcal{X}$ and $\mathcal{Y}$, if $\epsilon > 0$ and $f : \mathcal{X} \to \mathcal{Y}$ such that

$$\|f(x+y) - f(x) - f(y)\| \le \epsilon, \tag{1.1}$$

for all $x, y \in \mathcal{X}$, then there exist a unique additive mapping $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\|f(x) - T(x)\| \le \epsilon, \quad (x \in \mathcal{X}). \tag{1.2}$$

The work has been extended to quadratic functional equations. Consider $f : \mathcal{X} \to \mathcal{Y}$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$, for all $x \in \mathcal{X}$. Assume that there exist constants $\epsilon \geq 0$ and $p \in [0,1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p), \quad (x \in \mathcal{X}). \tag{1.3}$$

Th. M. Rassias in [3] showed with the above conditions for f, there exists a unique $\mathbb{R}$-linear mapping $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2-2^p}\|x\|^p, \quad (x \in \mathcal{X}). \tag{1.4}$$

Găvruţa then generalized the Rassias's result in [4].

A square norm on an inner product space satisfies the important parallelogram equality

$$\|x+y\|^2 + \|x-y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right). \tag{1.5}$$

Recall that the functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{1.6}$$

is called quadratic functional equation. In addition, every solution of functional eqaution (1.6) is said to be a *quadratic mapping*. A Hyers-Ulam stability problem for the quadratic functional equation was proved by Skof [5] for mappings $f : \mathcal{X} \to \mathcal{Y}$, where $\mathcal{X}$ is a normed space and $\mathcal{Y}$ is a Banach space. Cholewa [6] noticed that the theorem of Skof is still true if the relevant domain $\mathcal{X}$ is replaced by an abelian group. Indeed, Czerwik in [7] proved the Cauchy-Rassias stability of the quadratic functional equation. Since then, the stability problems of various functional equation have been extensively investigated by a number of authors (e.g, [8–13]).

One should remember that the functional equation is called *stable* if any approximately solution to the functional equation is near to a true solution of that functional equation, and is super *superstable* if every approximately solution is an exact solution of it (see [14]). Recently, the first and third authors in [15] investigated the stability of quadratic double centralizer: the maps which are quadratic and double centralizer. Later, Eshaghi Gordji et al. introduced a new concept of the quadratic double centralizer and the quadratic multipliers in [16], and established the stability of quadratic double centralizer and quadratic multipliers on Banach algebras. They also established the superstability for those which are weakly commutative and weakly without order. In this paper, we show that the hypothesis on Banach algebras being weakly commutative and weakly without order in [16] can be eliminated, and prove the superstability of quadratic double centralizers and quadratic multipliers on a Banach algebra by a method of fixed point.

2. Stability of Quadratic Double Centralizers

A linear mapping $L : \mathcal{A} \to \mathcal{A}$ is said to be *left centralizer* on $\mathcal{A}$ if $L(ab) = L(a)b$, for all $a, b \in \mathcal{A}$. Similarly, a linear mapping $R : \mathcal{A} \to \mathcal{A}$ satisfying $R(ab) = aR(b)$, for all $a, b \in \mathcal{A}$ is called *right centralizer* on $\mathcal{A}$. A *double centralizer* on $\mathcal{A}$ is a pair (L, R), where L is a left centralizer, R is a right centralizer and $aL(b) = R(a)b$, for all $a, b \in \mathcal{A}$. An operator $T : \mathcal{A} \to \mathcal{A}$ is said to be a *multiplier* if $aT(b) = T(a)b$, for all $a, b \in \mathcal{A}$.

Throughout this paper, let $\mathcal{A}$ be a complex Banach algebra. Recall that a mapping $L : \mathcal{A} \to \mathcal{A}$ is a quadratic left centralizer if L is a quadratic homogeneous mapping, that is L is quadratic and $L(\lambda a) = \lambda^2 L(a)$, for all $a \in \mathcal{A}$ and $\lambda \in \mathbb{C}$, and $L(ab) = L(a)b^2$, for all $a, b \in \mathcal{A}$. A mapping $R : \mathcal{A} \to \mathcal{A}$ is a quadratic right centralizer if R is a quadratic homogeneous mapping and $R(ab) = a^2 R(b)$, for all $a, b \in \mathcal{A}$. Also, a quadratic double centralizer of an algebra $\mathcal{A}$ is a pair (L, R) where L is a quadratic left centralizer, R is a quadratic right centralizer and $a^2 L(b) = R(a)b^2$, for all $a, b \in \mathcal{A}$ (see [16] for details).

It is proven in [8]; that for the vector spaces $\mathcal{X}$ and $\mathcal{Y}$ and the fixed positive integer k, the map $f : \mathcal{X} \to \mathcal{Y}$ is quadratic if and only if the following equality holds:

$$2f\left(\frac{kx + ky}{2}\right) + 2f\left(\frac{kx - ky}{2}\right) = k^2 f(x) + k^2 f(y). \tag{2.1}$$

We thus can show that f is quadratic if and only if for a fixed positive integer k, the following equality holds:

$$f(kx + ky) + f(kx - ky) = 2k^2 f(x) + 2k^2 f(y). \tag{2.2}$$

Before proceeding to the main results, we will state the following theorem which is useful to our purpose.

Theorem 2.1 (The alternative of fixed point [17]). *Suppose that we are given a complete generalized metric space (X, d) and a strictly contractive mapping $T : X \to X$ with Lipschitz constant L. Then for each given $x \in X$, either $d(T^n x, T^{n+1} x) = \infty$, for all $n \geq 0$, or else exists a natural number n_0 such that*

(1) *$d(T^n x, T^{n+1} x) < \infty$, for all $n \geq n_0$,*

(2) *the sequence $\{T^n x\}$ is convergent to a fixed point y^* of T,*

(3) *y^* is the unique fixed point of T in the set $\Lambda = \{y \in X : d(T^{n_0} x, y) < \infty\}$,*

(4) *$d(y, y^*) \leq (1/(1 - L)d)(y, Ty)$, for all $y \in \Lambda$.*

Theorem 2.2. *Let $f_j : \mathcal{A} \to \mathcal{A}$ be continuous mappings with $f_j(0) = 0$ $(j = 0, 1)$, and let $\phi : \mathcal{A}^6 \to [0, \infty)$ be continuous in the first and second variables such that*

$$\begin{aligned}\Big\| f_j(\lambda a + \lambda b + cd) + f_j(\lambda a - \lambda b + cd) - 2\lambda^2 \left[f_j(a) + f_j(b)\right] \\ -2\left[(1 - j)\left(f_j(c)d^2\right)^{1-j} + j\left(c^2 f_j(d)\right)^j\right] + u^2 f_0(v) - f_1(u)v^2 \Big\| \leq (a, b, c, d, u, v),\end{aligned} \tag{2.3}$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ *and, for all* $a, b, c, d, u, v \in \mathcal{A}$, $j = 0, 1$. *If there exists a constant* m, $0 < m < 1$ *such that*

$$\phi(a,b,c,d,u,v) \leq 4m \operatorname{Min}\left\{\phi\left(\frac{a}{2},\frac{b}{2},\frac{c}{2},d,\frac{u}{2},\frac{v}{2}\right),\phi\left(\frac{a}{2},\frac{b}{2},c,\frac{d}{2},\frac{u}{2},\frac{v}{2}\right)\right\}, \tag{2.4}$$

for all $a, b, c, d, u, v \in \mathcal{A}$, *then there exists a unique double quadratic centralizer* (L, R) *on* $\mathcal{A}$ *satisfying*

$$\left\|f_0(a) - L(a)\right\| \leq \frac{1}{4(1-m)}\phi(a,a,0,0,0,0), \tag{2.5}$$

$$\left\|f_1(a) - R(a)\right\| \leq \frac{1}{4(1-m)}\phi(a,a,0,0,0,0), \tag{2.6}$$

for all $a \in \mathcal{A}$.

Proof. From (2.4), it follows that

$$\lim_i 4^{-i}\phi\left(2^i a, 2^i b, 2^i c, d, 2^i u, 2^i v\right) = 0, \tag{2.7}$$

for all $a, b, c, d, u, v \in \mathcal{A}$. Putting $j = 0$, $\lambda = 1$, $a = b$, $c = d = u = v = 0$ and replacing a by $2a$ in (2.3), we get

$$\left\|f_0(2a) - 4f_0(a)\right\| \leq \phi(a,a,0,0,0,0), \tag{2.8}$$

for all $a \in \mathcal{A}$. By the above inequality, we have

$$\left\|\frac{1}{4}f_0(2a) - f_0(a)\right\| \leq \frac{1}{4}\phi(a,a,0,0,0,0), \tag{2.9}$$

for all $a \in \mathcal{A}$. Consider the set $X := \{g : \mathcal{A} \to \mathcal{A} \mid g(0) = 0\}$ and introduce the generalized metric on X:

$$d(h,g) := \inf\left\{C \in \mathbb{R}^+ : \left\|g(a) - h(a)\right\| \leq C\phi(a,a,0,0,0,0),\ \forall a \in \mathcal{A}\right\}. \tag{2.10}$$

It is easy to show that (X, d) is complete. Now, we define the linear mapping $Q : X \to X$ by

$$Q(h)(a) = \frac{1}{4}h(2a), \tag{2.11}$$

for all $a \in \mathcal{A}$. Given $g, h \in X$, let $C \in \mathbb{R}^+$ be an arbitrary constant with $d(g,h) \leq C$, that is

$$\left\|g(a) - h(a)\right\| \leq C\phi(a,a,0,0,0,0), \tag{2.12}$$

for all $a \in \mathcal{A}$. Substituting a by $2a$ in the inequality (2.12) and using (2.4) and (2.11), we have

$$\begin{aligned} \|(Qg)(a) - (Qh)(a)\| &= \frac{1}{4}\|g(2a) - h(2a)\| \\ &\leq \frac{1}{4}C\phi(2a, 2a, 0, 0, 0, 0) \\ &\leq Cm\phi(a, a, 0, 0, 0, 0), \end{aligned} \tag{2.13}$$

for all $a \in \mathcal{A}$. Hence, $d(Qg, Qh) \leq Cm$. Therefore, we conclude that $d(Qg, Qh) \leq md(g, h)$, for all $g, h \in X$. It follows from (2.9) that

$$d(Qf_0, f_0) \leq \frac{1}{4}. \tag{2.14}$$

By Theorem 2.1, Q has a unique fixed point $L : \mathcal{A} \to \mathcal{A}$ in the set $X_1 = \{h \in X, \ d(f_0, h) < \infty\}$. On the other hand,

$$\lim_{n\to\infty} \frac{f_0(2^n a)}{4^n} = L(a), \tag{2.15}$$

for all $a \in \mathcal{A}$. By Theorem 2.1 and (2.14), we obtain

$$d(f_0, L) \leq \frac{1}{1-m} d(Qf_0, L) \leq \frac{1}{4(1-m)}, \tag{2.16}$$

that is, the inequality (2.5) is true, for all $a \in \mathcal{A}$. Now, substitute $2^n a$ and $2^n b$ by a and b respectively, put $c = d = u = v = 0$ and $j = 0$ in (2.15). Dividing both sides of the resulting inequality by 2^n, and letting n goes to infinity, it follows from (2.7) and (2.3) that

$$L(\lambda a + \lambda b) + L(\lambda a - \lambda b) = 2\lambda^2 L(a) + 2\lambda^2 L(b), \tag{2.17}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Putting $\lambda = 1$ in (2.17) we have

$$L(a + b) + L(a - b) = 2L(a) + 2L(b), \tag{2.18}$$

for all $a, b \in \mathcal{A}$. Hence L is a quadratic mapping.

Letting $b = 0$ in (2.17), we get $L(\lambda a) = \lambda^2 L(a)$, for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. We can show from (2.18) that $L(ra) = r^2 L(a)$ for any rational number r. It follows from the continuity of f_0 and ϕ that for each $\lambda \in \mathbb{R}$, $L(\lambda a) = \lambda^2 L(a)$. So,

$$L(\lambda a) = L\left(\frac{\lambda}{|\lambda|}|\lambda| a\right) = \frac{\lambda^2}{|\lambda|^2} L(|\lambda| a) = \frac{\lambda^2}{|\lambda|^2}|\lambda|^2 L(a) = \lambda^2 L(a), \tag{2.19}$$

for all $a \in \mathcal{A}$ and $\lambda \in \mathbb{C}(\lambda \neq 0)$. Therefore, L is quadratic homogeneous. Putting $j = 0$, $a = b = u = v = 0$ in (2.3) and replacing $2^n c$ by c, we obtain

$$\left\| \frac{f_0(2^n cd)}{4^n} - \frac{f_0(2^n c)}{4^n} d^2 \right\| \leq \frac{1}{2} 4^{-n} \phi(0,0,2^n c,d,0,0). \tag{2.20}$$

By (2.7), the right hand side of the above inequality tends to zero as $n \to \infty$. It follows from (2.15) that $L(cd) = L(c)d^2$, for all $c, d \in \mathcal{A}$. Therefore L is a quadratic left centralizer. Also, one can show that there exists a unique mapping $R : \mathcal{A} \to \mathcal{A}$ which satisfies

$$\lim_{n \to \infty} \frac{f_1(2^n a)}{4^n} = R(a), \tag{2.21}$$

for all $a \in \mathcal{A}$. The same manner could be used to show that R is a quadratic right centralizer. If we substitute u and v by $2^n u$ and $2^n v$ in (2.3) respectively, and put $a = b = c = d = 0$, and divide both sides of the obtained inequality by 8^n, then we get

$$\left\| u^2 \frac{f_0(2^n v)}{2^n} - \frac{f_1(2^n u)}{2^n} v^2 \right\| \leq 8^{-n} \phi(0,0,0,0,2^n u,2^n v). \tag{2.22}$$

Passing to the limit as $n \to \infty$, and again from (2.7), we conclude that $u^2 L(v) = R(u)v^2$, for all $u, v \in \mathcal{A}$. Therefore (L, R) is a quadratic double centralizer on $\mathcal{A}$. This completes the proof of this theorem. □

Now, we establish the superstability of double quadratic centralizers on Banach algebras as follows.

Corollary 2.3. *Let $0 < m < 1$, $p < 2$ with $2^{p-2} \leq m$, let $f_j : \mathcal{A} \to \mathcal{A}$ be continuous mappings with $f_j(0) = 0$ $(j = 0, 1)$, and let*

$$\begin{aligned} &\Big\| f_j(\lambda a + \lambda b + cd) + f_j(\lambda a - \lambda b + cd) - 2\lambda^2 \left[f_j(a) + f_j(b) \right] \\ &\quad - 2\left[(1-j)\left(f_j(c) d^2 \right)^{1-j} + j\left(c^2 f_j(d) \right)^j \right] + u^2 f_0(v) - f_1(u) v^2 \Big\| \\ &\quad \leq \left(\|a\|^p + \|b\|^p + \|c\|^p + \|u\|^p + \|v\|^p \right) \|d\|^p, \end{aligned} \tag{2.23}$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and, for all $a, b, c, d, u, v \in \mathcal{A}, j = 0, 1$. Then (f_0, f_1) is a double quadratic centralizer on $\mathcal{A}$.

Proof. The result follows from Theorem 2.2 by putting $\phi(a,b,c,d,u,v) = (\|a\|^p + \|b\|^p + \|c\|^p + \|u\|^p + \|v\|^p)\|d\|^p$. □

3. Stability of Quadratic Multipliers

Assume that $\mathcal{A}$ is a complex Banach algebra. Recall that a mapping $T : \mathcal{A} \to \mathcal{A}$ is a *quadratic multiplier* if T is a quadratic homogeneous mapping, and $a^2T(b) = T(a)b^2$, for all $a, b \in \mathcal{A}$ (see [16]). We investigate the stability of quadratic multipliers.

Theorem 3.1. *Let $f : \mathcal{A} \to \mathcal{A}$ be a continuous mapping with $f(0) = 0$ and let $\phi : \mathcal{A}^4 \to [0, \infty)$ be a function which is continuous in the first and second variables such that*

$$\left\| f(\lambda a + \lambda b) + f(\lambda a - \lambda b) - 2\lambda^2 \left[f(a) + f(b) \right] + c^2 f(d) - f(c)d^2 \right\| \leq \phi(a, b, c, d), \tag{3.1}$$

for all $\lambda \in \mathbb{T}$ and all $a, b, c, d \in \mathcal{A}$. Suppose exists a constant m, $0 < m < 1$, such that

$$\phi(2a, 2b, 2c, 2d) \leq 4m\phi(a, b, c, d), \tag{3.2}$$

for all $a, b, c, d \in \mathcal{A}$. Then there exists a unique multiplier T on $\mathcal{A}$ satisfying

$$\| f(a) - T(a) \| \leq \frac{1}{4(1-m)} \phi(a, a, 0, 0), \tag{3.3}$$

for all $a \in \mathcal{A}$.

Proof. It follows from (3.2) that

$$\lim_{n\to\infty} \frac{\phi(2^n a, 2^n b, 2^n c, 2^n d)}{4^n} = 0, \tag{3.4}$$

for all $a, b, c, d \in \mathcal{A}$. Putting $\lambda = 1$, $a = b, c = d = 0$ in (3.1), we obtain

$$\| f(2a) - 4f(a) \| \leq \phi(a, a, 0, 0), \tag{3.5}$$

for all $a \in \mathcal{A}$. Thus

$$\left\| f(a) - \frac{1}{4} f(2a) \right\| \leq \frac{1}{4} \phi(a, a, 0, 0), \tag{3.6}$$

for all $a \in \mathcal{A}$. Now we set $X := \{h : \mathcal{A} \to \mathcal{A} \mid h(0) = 0\}$ and introduce the generalized metric on X as

$$d(g, h) := \inf\{C \in \mathbb{R}^+ : \| g(a) - h(a) \| \leq C\phi(a, a, 0, 0), \ \ \forall a \in \mathcal{A}\}. \tag{3.7}$$

It is easy to show that (X, d) is complete. Consider the mapping $\Phi : X \to X$ defined by $\Phi(h)(a) = 1/4h(2a)$, for all $a \in \mathcal{A}$. By the same reasoning as in the proof of Theorem 2.2, Φ is strictly contractive on X. It follows from (3.6) that $d(\Phi f, f) \leq (1/4)$. By Theorem 2.1, Φ has a unique fixed point in the set $X_1 := \{h \in X : d(f, h) < \infty\}$. Let T be the fixed point of Φ. Then

T is the unique mapping with $T(2a) = 4T(a)$, for all $a \in \mathcal{A}$ such that there exists $C \in (0, \infty)$ satisfying

$$\|T(x) - f(x)\| \le C\phi(a, a, 0, 0), \tag{3.8}$$

for all $a \in \mathcal{A}$. On the other hand, we have $\lim_{n\to\infty} d(\Phi^n(f), T) = 0$. Thus

$$\lim_{n\to\infty} \frac{1}{4^n} f(2^n x) = T(x), \tag{3.9}$$

for all $a \in \mathcal{A}$. Hence

$$d(f, T) \le \frac{1}{1-m} d(T, \Phi(f)) \le \frac{1}{4(1-m)}. \tag{3.10}$$

This implies the inequality (3.3). It follows from (3.1), (3.4) and (3.9) that

$$\begin{aligned} &\left\| T(\lambda a + \lambda b) + T(\lambda a - \lambda b) - 2\lambda^2 T(a) - 2\lambda^2 T(b) \right\| \\ &\quad = \lim_{n\to\infty} \frac{1}{4^n} \left\| T(2^n(\lambda a + \lambda b)) + T(2^n(\lambda a - \lambda b)) - 2\lambda^2 T(2^n a) - 2\lambda^2 T(2^n b) \right\| \\ &\quad \le \lim_{n\to\infty} \frac{1}{4^n} \phi(2^n a, 2^n b, 0, 0) = 0, \end{aligned} \tag{3.11}$$

for all $a, b \in \mathcal{A}$. Thus

$$L(\lambda a + \lambda b) + L(\lambda a - \lambda b) = 2\lambda^2 L(a) + 2\lambda^2 L(b), \tag{3.12}$$

for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Letting $b = 0$ in (3.14), we have $L(\lambda a) = \lambda^2 L(a)$, for all $a, b \in \mathcal{A}$ and $\lambda \in \mathbb{T}$. Now, it follows from the proof of Theorem 2.1 and continuity of f and ϕ that T is $\mathbb{C}$-linear. If we substitute c and d by $2^n c$ and $2^n d$ in (3.1), respectively, and put $a = b = 0$ and we divide the both sides of the obtained inequality by 8^n, we get

$$\left\| c^2 \frac{f(2^n d)}{4^n} - \frac{f(2^n c)}{4^n} d^2 \right\| \le \frac{\phi(0, 0, 2^n c, 2^n d)}{8^n}. \tag{3.13}$$

Passing to the limit as $n \to \infty$, and from (3.4) we conclude that $c^2 T(d) = T(c) d^2$, for all $c, d \in \mathcal{A}$. □

Using Theorem 3.1, we establish the superstability of quadratic multipliers on Banach algebras.

Corollary 3.2. *Let* $0 < m < 1$, $p < 2/3$ *with* $2^{3p-2} \leq m$, *and* $f : \mathcal{A} \to \mathcal{A}$ *be a continuous mapping with* $f(0) = 0$, *and let*

$$\left\| f(\lambda a + \lambda b) + f(\lambda a - \lambda b) - 2\lambda^2 [f(a) + f(b)] + c^2 f(d) - f(c) d^2 \right\| \leq \left(\|a\|^p + \|ab\|^p \right) \|c\|^p \|d\|^p, \tag{3.14}$$

for all $\lambda \in \mathbb{T} = \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ *and, for all* $a, b, c, d \in \mathcal{A}$. *Then* f *is a quadratic multiplier on* $\mathcal{A}$.

Proof. The results follows from Theorem 3.1 by putting $\phi(a, b, c, d) = (\|a\|^p + \|b\|^p)\|c\|^p\|d\|^p$. □

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