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Research Article

Robust Stability of Uncertain Systems over Network with Bounded Packet Loss

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This paper investigates the problem of robust stability of uncertain linear discrete-time system over network with bounded packet loss. A new Lyapunov functional is constructed. It can more fully utilize the characteristics of the packet loss; hence the established stability criterion is more effective to deal with the effect of packet loss on the stability. Numerical examples are given to illustrate the effectiveness and advantage of the proposed methods.

1. Introduction

A networked control system (NCS) is a system whose feedback loop or (and) control loop is (are) connected via a communication network, which may be shared with other devices. The main advantages of NCS are low cost, reduced weight, high reliability, simple installation, and maintenance. As a result, the NCSs have been applied in many fields, such as mobile sensor networks, manufacturing systems, teleoperation of robots, and aircraft systems [1].

However, the insertion of the communication networks in control loops will bring some new problems. One of the most common problems in NCSs, especially in wireless sensor networks, is packet dropout, that is, packets can be lost due to communication noise, interference, or congestion [2]. Some results on this issue have been available. Generally, in these results there are two types of packet-loss model. One is stochastic packet loss ([2–5]; etc), another is arbitrary but bounded packet loss ([6–9]; etc).

Here, we are concerned about the arbitrary but bounded packet loss. For this case, there are two approaches available. One approach is based on switched system theory;

another one is based on the theory of time-varying-delay system. Yu et al. [7] modeled the packet-loss process as an arbitrary but finite switching signal. This enables them to apply the theory from switched systems to stabilize the NCS. However, Yu et al. [7] adopted a common Lyapunov function and the results are quadratic. Xiong and Lam [9] utilized a packet-loss-dependent Lyapunov function to establish the stabilization condition, which is less conservative than that of Yu et al. [7]. Unfortunately, however, in the stability condition of their approaches the system matrices appear in the forms of power and cross-multiplication among them. Therefore, it is difficult to deal with the systems with parametric uncertainty by using these approaches. In contrast, if utilizing the delay system approach, the system matrices are affine in the stability condition. Hence, this approach suits the uncertain systems. However, it may be very conservative if directly using the existing delay system approaches (e.g., [10–14]) to deal with the bounded packet loss. The main reason is that the existing approaches can not fully utilize the characteristic of packet loss. Therefore, for the systems in the simultaneous presence of parameter uncertainties and bounded packet loss, the problem of robust stability has not been fully investigated and remains to be challenging, which motivates the present study.

In this paper, we study the robust stability problem for uncertain discrete-time systems with bounded networked packet loss. First, we transform the packet loss into a time-varying input delay. Second, we note that the considered time-varying delay has a new characteristic. It is different with the general time-varying delay, that is, the considered time delay will change with some laws in the interval of two consecutive successful transmissions of the network, which is not possessed by general time-varying delay. In order to utilize this characteristic, we define a new Lyapunov functional. It does not only depend on the bound of the delay, but also on the rate of its change. Due to more fully utilizing the properties of the packet loss (that is the time-varying delay induced by the packet loss), the established stability criterion shows its less conservativeness. The construction of Lyapunov functional is inspired by Fridman [15], where the stability of sampled-data control systems is considered. It does not mean that the method developed in this paper is trivial. In fact, as it is shown in the Section 3 of this paper that the properties of induced-delay are more complicate than that in Fridman [15], such that the method of Fridman [15] cannot directly be applied to the problem considered in this paper. Finally, three examples are provided to illustrate the effectiveness of the developed results.

2. Problem Formulation

The framework of NCSs considered in the paper is depicted in Figure 1. The plant to be controlled is modeled by linear discrete-time system:

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k), \tag{2.1}$$

where $k \in \mathbb{Z}_+$ is the time step, $x(k) \in \mathbb{R}^{n_x}$ and $u(k) \in \mathbb{R}^{n_u}$ are the system state and control input, respectively. $x_0 \triangleq x(0)$ is the initial state. A and B are known real constant matrices with appropriate dimension. ΔA and ΔB are unknown matrices describing parameter uncertainties.

In this paper, the parameter uncertainties are assumed to be of the form $\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = DF(k)\begin{bmatrix} E_1 & E_2 \end{bmatrix}$, where D, E_1, and E_2 are known real constant matrices of appropriate

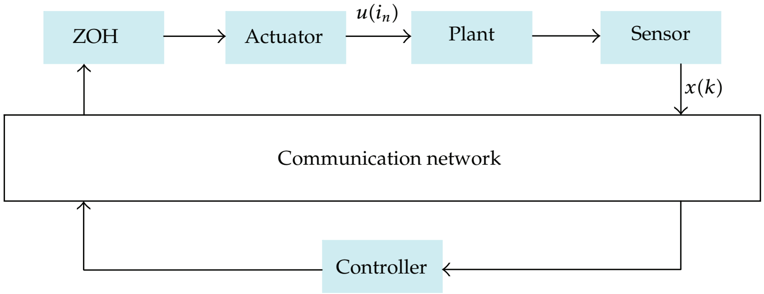


Figure 1: Networked control systems with packet loss.

dimensions, and $F(k)$ is an unknown real-valued time-varying matrix satisfying $F(k)F^T(k) \leq I$.

Networks exist between sensor and controller and between controller and zero-order holder (ZOH). The sensor is clock driven, the controller and ZOH are event driven and the data are transmitted in a single packet at each time step. As have been mentioned in Section 1, this paper only considers the network packet loss. Then it is assumed that there is not any network-induced delay.

Let $\mathbb{S} \triangleq \{i_1, i_2, i_3, \ldots\} \subset \{0, 1, 2, 3, \ldots\}$ denote the sequence of time points of successful data transmissions from the sensor to the zero-order hold at the actuator side and $i_n < i_{n+1}$ for any $n = 1, 2, 3, \ldots$.

Assumption 2.1. The number of consecutive packet loss in the network is less than $\overline{m}$, that is

$$i_{n+1} - i_n - 1 \leq \overline{m}, \quad \forall n \in \{1, 2, 3, \ldots\}. \tag{2.2}$$

Remark 2.2. Assumption 2.1 is similar to that in Liu et al. [8]. From the physical point of view, it is natural to assume that only a finite number of consecutive packet losses can be tolerated in order to avoid the NCS becoming open loop. Thus, the number of consecutive packet loss in the networks should be less than the finite number $\overline{m}$.

The networked controller is a state-feedback controller:

$$u = Kx. \tag{2.3}$$

From the viewpoint of the ZOH, the control input is

$$u(k) = u(i_n) = Kx(i_n), \quad i_n \leq k < i_{n+1}. \tag{2.4}$$

The initial inputs are set to zero: $u(k) = 0$, $0 \leq k < i_1$. Hence the closed-loop system becomes

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)Kx(i_n), \tag{2.5}$$

for $k \in [i_n, i_{n+1})$. The objective of this paper is to analyze the robust stability of NCS (2.5).

Remark 2.3. The packet loss process can take place in the sensor-controller link and the controller-actuator link. Since the considered controller is static in this paper, it is equivalent to incorporate the double-sided packet loss as a single-packet loss process. This is just the reason that this paper only considers the single-packet loss process. However, if the controller is online implemented, then one should clearly consider the double-sided packet loss rather than incorporate them as a single-packet loss process. For such case, readers can be referred to Ding [16], which systematically addressed the modeling and analysis methods for double-sided packet loss process.

3. Stability of Networked Control Systems

In this section, we analyze the stability property of NCSs. Here we firstly investigate the stability of NCS (2.5) when the plant (2.1) without any uncertainty, that is, $\Delta A = 0$ and $\Delta B = 0$. we have the following result.

Theorem 3.1. *Assuming $\Delta A = 0$ and $\Delta B = 0$, NCS (2.5) with arbitrary packet-loss process is asymptotically stable if there exist matrices $P > 0$, $Z > 0$, Q_1, Q_2, M, N, and S such that the following LMI holds*

$$\Phi_1 \triangleq \Xi_1 + \Xi_2 + \Xi_2^T + \overline{m}(\Xi_3 + \Xi_5) - (\overline{m}+1)\Xi_4 < 0, \tag{3.1}$$

$$\begin{bmatrix} \widetilde{\Phi}_2 & N \\ * & -\dfrac{1}{\overline{m}-1}Z \end{bmatrix} < 0, \tag{3.2}$$

$$\Phi_3 \triangleq \Xi_1 + \Xi_6 + \Xi_6^T - \Xi_4 - \frac{1}{\overline{m}}\Xi_7 < 0, \tag{3.3}$$

$$\begin{bmatrix} P + (\overline{m}+1)\Lambda_1 & (\overline{m}+1)\Lambda_2 \\ * & (\overline{m}+1)\Lambda_3 \end{bmatrix} > 0, \tag{3.4}$$

where $\widetilde{\Phi}_2 \triangleq \Xi_1 + \Xi_2 + \Xi_2^T + \Xi_3 + \Xi_5 - 2\Xi_4$ *and*

$$\begin{gathered}
\Xi_1 = \operatorname{diag}\{P, -P, 0\}, \qquad \Xi_6 = \begin{bmatrix} S & -SA & -SBK \end{bmatrix}, \\
\Xi_2 = \begin{bmatrix} M & -MA+N & -MBK & -N \end{bmatrix}, \quad \Lambda_1 = \frac{Q_1 + Q_1^T}{2}, \\
\Lambda_2 = -Q_1 + Q_2, \quad \Lambda_3 = \frac{Q_1 + Q_1^T}{2} - Q_2 - Q_2^T, \\
\Xi_3 = \begin{bmatrix} \Lambda_1 & 0 & \Lambda_2 \\ * & 0 & 0 \\ * & * & \Lambda_3 \end{bmatrix}, \qquad \Xi_4 = \begin{bmatrix} 0 & 0 & 0 \\ * & \Lambda_1 & \Lambda_2 \\ * & * & \Lambda_3 \end{bmatrix}, \\
\Xi_5 = \begin{bmatrix} Z & -Z & 0 \\ * & Z & 0 \\ * & * & 0 \end{bmatrix}, \qquad \Xi_7 = \begin{bmatrix} 0 & 0 & 0 \\ * & Z & -Z \\ * & * & Z \end{bmatrix}.
\end{gathered} \tag{3.5}$$

Proof. Define

$$d_k = k - i_n, \quad i_n \le k < i_{n+1}, \tag{3.6}$$

then the NCS (2.5) can be represented as a delay system:

$$x(K+1) = Ax(K) + BKx(k-d_k), \quad k \in [i_n, i_{n+1}). \tag{3.7}$$

Inspired by Fridman [15], we construct the following new functional candidate as:

$$V(k) \triangleq V_1(k) + V_2(k) + V_3(k), \tag{3.8}$$

with $V_1(k) = x^T(k)Px(k)$ and

$$\begin{aligned} V_2(k) =& (i_{n+1} - i_n - d_k) \begin{bmatrix} x(k) \\ x(k-d_k) \end{bmatrix}^T \\ &\times \begin{bmatrix} \dfrac{Q_1 + Q_1^T}{2} & -Q_1 + Q_2 \\ * & \dfrac{Q_1 + Q_1^T}{2} - Q_2 - Q_2^T \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d_k) \end{bmatrix}, \\ V_3(k) =& (i_{n+1} - i_n - d_k) \sum_{i=k-d_k}^{k-1} \eta^T(i) Z \eta(i), \end{aligned} \tag{3.9}$$

where $\eta(i) = x(i+1) - x(i)$ and $P > 0$, $Z > 0$, Q_1, Q_2 are to be determined.

From (2.2) and (3.6), we know that $d_k \le \overline{m}$. Therefore, similar with the discussion of Fridman [15], it can be seen that (3.4) guarantees (3.8) to be a Lyapunov functional. For $k \in [i_n, i_{n+1})$, we, respectively, calculate the forward difference of the functional (3.8) along the solution of system (3.7) by two cases.

Case 1 ($i_n \le k < i_{n+1} - 1$). In this case, we have $d_{k+1} = d_k + 1$. Then,

$$\Delta V_1(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k), \tag{3.10}$$

$$\begin{aligned} \Delta V_2(k) =& (i_{n+1} - i_n - d_k - 1)\xi^T(k)\Xi_3\xi(k) \\ &- (i_{n+1} - i_n - d_k)\xi^T(k)\Xi_4\xi(k), \\ \Delta V_3(k) =& (i_{n+1} - i_n - d_k - 1)[x(k+1) - x(k)]^T \\ &\times Z[x(k+1) - x(k)] - \sum_{i=k-d_k}^{k-1} \eta^T(i) Z \eta(i), \end{aligned} \tag{3.11}$$

where $\xi(k) = [x\ (k+1)^T\ x(k)^T\ x(k-d_k)^T]^T$.

In addition, for any appropriately dimensioned matrices M and N the following relationships always hold:

$$0 = 2\xi^T(k)\mathrm{M}[x(k+1) - Ax(k) - BKx(k-d_k)] \;, \tag{3.12}$$

$$\begin{aligned} 0 &\le 2\xi^T(k)N[x(k) - x(k-d_k)], \\ &+ d_k\xi^T(k)NZ^{-1}N^T\xi(k) + \sum_{i=k-d_k}^{k-1}\eta^T(i)Z\eta(i). \end{aligned} \tag{3.13}$$

Then, from (3.10)–(3.13), we have

$$\Delta V(k) \le \xi^T(k)\Omega\xi(k), \tag{3.14}$$

where $\Omega = \Xi_1 + \Xi_2 + \Xi_2^T + (i_{n+1} - i_n - d_k - 1)(\Xi_3 + \Xi_5) - (i_{n+1} - i_n - d_k)\Xi_4 + d_kNZ^{-1}N^T$.

Now we prove (3.1) and (3.2) guaranteeing that $\Omega < 0$. By Schur complement, (3.2) is equivalent to

$$\Phi_2 \triangleq \widetilde{\Phi}_2 + (\overline{m} - 1)NZ^{-1}N^T < 0. \tag{3.15}$$

Then from (3.1) and (3.15), we know that $\Phi_1 < 0$ and $\Phi_2 < 0$. Hence for any scalar $\alpha \in [0,1]$, the following inequality holds:

$$\alpha\Phi_1 + (1-\alpha)\Phi_2 < 0. \tag{3.16}$$

Noting that in this case $0 \le d_k \le i_{n+1} - i_n - 2$, then we have $0 \le i_{n+1} - i_n - 2 - d_k \le \overline{m} - 1$. Therefore, $0 \le (i_{n+1} - i_n - d_k - 2)/(\overline{m} - 1) \le 1$. By setting $\alpha = (i_{(n+1)} - i_n - d_k - 2)/(\overline{m} - 1)$, from (3.16) we obtain that $\Omega + (\overline{m} - (i_{n+1} - i_n - 1))NZ^{-1}N^T < 0$. Due to $\overline{m} - (i_{n+1} - i_n - 1) \ge 0$, the inequality above implies $\Omega < 0$ holds. Therefore, in this case $\Delta V(k) < 0$ holds.

Case 2 ($k = i_{n+1} - 1$). In this case, we have $d_k = i_{n+1} - i_n - 1$ and $d_{k+1} = 0$. The

$$\Delta V_1(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k), \tag{3.17}$$

$$\begin{aligned} \Delta V_2(k) &= -\xi^T(k)\Xi_4\xi(k), \\ \Delta V_3(k) &= -\sum_{i=k-d_k}^{k-1}\eta^T(i)Z\eta(i). \end{aligned} \tag{3.18}$$

By the Jensen's inequality [17], we have

$$\begin{aligned} &-\sum_{i=k-d_k}^{k-1}\eta^T(i)Z\eta(i) \\ &\quad\le -\frac{1}{\overline{m}}(x(k) - x(k-d_k))^T Z(x(k) - x(k-d_k)). \end{aligned} \tag{3.19}$$

In addition, for any appropriately dimensioned matrix S the following relationship always holds:

$$0 = 2\xi^T(k)S[x(k+1) - Ax(k) - BKx(k-d_k)]. \tag{3.20}$$

Then, from (3.17)–(3.20), we have

$$\Delta V(k) \leq \xi^T(k)\left[\Xi_1 + \Xi_6 + \Xi_6^T - \Xi_4 - \frac{1}{\overline{m}}\Xi_7\right]\xi^T(k). \tag{3.21}$$

In this case, we can see that (3.3) guarantees $\Delta V(k) < 0$.

From both Cases 1 and 2, we can conclude $\Delta V(k) < 0$ for $k \in [i_n, i_{n+1})$, for all $n \in \{1, 2, 3, \ldots\}$. Then, from the Lyapunov stability theory, the NCS (2.5) with arbitrary packet-loss process satisfying (2.2) is asymptotically stable. □

Remark 3.2. The proposed stability criterion in Theorem 3.1 is dependent on the bound of the packet loss. Furthermore, from the proof of Theorem 3.1, we can see that the varying rate of packet-loss-induced delays is fully utilized to obtain the stability condition. According to the difference of induced delays' varying rates, we separate $k \in [i_n, i_{n+1})$ into two parts, that is $i_n \leq k < i_{n+1} - 1$ and $k = i_{n+1} - 1$. For the two cases, we, respectively, calculate the forward difference of the functional and guarantee it less than zero, such that the NCS is asymptotically stable. Theorem 3.1 is more effective to deal with packet loss than the existing time-varying delay system approaches in the sense that Theorem 3.1 can allow a larger upper bound of the packet loss, which will be demonstrated in an example in next section.

Remark 3.3. In Fridman [15], the continuous-time sampled control system is considered. The varying rate of sampling-induced delays is constant when the derivative of the Lyapunov functional is calculated. However, in our paper, the varying rate of packet-loss-induced delays will be changing when the difference of the Lyapunov functional is calculated. Therefore, the method of Fridman [15] for continuous-time domain cannot directly be applied to the problem of discrete-time domain considered in this paper.

Note that in LMIs (3.1)–(3.3) the system matrices A and B appear in affine form, thus the stability condition presented in Theorem 3.1 can be readily extended to cope with uncertain systems (2.1). By using Theorem 3.4 and the well-known S-procedure, we can easily obtain the following theorem, and hence its proof is omitted.

Theorem 3.4. *NCS* (2.5) *is robustly asymptotically if there exist matrices $P > 0$, $Z > 0$, Q_1, Q_2, M, N, S and scalar ε_1, ε_2, ε_3 satisfying* (3.4) *and the following LMIs:*

$$\begin{bmatrix} \Phi_1 & MD & \varepsilon_1\Pi^T \\ * & -\varepsilon_1 I & 0 \\ * & * & -\varepsilon_1 I \end{bmatrix} < 0, \qquad \begin{bmatrix} \Phi_3 & SD & \varepsilon_3\Pi^T \\ * & -\varepsilon_3 I & 0 \\ * & * & -\varepsilon_3 I \end{bmatrix} < 0, \qquad \begin{bmatrix} \widetilde{\Phi}_2 & N & MD & \varepsilon_2\Pi^T \\ * & -\dfrac{1}{\overline{m}-1}Z & 0 & 0 \\ * & * & -\varepsilon_2 I & 0 \\ * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{3.22}$$

where Φ_1, $\widetilde{\Phi}_2$, Φ_3 are given in (3.5) *and* $\Pi = \begin{bmatrix} 0 & -E_1 & -E_2K \end{bmatrix}$.

Remark 3.5. Because the LMIs of Theorem 3.1 are affine in the system matrices A and B, it is readily extended to deal with the systems with norm-bounded parameter uncertainty (i.e., Theorem 3.4). With similar reason it can also be easily extended to deal with the systems with polytopic-type uncertainty. The reason why we only consider one of the cases is to avoid the paper being too miscellaneous.

Remark 3.6. It is worth to reiterate that if there is only packet loss, the method of this paper is more suitable than the general time-delay method. However, if there simultaneously exist network-induced delay and packet loss, the method of this paper is not applicable, but the general time-delay method is still valid. For example, Yue et al. [18], Gao and Chen [19], and Huang and Nguang [20] considered the networked control systems with both network-induced delay and packet loss, where Yue et al. [18] and Gao and Chen [19] are the methods of continuous-time domain, Huang and Nguang [20] is the method of discrete-time domain. Yue et al. [18] investigated the H_∞ regulating control for network-based uncertain systems. Gao and Chen [19] studied the H_∞ output tracking control for network-based uncertain systems. For the uncertain networked control system with random time delays, Huang and Nguang [20] analyzed robust disturbance attenuation performance and proposed the corresponding design method for the controllers.

4. Numerical Examples

In this section, three examples are provided to illustrate the effectiveness and advantage of the proposed stability results.

Example 4.1. Borrow the system considered by Gao and Chen [10], where $\Delta A = 0$, $\Delta B = 0$ and

$$A = \begin{bmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.0001 \\ -0.0053 \end{bmatrix}. \tag{4.1}$$

Here we are interested in the allowable maximum bound of dropout loss that guarantees the asymptotic stability of the closed-loop system. For extensive comparison purpose, we let the controller gain matrices take two different values: $K_1 = \begin{bmatrix} 105.2047 & 25.3432 \end{bmatrix}$ and $K_2 = \begin{bmatrix} 110.6827 & 34.6980 \end{bmatrix}$. By using different methods, the calculated results are presented in Table 1. From the table, it is easy to see that the method proposed in this paper is more effective than the others. But it is never to say that the proposed method in this paper is more suitable to deal with the time-delay; it is only to show that the proposed method is more suitable to deal with the packet-loss than the general time-delay methods.

Example 4.2. Borrow the system considered by Wang et al. [14], where $\Delta A = 0$, $\Delta B = 0$ and

$$A = \begin{bmatrix} 1 & 0.01 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.01 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 0.01 & 0 \\ 0 & 0 \\ 0 & 0.01 \end{bmatrix},$$
$$K = \begin{bmatrix} -0.0166 & -0.2248 & 0.0006 & 0.0016 \\ 0.0004 & 0.0016 & -0.0165 & -0.2271 \end{bmatrix}. \tag{4.2}$$

Table 1: Calculated maximum bound of dropout loss.

Controller gain matrix	$K = K_1$	$K = K_2$
[10, Theorem 1]	7	5
[11, Theorem 1]	6	5
[12, Theorem 1]	6	4
[13, Theorem 1]	7	5
Theorem 3.1 in this paper	13	10

When lower bound of the equivalent delay is 0, the allowable maximum upper bound of the equivalent delay is 13 as reported in Wang et al. [14]. Therefore, if there is only bounded packet loss, by using the method of Wang et al. [14], the allowable maximum bound of dropout loss is 13. However, by using Theorem 3.1 of this paper, one can obtain that the allowable maximum bound of dropout loss is 190. This example shows again that the proposed method is more suitable to deal with the packet-loss than the general time-delay methods.

Example 4.3. Consider the following uncertain system:

$$x(k+1) = \begin{bmatrix} 1.0078 + \alpha(k) & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix} x(k) + \begin{bmatrix} -0.1 \\ -5.3 + \alpha(k) \end{bmatrix} u(k), \tag{4.3}$$

where $|\alpha(k)| \leq \overline{\alpha}$. The system matrices can be written in the form of (2.1) with matrices given by

$$A = \begin{bmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ -5.3 \end{bmatrix}, \quad D = \overline{\alpha},$$
$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad F(k) = \frac{\alpha(k)}{\overline{\alpha}}. \tag{4.4}$$

Now assume that the controller gain matrix is $K = \begin{bmatrix} 0.1052 & 0.0253 \end{bmatrix}$, and our purpose is to determine the upper value of $\overline{\alpha}$ such that the closed-loop system is robustly stable. By using Theorem 3.4, the detail calculated result is shown in Table 2.

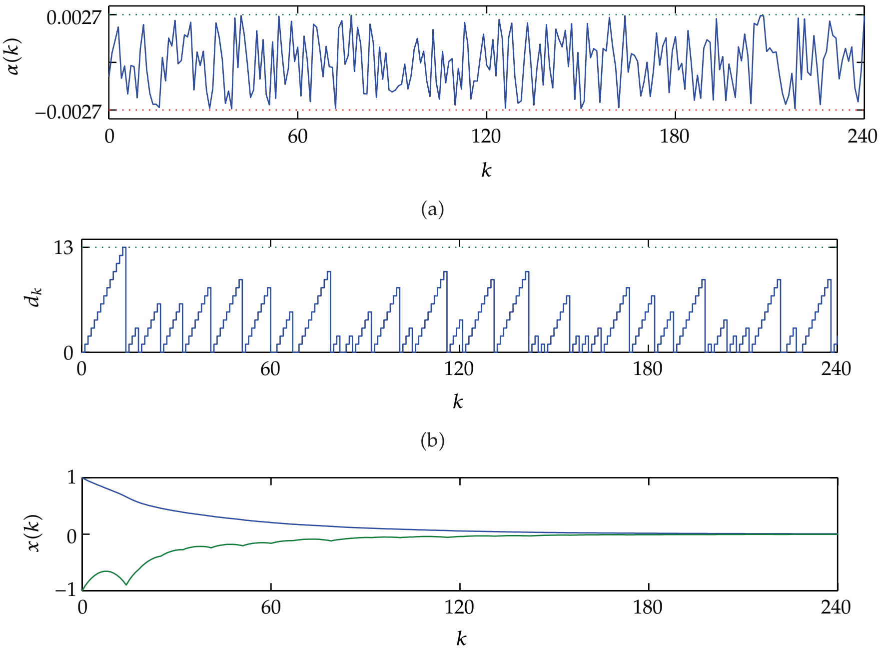
In the following, we will present some simulation results. Assume the initial condition to be $x(k) = [1 \; -1]^T$ for $k \leq 0$. Let $\alpha(k)$ changes randomly between -0.0027 and 0.0027, which is shown in Figure 2(a). In addition, let the upper of dropout loss is 13, which is shown in Figure 2(b). Then, the state response of the close-loop system is given in Figure 2(c). It can be seen from this figure that the system is robustly asymptotically stable, which shows the validity of the method proposed in this paper.

5. Conclusions

The problem of robust stability analysis for uncertain systems over network with bounded packet loss has been considered in this paper. A new Lyapunov functional is constructed.

Table 2: Calculated upper values of $\overline{\alpha}$ for different cases.

$\overline{m}$	3	5	7	9	11	13
Upper value of $\overline{\alpha}$	0.0105	0.0104	0.0101	0.0096	0.0077	0.0027



(c)

Figure 2: Simulation results.

This Lyapunov functional not only utilizes the bound of the packet loss but also utilizes the varying rate of the packet-loss-induced delays, which aims at reducing the conservatism of the results. Numerical examples are also presented to demonstrate the effectiveness and advantages of the proposed approach.

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