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IN MEMORIAM

PÁL ERDŐS
(March 26, 1913 - September 20, 1996)

LAJOS TAKÁCS[1]
Case Western Reserve University
Department of Mathematics
Cleveland, OH 44106 USA

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Pál Erdős was born in Budapest, Hungary, on March 26, 1913. His father, Lajos Erdős was a high school teacher of mathematics and physics. His mother, Anna Wilhelm, was also a trained mathematics teacher, but did not have a teaching job. Shortly before Pali (as he was called in his family) was born, his two sisters died of scarlet fever. He was raised as an only child. He was a child prodigy. When he was four, he could multiply three-digit numbers in his head. He did not like public schools and was educated mostly at home. At seventeen he enrolled at the Péter Pázmány Tudományegyetem (University of Budapest), studying mathematics. He wrote his first paper in mathematics at eighteen: Beweis eines Satzes von Tschebyschef, *Acta Scientiarum Mathematicarum* (Szeged) **5** (1932), 194-198. This paper contains a simple elementary proof of a theorem which states that there is at least one prime number between a and $2a$ if $a \geq 2$. In 1845, J. Bertrand actually checked that this theorem is true for $2 \leq a \leq 3 \times 10^6$ and in 1852, P.L. Chebyshev proved that it is true for every $a \geq 2$. Actually, Chebyshev obtained this result as a by-product of a more general theorem on the distribution of primes. Erdős' proof is based on the inequalities

$$\frac{1}{2a} < \binom{2a}{a}\frac{1}{2^{2a}} < \frac{1}{4},$$

valid for $a \geq 5$. In 1934, he obtained his doctor's degree with his thesis "On the prime numbers in certain arithmetic sequences." His advisor was Lipót Fejér. At the invitation of Professor Louis Mordell, he spent four years at the University of Manchester. During these years his main work was on number theory. In 1938, he left Manchester and joined the Institute for Advanced Study at Princeton. He spent the war years and the following years at various universities in the U.S. These were very productive years for Erdős. He worked on many research problems in number theory, geometry, set theory, combinatorics, graph theory, probability theory, and related fields and achieved outstanding results. In 1939, Mark Kac and Erdős proved that if $\nu(m)$ denotes the number of prime divisors of m and if $K_n(x)$ is the number of integers $m = 1, 2, \ldots, n$ for which

$$\nu(m) < \log \log n + x\sqrt{\log \log n},$$

then

$$\lim_{n \to \infty} \frac{K_n(x)}{n} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du$$

for any real x.

In 1949, both Atle Selberg and Erdős gave completely elementary proofs for the prime number theorem, which states that if $\pi(x)$ denotes the number of primes $\leq x$, then

$$\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1.$$

[1]Postal address: 2410 Newbury Drive, Cleveland Heights, OH 44118, USA.

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Their remarkable proofs surprised the mathematical world and for their achievements they were awarded in 1950 the Frank Nelson Cole prize in number theory. This prize is awarded by the American Mathematical Society every five years for an outstanding research memoir which has appeared in the previous five years.

Pál Erdős was an ardent problem solver and he proposed many problems for solution to interested mathematicians. He had a great repertoire of problems which can be stated simply, but whose solutions are extremely difficult. He offered monetary rewards for the solutions of various problems. His offers ranged from $25 to $3000. He once paid $1000 to E. Szemerédi who demonstrated that a sequence of positive integers of positive density always contains arbitrarily long arithmetic progressions. This was the largest amount ever paid by Erdős. The $3000 problem, which is still open, is as follows: If you have an infinite sequence of positive integers, the sum of whose reciprocals diverges, does the sequence contain arbitrarily long arithmetic progressions? An affirmative answer to this problem would imply that the sequence of primes contains arithmetic progressions of any length $k = 3, 4, \ldots$. The longest known arithmetic progression of primes consists of $k = 19$ primes. This was found by Paul A. Pritchard in 1984 by an extensive computer search consuming almost 14,000 hours of computer time.

After spending 16 years in the U.S., Erdős returned to Hungary in 1954 and became a member of the Hungarian Academy of Sciences. The Academy provided him with regular financial support. Simultaneously, he had a visiting professorship at the Technion in Haifa, Israel. He was a traveling scientist who never had a regular academic appointment. He traveled from one mathematical center to the next one, exchanging ideas with his fellow mathematicians on many subjects, and writing joint papers. His itineraries varied, but included mostly the U.S., Canada, England, Hungary, Israel, and Poland. He published about 1500 papers, more than any other mathematician who ever lived. Erdős had over 400 coauthors who are proud to have the Erdős number of 1. If a person is not a coauthor of Erdős, but publishes a joint paper with somebody who is a coauthor of Erdős, then he or she has Erdős number of 2. Albert Einstein has Erdős number of 2, the late Ernst Straus being the link.

Erdős died on September 20, 1996, while attending a mathematical meeting in Warsaw. Erdős used to sign his name as P. Erdős PGOM. The abbreviation means "poor good old man." Indeed he was a good man. He took special interest in child prodigies. In 1959, he discovered a 12-year old Hungarian child prodigy, Lajos Pósa. Erdős encouraged Pósa to work in graph theory, and at 14 Pósa became the youngest person who had Erdős number of 1.

Besides receiving the Cole prize in 1950, Erdős received a Kossuth prize in 1958 in Hungary and shared a Wolf Foundation prize in Israel in 1983. In 1991, he received an honorary doctor's degree from Cambridge University. He was a member of several academies. Besides the Hungarian Academy of Sciences, he was a member of the Royal Society, the National Academy of Sciences in the U.S.A., the Netherland Academy, the Australian Academy, and the Academy of Sciences in India.

Before finishing this short reminiscence, I must say a few words about the Erdősian language. For example, "wine, women and song" in Erdősian translates as "poison, bosses and noise." He called children "epsilons" and men "slaves."

Erdős never married. In his words, "no boss captured him." He was always in a happy mood and he devoted his whole life to the mathematical sciences. His aim was to find simple, elegant solutions to difficult problems - solutions which are in the BOOK. The BOOK in Erdős' parlance contains the best solution of every problem. The greatest praise that Erdős had for a solution was "This solution is from the BOOK."

Young and old mathematicians will miss his remarkable contribution to the progress of the mathematical sciences. Thanks to the unselfish devotion of Pál Erdős, we are all enriched in our mathematical knowledge.