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Research Article

Generalized Fuzzy Quasi-Ideals of an Intraregular Abel-Grassmann's Groupoid

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We have introduced a new nonassociative class of Abel-Grassmann's groupoid, namely, intraregular and characterized it in terms of its $(\in, \in \vee_q)$-fuzzy quasi-ideals.

1. Introduction

Fuzzy set theory and its applications in several branches of science are growing day by day. These applications can be found in various fields such as computer science, artificial intelligence, operation research, management science, control engineering, robotics, expert systems, and many others. Fuzzy mappings are used in fuzzy image processing, fuzzy data bases, fuzzy decision making, and fuzzy linear programming. It has wide range of applications in engineering such as civil engineering, mechanical engineering, industrial engineering, and computer engineering. Moreover, the usage of fuzzification can be found in mechanics, economics, fuzzy systems, and genetic algorithms.

In [1], Mordeson has discovered the grand exploration of fuzzy semigroups, where theoretical exploration of fuzzy semigroups and their applications used in fuzzy coding, fuzzy finite state mechanics, and fuzzy languages. The use of fuzzification in automata and formal language has widely been explored.

Fuzzy set theory on semigroups has already been developed. In [2], Murali defined the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on a fuzzy subset. The idea of quasicoincidence of a fuzzy point with a fuzzy set is defined in [3]. Bhakat and Das [4, 5] gave the concept of (α, β)-fuzzy subgroups by using the "belongs to" relation $\in$ and "quasicoincident with" relation q between a fuzzy point and a fuzzy subgroup, and introduced the concept of an $(\in, \in \vee q)$-fuzzy subgroups, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$. Davvaz defined $(\in, \in \vee q)$-fuzzy subnearrings and ideals of a near ring in [6]. Jun and Song initiated the study of (α, β)-fuzzy interior ideals of a semigroup in [7]. In [8] regular semigroups are characterized by the properties of their $(\in, \in \vee q)$-fuzzy ideals. In [9] semigroups are characterized by the properties of their $(\in, \in \vee q)$-fuzzy ideals.

In this paper, we have introduced $(\in, \in \vee q)$-fuzzy ideals in a new nonassociative algebraic structure, that is, in an AG-groupoid and developed some new results. We have defined an intraregular AG-groupoid and characterized it by the properties of its $(\in, \in \vee q)$-fuzzy ideals.

2. AG-groupoids

A groupoid is called an AG-groupoid if it satisfies the left invertive law, that is, $(ab)c = (cb)a$. Every AG-groupoid satisfies the medial law $(ab)(cd) = (ac)(bd)$. It is basically a nonassociative algebraic structure in between a groupoid and a commutative semigroup. It is important to mention here that if an AG-groupoid contains identity or even right identity, then it becomes a commutative monoid. An AG-groupoid not necessarily contains a left identity, and if it contains a left identity, then it is unique [10]. An AG-groupoid S with left identity satisfies the paramedial law, that is, $(ab)(cd) = (db)(ca)$ and $S = S^2$. Moreover, S satisfies the following law:

$$a(bc) = b(ac), \quad \forall a, b, c, d \in S. \tag{2.1}$$

Let S be an AG-groupoid. By an AG-subgroupoid of S, we mean a nonempty subset A of S such that $A^2 \subseteq A$. A nonempty subset A of an AG-groupoid S is called a left (right) ideal of S if $SA \subseteq A(AS \subseteq A)$, and it is called a two-sided ideal if it is both left and a right ideal of S. A nonempty subset A of an AG-groupoid S is called quasi-ideal of S if $SQ \cap QS \subseteq Q$. A nonempty subset A of an AG-groupoid S is called a generalized bi-ideal of S if $(AS)A \subseteq A$, and an AG-subgroupoid A of S is called a bi-ideal of S if $(AS)A \subseteq A$. A nonempty subset A of an AG-groupoid S is called an interior ideal of S if $(SA)S \subseteq A$. A subset A of an AG-groupoid S is called semiprime if for all $a \in S, a^2 \in A$ implies that $a \in A$.

If S is an AG-groupoid with left identity e, then $S = S^2$. It is easy to see that every one sided ideal of S is quasi-ideal of S. In [11], it is given that $L[a] = a \cup Sa$, $I[a] = a \cup Sa \cup aS$ and $Q[a] = a \cup (aS \cap Sa)$ are principal left ideal, principal two-sided ideal, and principal quasi-ideal of S generated by a. Moreover using (2.1), left invertive law, paramedial law, and medial law, we get the following equations:

$$a(Sa) = S(aa) = Sa^2, \quad (Sa)a = (aa)S = a^2S, \quad (Sa)(Sa) = (SS)(aa) = Sa^2. \tag{2.2}$$

To obtain some more useful equations, we use medial, paramedial laws, and (2.1), we get the following:

$$\begin{aligned}(Sa)^2 &= (Sa)(Sa) = (SS)a^2 = (aa)(SS) = S((aa)S) \\ &= (SS)((aa)S) = \left(Sa^2\right)SS = \left(Sa^2\right)S.\end{aligned} \tag{2.3}$$

Therefore,

$$Sa^2 = a^2S = \left(Sa^2\right)S. \tag{2.4}$$

The following definitions are available in [1].

A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called a fuzzy AG-subgroupoid of $\mathcal{S}$ if $f(xy) \geq f(x) \wedge f(y)$ for all $x, y \in \mathcal{S}$. A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called a fuzzy left (right) ideal of $\mathcal{S}$ if $f(xy) \geq f(y)(f(xy) \geq f(x))$ for all $x, y \in \mathcal{S}$.

A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called a fuzzy two-sided ideal of $\mathcal{S}$ if it is both a fuzzy left and a fuzzy right ideal of $\mathcal{S}$. A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called a fuzzy quasi-ideal of S if $f \circ S \cap S \circ f \subseteq f$. A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called a fuzzy generalized bi-ideal of $\mathcal{S}$ if $f((xa)y) \geq f(x) \wedge f(y)$, for all x, a and $y \in \mathcal{S}$. A fuzzy AG-subgroupoid f of an AG-groupoid $\mathcal{S}$ is called a fuzzy bi-ideal of $\mathcal{S}$ if $f((xa)y) \geq f(x) \wedge f(y)$, for all x, a, and $y \in \mathcal{S}$. A fuzzy AG-subgroupoid f of an AG-groupoid $\mathcal{S}$ is called a fuzzy interior ideal of $\mathcal{S}$ if $f((xa)y) \geq f(a)$, for all x, a and $y \in \mathcal{S}$.

A fuzzy subset f of an AG-groupoid S is called fuzzy semiprime if $f(a) \geq f(a^2)$, for all $a \in S$.

Let f and g be any two fuzzy subsets of an AG-groupoid $\mathcal{S}$. Then, the product $f \circ g$ is defined by

$$(f \circ g)(a) = \begin{cases} \bigvee\limits_{a=bc} \{f(b) \wedge g(c)\}, & \text{if there exist } b, c \in \mathcal{S}, \text{ such that } a = bc, \\ 0, & \text{otherwise.} \end{cases} \tag{2.5}$$

The symbols $f \cap g$ and $f \cup g$ will mean the following fuzzy subsets of $\mathcal{S}$:

$$\begin{aligned} (f \cap g)(x) &= \min\{f(x), g(x)\} = f(x) \wedge g(x), \quad \forall x \text{ in } \mathcal{S}, \\ (f \cup g)(x) &= \max\{f(x), g(x)\} = f(x) \vee g(x), \quad \forall x \text{ in } \mathcal{S}. \end{aligned} \tag{2.6}$$

Let f be a fuzzy subset of an AG-groupoid S and $t \in (0, 1]$. Then, $x_t \in f$ means $f(x) \geq t$, $x_t q f$ means $f(x) + t > 1$, $x_t \alpha \vee \beta f$ means $x_t \alpha f$ or $x_t \beta f$, where α, β denotes any one of $\in, q, \in \vee q, \in \wedge q$. $x_t \alpha \wedge \beta f$ means $x_t \alpha f$ and $x_t \beta f$, $x_t \overline{\alpha} f$ means $x_t \alpha f$ does not holds.

Let f and g be any two fuzzy subsets of an AG-groupoid $\mathcal{S}$. Then, the product $f \circ_{0.5} g$ is defined by

$$(f \circ_{0.5} g)(a) = \begin{cases} \bigvee\limits_{a=bc} \{f(b) \wedge g(c)\}, & \text{if there exist } b, c \in \mathcal{S}, \text{ such that } a = bc, \\ 0, & \text{otherwise.} \end{cases} \tag{2.7}$$

The following definitions for AG-groupoids are same as for semigroups in [8].

Definition 2.1. A fuzzy subset δ of an AG-groupoid S is called an $(\in, \in \vee q)$-fuzzy AG-subgroupoid of S if for all $x, y \in S$ and $t, r \in (0, 1]$, it satisfies the following:

$$x_t \in \delta, \; y_r \in \delta \text{ implies that } (xy)_{\min\{t,r\}} \in \vee q \delta. \tag{2.8}$$

Definition 2.2. A fuzzy subset δ of S is called an $(\in, \in \vee q)$-fuzzy left (right) ideal of S if for all $x, y \in S$ and $t, r \in (0,1]$, it satisfies the following:

$$x_t \in \delta \text{ implies } (yx)_t \in \vee q\delta \quad (x_t \in \delta \text{ implies } (xy)_t \in \vee q\delta). \tag{2.9}$$

Definition 2.3. A fuzzy AG-subgroupoid f of an AG-groupoid S is called an $(\in, \in \vee q)$-fuzzy interior ideal of S if for all $x, y, z \in S$ and $t, r \in (0,1]$, the following condition holds:

$$y_t \in f \text{ implies } ((xy)z)_t \in \vee qf. \tag{2.10}$$

Definition 2.4. A fuzzy subset f of an AG-groupoid $\mathcal{S}$ is called an $(\in, \in \vee q)$-fuzzy quasi-ideal of S if it satisfies $f(x) \geq \min(f \circ C_S(x), C_S \circ f(x), 0.5)$, where C_S is the fuzzy subset of S mapping every element of S on 1.

Definition 2.5. A fuzzy subset f of an AG-groupoid S is called an $(\in, \in \vee q)$-fuzzy generalized bi-ideal of S if $x_t \in f$ and $z_r \in S$ implies $((xy)z)_{\min\{t,r\}} \in \vee qf$, for all $x, y, z \in S$ and $t, r \in (0,1]$.

Definition 2.6. A fuzzy subset f of an AG-groupoid S is called an $(\in, \in \vee q)$-fuzzy bi-ideal of S if for all $x, y, z \in S$ and $t, r \in (0,1]$, the following conditions hold.

(i) If $x_t \in f$ and $y_r \in S$, then $(xy)_{\min\{t,r\}} \in \vee qf$,

(ii) If $x_t \in f$ and $z_r \in S$, then $((xy)z)_{\min\{t,r\}} \in \vee qf$.

Definition 2.7. A fuzzy subset f of an AG-groupoid S is said to be $(\in, \in \vee q)$-fuzzy semiprime if it satisfies the following:

$$x_t^2 \in f \Longrightarrow x_t \in \vee qf \tag{2.11}$$

for all $x \in S$ and $t \in (0,1]$.

The proofs of the following four theorems are same as in [8].

Theorem 2.8. *Let δ be a fuzzy subset of S. Then, δ is an $(\in, \in \vee q)$-fuzzy AG-subgroupoid of S if $\delta(xy) \geq \min\{\delta(x), \delta(y), 0.5\}$.*

Theorem 2.9. *A fuzzy subset δ of an AG-groupoid S is called an $(\in, \in \vee q)$-fuzzy left (right) ideal of S if*

$$\delta(xy) \geq \min\{\delta(y), 0.5\} (\delta(xy) \geq \min\{\delta(x), 0.5\}). \tag{2.12}$$

Theorem 2.10. *A fuzzy subset f of an AG-groupoid S is an$(\in, \in \vee q)$-fuzzy interior ideal of S if and only if it satisfies the following conditions:*

(i) $f(xy) \geq \min\{f(x), f(y), 0.5\}$ *for all* $x, y \in S$,

(ii) $f((xy)z) \geq \min\{f(y), 0.5\}$ *for all* $x, y, z \in S$.

Theorem 2.11. *Let f be a fuzzy subset of S. Then, f is an $(\in, \in \vee q)$-fuzzy bi-ideal of S if and only if*

(i) $f(xy) \geq \min\{f(x), f(y), 0.5\}$ *for all* $x, y \in S$,

(ii) $f((xy)z) \geq \min\{f(x), f(z), 0.5\}$ *for all* $x, y, z \in S$.

Theorem 2.12. *A fuzzy subset f of an AG-groupoid S is $(\in, \in \vee q)$-fuzzy semiprime if and only if $f(x) \geq f(x^2) \wedge 0.5$, for all $x \in S$.*

Proof. It is easy. □

Here we begin with examples of AG-groupoids:

Example 2.13. Let us consider an AG-groupoid $S = \{1, 2, 3\}$ in the following multiplication table.

$$\begin{array}{c|ccc} . & 1 & 2 & 3 \\ \hline 1 & 2 & 3 & 2 \\ 2 & 2 & 2 & 2 \\ 3 & 2 & 2 & 2. \end{array} \tag{2.13}$$

Clearly S is noncommutative and nonassociative, because $1.2 \neq 2.1$ and $(1.2).2 \neq 1.(2.2)$. Note that S has no left identity. Define a fuzzy subset $F : S \rightarrow [0, 1]$ as follows:

$$F(x) = \begin{cases} 0.8 & \text{for } x = 1 \\ 0.7 & \text{for } x = 2 \\ 0.6 & \text{for } x = 3. \end{cases} \tag{2.14}$$

Then, clearly F is an $(\in, \in \vee q)$-fuzzy ideal of S.

Example 2.14. Let $S = \{1, 2, 3, 4\}$, and the binary operation "$\cdot$" be defined on S as follows:

$$\begin{array}{c|cccc} . & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 3 & 3 \\ 3 & 3 & 3 & 3 & 3 \\ 4 & 2 & 3 & 3 & 3. \end{array} \tag{2.15}$$

Then, $(S, \cdot)$ is an AG-groupoid with left identity 1. Define a fuzzy subset $F : S \rightarrow [0, 1]$ as follows:

$$F(x) = \begin{cases} 0.9 & \text{for } x = 1 \\ 0.7 & \text{for } x = 2 \\ 0.6 & \text{for } x = 3 \\ 0.6 & \text{for } x = 3. \end{cases} \tag{2.16}$$

Then, clearly F is an $(\in, \in \vee q)$-fuzzy ideal of S.

Lemma 2.15. *Intersection of two ideals of an AG-groupoid with left identity is either empty or an ideal.*

Proof. It is straightforward. □

3. $(\in, \in \vee q)$-Fuzzy Quasi-Ideals of an Intraregular AG-groupoid

An element a of an AG-groupoid S is called intraregular if there exist $x, y \in S$ such that $a = (xa^2)y$, and S is called intraregular if every element of S is intraregular.

Example 3.1. Let $S = \{a, b, c, d, e\}$, and the binary operation "·" be defined on S as follows:

$$\begin{array}{c|cccccc} \cdot & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 6 & 3 & 4 & 5 \\ 4 & 1 & 1 & 5 & 6 & 3 & 4 \\ 5 & 1 & 1 & 4 & 5 & 6 & 3 \\ 6 & 1 & 1 & 3 & 4 & 5 & 6. \end{array} \tag{3.1}$$

It can be easily checked by the test given in [12] that $(S, \cdot)$ is an AG-groupoid. Also, $1 = (1 \cdot 1^2) \cdot 1, 2 = (2 \cdot 2^2) \cdot 2, 3 = (4 \cdot 3^2) \cdot 5, 4 = (5 \cdot 4^2) \cdot 5, 5 = (3 \cdot 5^2) \cdot 4$ and $6 = (6 \cdot 6^2) \cdot 6$. Therefore, $(S, \cdot)$ is an intraregular AG-groupoid. Define a fuzzy subset $f : S \to [0, 1]$ as follows:

$$f(x) = \begin{cases} 0.9 & \text{for } x = 1 \\ 0.8 & \text{for } x = 2 \\ 0.7 & \text{for } x = 3 \\ 0.7 & \text{for } x = 4 \\ 0.6 & \text{for } x = 5 \\ 0.5 & \text{for } x = 6. \end{cases} \tag{3.2}$$

Then, clearly f is an $(\in, \in \vee q)$-fuzzy quasi-ideal of S.

Theorem 3.2 (See [13]). *For an intraregular AG-groupoid S with left identity, the following statements are equivalent:*

(i) *A is a left ideal of S,*

(ii) *A is a right ideal of S,*

(iii) *A is an ideal of S,*

(iv) *A is a bi-ideal of S,*

(v) *A is a generalized bi-ideal of S,*

(vi) *A is an interior ideal of S,*

(vii) *A is a quasi-ideal of S,*

(viii) *$AS = A$ and $SA = A$.*

Theorem 3.3 (See [14]). *In intraregular AG-groupoid S with left identity, the following are equivalent.*

(i) *A fuzzy subset f of S is an $(\in, \in \vee q)$-fuzzy right ideal.*

(ii) *A fuzzy subset f of S is an $(\in, \in \vee q)$-fuzzy left ideal.*

(iii) *A fuzzy subset f of S is an $(\in, \in \vee q)$-fuzzy bi-ideal.*

(iv) *A fuzzy subset f of S is an $(\in, \in \vee q)$-fuzzy interior ideal.*

(v) *A fuzzy subset f of S is an $(\in, \in \vee q)$-fuzzy quasi-ideal.*

Definition 3.4. Let f and g be fuzzy subsets of an AG-groupoid S. We define the fuzzy subsets $f_{0.5}$, $f \wedge_{0.5} g$, and $f \circ_{0.5} g$ of S as follows:

(i) $f_{0.5}(a) = f(a) \wedge 0.5$,

(ii) $(f \wedge_{0.5} g)(a) = (f \wedge g)(a) \wedge 0.5$.

Definition 3.5. Let A be any subset of an AG-groupoid S. Then, the characteristic function $(C_A)_{0.5}$ is defined as

$$(C_A)_{0.5}(a) = \begin{cases} 0.5 & \text{if } a \in A, \\ 0, & \text{otherwise.} \end{cases} \tag{3.3}$$

Lemma 3.6 (See [14]). *The following properties hold in an AG-groupoid S.*

(i) *A is an AG-subgroupoid of S if and only if $(C_A)_{0.5}$ is an $(\in, \in \vee q)$-fuzzy AG-subgroupoid of S.*

(ii) *A is a left (right, two sided) ideal of S if and only if $(C_A)_{0.5}$ is an $(\in, \in \vee q)$-fuzzy left (right, two-sided) ideal of S.*

(iii) *A is left (quasi) ideal of an AG-groupoid S if and only if $(C_A)_{0.5}$ is $(\in, \in \vee q)$-fuzzy left (quasi)-ideal.*

(iv) *For any nonempty subsets A and B of $S, C_A \circ_{0.5} C_B = (C_{AB})_{0.5}$ and $C_A \wedge_{0.5} C_B = (C_{A \cap B})_{0.5}$.*

(v) *A nonempty subsets A of S semiprime if and only if $(C_A)_{0.5}$ is semiprime.*

Lemma 3.7. *If S is an AG-groupoid with left identity, then Sa is quasi-ideal of S.*

Proof. Using paramedial, medial, and left invertive laws, we get:

$$S(Sa) \cap (Sa)S \subseteq S(Sa) = (SS)(Sa) = (aS)(SS) = (aS)S = (SS)a = Sa. \tag{3.4}$$

Hence, Sa is a quasi-ideal of S. □

Theorem 3.8. *For an AG-groupoid with left identity e, the following are equivalent:*

(i) *S is intraregular,*

(ii) *$I \cap J = IJ$ $(I \cap J \subseteq IJ)$, for all quasi-ideals I and J,*

(iii) *$f \wedge_{0.5} g = f \circ_{0.5} g$ $(f \wedge_{0.5} g \leq f \circ_{0.5} g)$, for all $(\in, \in \vee q)$-fuzzy quasi-ideals f and g.*

Proof. (i)⇒(iii) Let f and g be $(\in, \in \vee q)$-fuzzy quasi-ideals of an intraregular AG-groupoid S with left identity. Then, by Theorem 3.3, f and g become $(\in, \in \vee q)$-fuzzy ideals of S. For each a in S, there exists x, y in S such that $a = (xa^2)y$ and since $S = S^2$, so for y in S there exists u, v in S such that $y = uv$. Now, using paramedial law, medial law, and (2.1), we get the following:

$$a = \left(xa^2\right)y = \left(xa^2\right)(uv) = (vu)\left(a^2x\right) = a^2((vu)x) = (a(vu))(ax). \tag{3.5}$$

Then,

$$\begin{aligned}(f \circ_{0.5} g)(a) &= \bigvee_{a=pq} \{f(p) \wedge g(q) \wedge 0.5\} \\ &= \bigvee_{a=(a(vu))(ax)} \{f(a(vu)) \wedge g(ax) \wedge 0.5\} \\ &\geq f(a) \wedge g(a) \wedge 0.5 = f \wedge_{0.5} g(a).\end{aligned} \tag{3.6}$$

Therefore, $f \circ_{0.5} g \geq f \wedge_{0.5} g$. Also, one has

$$\begin{aligned}f \circ_{0.5} g(a) &= f \circ g(a) \wedge 0.5 \\ &= \bigvee_{a=bc} f(b) \wedge g(c) \wedge 0.5 \\ &= \bigvee_{a=bc} (f(b) \wedge 0.5) \wedge (g(c) \wedge 0.5) \wedge 0.5 \\ &\leq \bigvee_{a=bc} f(bc) \wedge g(bc) \wedge 0.5 = f \wedge_{0.5} g(a).\end{aligned} \tag{3.7}$$

Therefore, $f \circ_{0.5} g \leq f \wedge_{0.5} g$. Hence, $f \circ_{0.5} g(a) = f \wedge_{0.5} g(a)$.

(iii)⇒(ii) Let I and J be the quasi-ideals of an AG-groupoid S with left identity and let $a \in I \cap J$. Then, by hypothesis and Lemma 3.6, we get the following:

$$\begin{aligned}\left(C_{IJ}\right)_{0.5}(a) &= \left(C_I \circ_{0.5} C_J\right)(a) = \left(C_I \wedge_{0.5} C_J\right)(a) \\ &= \left(C_{I \cap J}\right)_{0.5}(a) = 0.5.\end{aligned} \tag{3.8}$$

Therefore, $a \in IJ$. Now, if $a \in IJ$, then

$$\left(C_{I \cap J}\right)_{0.5}(a) = \left(C_{IJ}\right)_{0.5}(a) = 0.5. \tag{3.9}$$

Therefore, $a \in I \cap J$. Thus, $IJ = I \cap J$.

(ii)⇒(i) Since Sa is a quasi-ideal of an AG-groupoid S with left identity containing a, by (ii), medial law, left invertive law, and paramedial law, we obtain that

$$\begin{aligned}Sa \cap Sa = (Sa)(Sa) &= (SS)(aa) = \left(a^2S\right)S = ((aa)(SS))S \\ &= ((SS)(aa))S = \left(Sa^2\right)S.\end{aligned} \tag{3.10}$$

Hence, S is intraregular. □

Corollary 3.9. *Let S be an AG-groupoid with left identity e, then S is intraregular if and only if every quasi-ideal of S is idempotent.*

Corollary 3.10. *For an AG-groupoid S with left identity, the following conditions are equivalent.*

(i) *S is intraregular.*

(ii) *$(f\wedge_{0.5} g)\wedge_{0.5} h \le (f\circ_{0.5} g)\circ_{0.5} h$, for all $(\in,\in\vee q)$-fuzzy quasi-ideals f, g and $(\in,\in\vee q)$-fuzzy left ideal h of S.*

(iii) *$(f\wedge_{0.5} g)\wedge_{0.5} h \le (f\circ_{0.5} g)\circ_{0.5} h$, for all $(\in,\in\vee q)$-fuzzy quasi-ideals f, g and h of S.*

Proof. (i)⇒(iii) Let f and g be $(\in,\in\vee q)$ fuzzy quasi-ideals and h be an $(\in,\in\vee q)$-fuzzy left ideal of an intraregular AG-groupoid with left identity e. Since S is intraregular. Therefore, for $a\in S$, there exists x, y in S such that $a=(xa^2)y$. Now, using (2.1), left invertive law, paramedial and medial laws, we get the following:

$$\begin{aligned} a &= \left(xa^2\right)y = (x(aa))y = (a(xa))y = (y(xa))a \\ &= \left(y\left(x\left(\left(xa^2\right)y\right)\right)\right)a = \left(y\left(\left(xa^2\right)(xy)\right)\right)a = \left(\left(xa^2\right)(y(xy))\right)a = \left(\left(xa^2\right)\left(xy^2\right)\right)a \\ &= \left(\left(y^2x\right)\left(a^2x\right)\right)a = \left(a^2\left(\left(y^2x\right)x\right)\right)a = \left(\left(a\left(y^2x\right)\right)(ax)\right)a. \end{aligned} \tag{3.11}$$

Then,

$$\begin{aligned} ((f\circ_{0.5} g)\circ_{0.5} h)(a) &= \bigvee_{a=pq} (f\circ_{0.5} g)(p)\wedge h(q)\wedge 0.5 \\ &= \bigvee_{a=((a(y^2x))(ax))a} (f\circ_{0.5} g)\left(\left(a\left(y^2x\right)\right)(ax)\right)\wedge h(a)\wedge 0.5 \\ &\ge (f\circ_{0.5} g)\left(\left(a\left(y^2x\right)\right)(ax)\right)\wedge h(a)\wedge 0.5 \\ &= \bigvee_{(a(y^2x))(ax)=cd} f(c)\wedge g(d)\wedge h(a)\wedge 0.5 \\ &\ge f\left(a\left(y^2x\right)\right)\wedge g(ax)\wedge h(a)\wedge 0.5 \\ &= (f\wedge_{0.5} g\wedge_{0.5} h)(a). \end{aligned} \tag{3.12}$$

(iii)⇒(ii) It is obvious.

(ii)⇒(i) Let f and g be $(\in,\in\vee q)$-fuzzy quasi-ideals of an AG-groupoid S with left identity. Then,

$$\begin{aligned} ((C_S\wedge_{0.5} f)\wedge_{0.5} g)(a) &= (C_S\wedge_{0.5} f)(a)\wedge g(a)\wedge 0.5 \\ &= C_S(a)\wedge f(a)\wedge 0.5\wedge g(a)\wedge 0.5 \\ &= 1\wedge f(a)\wedge g(a)\wedge 0.5 = f\wedge_{0.5} g(a). \end{aligned} \tag{3.13}$$

Thus, $((C_S \wedge_{0.5} f) \wedge_{0.5} g) = f \wedge_{0.5} g$. Also,

$$\begin{aligned} C_S \circ_{0.5} f(a) &= \bigvee_{a=pq} C_S(p) \wedge f(q) \wedge 0.5 \\ &= \bigvee_{a=ea} C_S(e) \wedge f(a) \wedge 0.5 \\ &= f(a) \wedge 0.5 \leq f(a). \end{aligned} \tag{3.14}$$

Thus, $C_S \circ_{0.5} g \leq f$. Now, using (ii), we get the following:

$$(f \wedge_{0.5} g)(a) = ((C_S \wedge_{0.5} f) \wedge_{0.5} g)(a) \leq ((C_S \circ_{0.5} f) \circ_{0.5} g)(a) \leq f \circ_{0.5} g(a). \tag{3.15}$$

Therefore, by Theorem 3.8, S is intraregular. □

Lemma 3.11. *Let S be an AG-groupoid with left identity, then $(aS)a^2 \subseteq (aS)a$, for some a in S.*

Proof. Using paramedial law, medial law, left invertive law, and (2.1), we get the following:

$$\begin{aligned} (aS)a^2 &= (aa)(Sa) = [(Sa)a]a = [(aa)(SS)]a = [(SS)(aa)]a \\ &= [a\{(SS)a\}]a \subseteq (aS)a. \end{aligned} \tag{3.16}$$

□

Lemma 3.12. *Let S be an AG-groupoid with left identity, then $(aS)[(aS)a] \subseteq (aS)a$, for some a in S.*

Proof. Using left invertive law and (2.1), we get the following:

$$(aS)[(aS)a] = [\{(aS)a\}S]a = [(Sa)(aS)]a = [a(Sa)]Sa \subseteq (aS)a. \tag{3.17}$$

□

Lemma 3.13. *If S is an AG-groupoid with left identity, then $(aS)(Sa) = (aS)a$, for some a in S.*

Proof. Using paramedial law, medial law, and (2.1), we get the following:

$$(aS)(Sa) = [(Sa)S]a = [(Sa)(SS)]a = [(SS)(aS)]a = (a(SS))a = (aS)a. \tag{3.18}$$

□

Theorem 3.14. *Let S be an AG-groupoid with left identity, then $B[a] = a \cup a^2 \cup (aS)a$ is a bi-ideal of S.*

Proof. Using Lemmas 3.13, 3.11, 3.12, left invertive law, and (2.1), we get the following:

$$\begin{aligned}(B[a]S)B[a] &= \left[\left\{a \cup a^2 \cup (aS)a\right\}S\right]\left[a \cup a^2 \cup (aS)a\right] \\ &= \left[aS \cup a^2S \cup ((aS)a)S\right]\left[a \cup a^2 \cup (aS)a\right] \\ &= \left[aS \cup a^2S \cup (Sa)(aS)\right]\left[a \cup a^2 \cup (aS)a\right] \\ &= \left[aS \cup a^2S \cup a((Sa)S)\right]\left[a \cup a^2 \cup (aS)a\right] \\ &\subseteq [aS \cup aS \cup aS]\left[a \cup a^2 \cup (aS)a\right] \\ &= (aS)\left(a \cup a^2 \cup (aS)a\right) \\ &= (aS)a \cup (aS)a^2 \cup (aS)\{(aS)a\} \\ &\subseteq (aS)a \cup (aS)a^2 \cup (aS)(Sa) \\ &\subseteq (aS)a \subseteq \left(a \cup a^2 \cup (aS)a\right).\end{aligned} \tag{3.19}$$

Thus $a \cup a^2 \cup (aS)a$ is a bi-ideal. □

Theorem 3.15. *For an AG-groupoid with left identity e, the following are equivalent.*

(i) *S is intraregular.*

(ii) *$Q[a] \cap B[a] = Q[a]B[a]$, for some a in S.*

(iii) *$Q \cap B = QB$, for every quasi-ideal Q and bi-ideal B of S.*

Proof. (i)⇒(iii) Let Q be a quasi-ideal and B be a bi-ideal of an intraregular AG-groupoid S with left identity. Then, by Theorem 3.2, Q and B become ideals of S. Let $a \in Q \cap B$. Now since S is intraregular so for each a in S there exists x, y in S such that $a = (xa^2)y$. Now, since $a = (a(vu))(ax)$; thus,

$$a = (a(vu))(ax) \in (Q(SS))(BS) \subseteq QB. \tag{3.20}$$

Therefore, $Q \cap B \subseteq QB$. Next let $qb \in QB$, for some $q \in Q$ and $b \in B$. Then $qb \in QS \subseteq Q$ and $qb \in SB \subseteq B$. Thus $QB \subseteq Q \cap B$. Hence, $Q \cap B = QB$.

(iii)⇒(ii) It is obvious.

(ii)⇒(i) For a in S,$B[a] = a \cup a^2 \cup (aS)a$ and $Q[a] = a \cup (Sa \cap aS)$ are bi- and quasi-ideals of S generated by a. Therefore using left invertive law, medial law, and (ii), we get the following:

$$\begin{aligned}[a \cup (Sa \cap aS)] \cap a \cup a^2 \cup (aS)a &= [a \cup (Sa \cap aS)]\left[a \cup a^2 \cup (aS)a\right] \\ &\subseteq [Sa]\left[a \cup a^2 \cup (aS)a\right] \\ &\subseteq (Sa)a \cup (Sa)a^2 \cup (Sa)[(aS)a] \\ &= Sa^2 \cup [S(aS)](aa) \subseteq Sa^2.\end{aligned} \tag{3.21}$$

Hence by (2.4), S is intraregular. □

Theorem 3.16. *For an AG-groupoid with left identity e, the following are equivalent.*

(i) *S is intraregular.*

(ii) *$f\wedge_{0.5} g = f\circ_{0.5} g$, where f is any $(\in, \in\vee q)$-fuzzy quasi-ideal and g is any $(\in, \in\vee q)$-fuzzy bi-ideal.*

Proof. (i)⇒(ii) Let f be an $(\in, \in\vee q)$-fuzzy quasi-ideal and g be an $(\in, \in\vee q)$-fuzzy bi-ideal of an intraregular AG-groupoid S with left identity. Then, by Theorem 3.3, f and g become $(\in, \in\vee q)$-fuzzy ideals of S. Since S is intraregular, so for each a in S there exists x, y in S such that $a = (xa^2)y$. Now, since $a = (a(vu))(ax)$,

$$\begin{aligned}(f\circ_{0.5} g)(a) &= \bigvee_{a=pq}\{f(p)\wedge g(q)\wedge 0.5\}\\ &\geq f(a(vu))\wedge g(ax)\wedge 0.5\\ &\geq f(a)\wedge g(a)\wedge 0.5 = f\wedge_{0.5} g(a).\end{aligned} \tag{3.22}$$

Thus, $f\circ_{0.5} g \geq f\wedge_{0.5} g$. Also

$$\begin{aligned}f\circ_{0.5} g(a) = f\circ g(a)\wedge 0.5 &= \bigvee_{a=bc} f(b)\wedge g(c)\wedge 0.5\\ &= \bigvee_{a=bc}(f(b)\wedge 0.5)\wedge(g(c)\wedge 0.5)\wedge 0.5\\ &\leq \bigvee_{a=bc} f(bc)\wedge(g(bc)\wedge 0.5) = f\wedge_{0.5} g(a).\end{aligned} \tag{3.23}$$

Therefore, $f\circ_{0.5} g \leq f\wedge_{0.5} g$. Hence, $f\circ_{0.5} g(a) = f\wedge_{0.5} g(a)$.

(ii)⇒(i) Let $a\in Q\cap B$. Then, by hypothesis and Lemma 3.6, we get the following:

$$\begin{aligned}(C_{QB})_{0.5}(a) &= (C_Q\circ_{0.5} C_B)(a) = (C_Q\wedge_{0.5} C_B)(a)\\ &= (C_{Q\cap L})_{0.5}(a) = 0.5.\end{aligned} \tag{3.24}$$

Therefore, $a\in QB$. Now, if $a\in QB$, then

$$(C_{Q\cap B})_{0.5}(a) = (C_{QB})_{0.5}(a) = 0.5. \tag{3.25}$$

Therefore, $a\in Q\cap B$. Thus, $QB = Q\cap B$. Hence by Theorem 3.15, S is intraregular. □

Lemma 3.17. *If I is an ideal of an intraregular AG-groupoid S with left identity, then $I = I^2$.*

Proof. It is straightforward. □

Theorem 3.18. *For an AG-groupoid S with left identity, the following are equivalent.*

(i) *S is intraregular.*

(ii) *$Q[a]\cap L[a]\cap B[a] = (Q[a]L[a])B[a]$, for some a in S.*

(iii) *$Q\cap L\cap B = (QL)B$, for any quasi-ideal Q, left ideal L, and bi-ideal B of S.*

Proof. (i)⇒(iii) Let Q be a quasi-ideal, L be a left ideal, and B be a bi-ideal of an intraregular AG-groupoid S with left identity. Since S is intraregular, for each $a \in S$, there exist $x, y \in S$ such that $a = (xa^2)y$. Then, by Theorem 3.2, Q, L, and B become ideals of S. Then, using (2.1), left invertive law, paramedial and medial law, we obtain that

$$\begin{aligned}
a &= \left(xa^2\right)y = (x(aa))y = (a(xa))y = (y(xa))a \\
&= (y(xa))((y(xa))a) = (a(y(xa)))((xa)y) \in Q(S(SL))((SB)S) \\
&\subseteq (QL)B.
\end{aligned} \tag{3.26}$$

(iii)⇒(ii) It is obvious.

(ii)⇒(i) For a in S,$L[a] = a \cup Sa$, $Q[a] = a \cup (Sa \cap aS)$ and $B[a] = a \cup a^2 \cup (aS)a$ are left, quasi and bi-ideals of S generated by a. Therefore using medial law, left invertive law and (ii), we get the following:

$$\begin{aligned}
[a \cup (Sa \cap aS)] \cap [a \cup Sa] \cap \left[a \cup a^2 \cup (aS)a\right] &= ([a \cup (Sa \cap aS)][a \cup Sa])\left[a \cup a^2 \cup (aS)a\right] \\
&\subseteq \{(Sa)(Sa)\}\left[a \cup a^2 \cup (aS)a\right] \\
&= \left(Sa^2\right)\left[a \cup a^2 \cup (aS)a\right] \\
&= \left(Sa^2\right)a \cup \left(Sa^2\right)a^2 \cup \left(Sa^2\right)[(aS)a] \subseteq Sa^2.
\end{aligned} \tag{3.27}$$

Hence by (2.4), S is intraregular. □

Theorem 3.19. *For an AG-groupoid S with left identity, the following are equivalent.*

(i) *S is intraregular.*

(ii) $(f \wedge_{0.5} g) \wedge_{0.5} h = (f \circ_{0.5} g) \circ_{0.5} h$, *for $(\in, \in \vee q)$-fuzzy quasi ideal f, $(\in, \in \vee q)$-fuzzy left-ideal g and $(\in, \in \vee q)$-fuzzy bi-ideal h of S.*

Proof. (i)⇒(ii) Let f be an $(\in, \in \vee q)$-fuzzy quasi-ideal, g be an $(\in, \in \vee q)$-fuzzy left ideal, and h be an $(\in, \in \vee q)$-fuzzy bi-ideal of an intraregular AG-groupoid S with left identity. Since S is intraregular, for each $a \in S$ there exist $x, y \in S$ such that $a = (xa^2)y$. Then, by Theorem 3.3, f, g, and h become $(\in, \in \vee q)$-fuzzy ideals of S. Then, using (2.1), left invertive law, paramedial and medial law, we obtain that

$$\begin{aligned}
a &= \left(xa^2\right)y = (x(aa))y = (a(xa))y = (y(xa))a \\
&= (y(xa))((y(xa))a) = (a(y(xa)))((xa)y).
\end{aligned} \tag{3.28}$$

Now,

$$
\begin{aligned}
((f\circ_{0.5}g)\circ_{0.5}h)(a) &= \bigvee_{a=pq}(f\circ_{0.5}g)(p)\wedge h(q)\wedge 0.5\\
&= \bigvee_{a=(a(y(xa)))((xa)y)} f\circ_{0.5}g(a(y(xa)))\wedge h((xa)y)\wedge 0.5\\
&\geq (f\circ_{0.5}g)(a(y(xa)))\wedge h(a)\wedge 0.5\\
&= \left\{\bigvee_{a(y(xa))=uv} f(u)\wedge g(v)\right\}\wedge h(a)\wedge 0.5\\
&= \left\{\bigvee_{a(y(xa))=uv} f(a)\wedge g(y(xa))\wedge 0.5\right\}\wedge h(a)\wedge 0.5\\
&\geq \{f(a)\wedge g(a)\wedge 0.5\}\wedge h(a)\wedge 0.5\\
&= f(a)\wedge g(a)\wedge h(a)\wedge 0.5 = [(f\wedge_{0.5}g)\wedge_{0.5}h](a).
\end{aligned}
\tag{3.29}
$$

Therefore, $(f\wedge_{0.5}g)\wedge_{0.5}h \leq (f\circ_{0.5}g)\circ_{0.5}h$. Also

$$
\begin{aligned}
((f\circ_{0.5}g)\circ_{0.5}h)(a) &= \bigvee_{a=pq}(f\circ_{0.5}g)(p)\wedge h(q)\wedge 0.5\\
&= \bigvee_{a=pq}\left\{\bigvee_{p=cd} f(c)\wedge g(d)\wedge 0.5\right\}\wedge h(q)\wedge 0.5\\
&\leq \bigvee_{a=pq}\left\{\bigvee_{p=cd} f(cd)\wedge g(cd)\wedge 0.5\right\}\wedge h(pq)\wedge 0.5\\
&= \bigvee_{a=pq}\{f(p)\wedge g(p)\wedge 0.5\}\wedge h(pq)\wedge 0.5\\
&\leq \bigvee_{a=pq} f(pq)\wedge g(pq)\wedge h(pq)\wedge 0.5\\
&= f(a)\wedge g(a)\wedge h(a)\wedge 0.5\\
&= [(f\wedge_{0.5}g)\wedge_{0.5}h](a).
\end{aligned}
\tag{3.30}
$$

Therefore, $(f\wedge_{0.5}g)\wedge_{0.5}h \geq (f\circ_{0.5}g)\circ_{0.5}f$. Hence, $(f\wedge_{0.5}g)\wedge_{0.5}h = (f\circ_{0.5}g)\circ_{0.5}h$.

(ii)⇒(i) Let Q be a quasi-ideal, L be a left ideal, and B be a bi-ideal of an AG-groupoid S. Then, by Lemma 3.6, $(C_Q)_{0.5}$, $(C_L)_{0.5}$, and $(C_B)_{0.5}$ are $(\in,\in\vee q)$-fuzzy quasi, left, and bi-ideals of S. Then, using (ii), we have

$$
(C_{Q\cap L\cap B})_{0.5} = [(C_Q\wedge_{0.5}C_L)\wedge_{0.5}C_L] = (C_Q\circ_{0.5}C_L)\circ_{0.5}C_B = (C_{(QL)B})_{0.5}. \tag{3.31}
$$

This implies that $Q\cap L\cap B = (QL)B$. Hence by Theorem 3.18, S is intraregular. □

Theorem 3.20. *Let S be an AG-groupoid with left identity, then the following conditions are equivalent.*

(i) *S is intraregular.*

(ii) *For every bi-ideal B and quasi-ideal Q of S, BQ = QB and B & Q are semiprime.*

Proof. (i)⇒(ii) Let B be a bi-ideal and Q be a quasi-ideal of an intraregular AG-groupoid S with left identity. Then, by Theorem 3.2, Q and B become ideals of S. Let $b \in B$ and $q \in Q$. Since S, is intraregular so for each b in S, there exists x, y, in S such that $b = (xb^2)y$. Thus by left invertive law, we get the following:

$$bq = \left[\left(xb^2\right)y\right]q = (qy)\left(xb^2\right) \in (QS)(SB) \subseteq QB. \tag{3.32}$$

Similarly we can prove that $QB \subseteq BQ$. Now let $b^2 \in B$. Then $b = (xb^2)y \in (SB)S \subseteq B$. Thus B is semiprime. Similarly we can prove that Q is semiprime.

(ii)⇒(i) For a in S,$Q[a] = a \cup (Sa \cap aS)$ and $B[a] = a \cup a^2 \cup (aS)a$ are quasi and bi-ideals of S generated by a. Therefore, using (2.1), (2.4), medial law, and (ii), we get

$$\begin{aligned} a^2 &\in \left[a \cup a^2 \cup (aS)a\right][a \cup (Sa \cap aS)] \\ &\subseteq \left[a \cup a^2 \cup (aS)a\right][Sa] \\ &\subseteq a(Sa) \cup a^2(Sa) \cup [(aS)a](Sa) \\ &\subseteq a^2S \cup [(aS)S](aa) \subseteq Sa^2. \end{aligned} \tag{3.33}$$

Clearly Sa^2 is a bi-ideal of S, so, by (ii), it is semiprime. Thus, $a \in Sa^2$. Hence by (2.4), S is intraregular. □

The proofs of following two Lemmas are easy and therefore omitted.

Lemma 3.21. *For any fuzzy subset f of an AG-groupoid S, $S \circ_{0.5} f \leq f$ and for any fuzzy right ideal g, $g \circ_{0.5} S \leq g$.*

Lemma 3.22. *Let f and g be $(\in, \in \vee q)$-fuzzy ideals of an AG-groupoid S with left identity, then $f \circ_{0.5} g$ is an $(\in, \in \vee q)$-fuzzy ideal of S.*

Theorem 3.23. *LetS be an AG-groupoid with left identity, then the following conditions are equivalent.*

(i) *S is intraregular.*

(ii) *For every $(\in, \in \vee q)$-fuzzy quasi-ideal f and $(\in, \in \vee q)$-fuzzy bi-ideal g, $f \circ_{0.5} g = g \circ_{0.5} f$, and f and g are semiprime.*

Proof. (i)⇒(ii) Let f be an $(\in, \in \vee q)$-fuzzy quasi-ideal and g be an $(\in, \in \vee q)$-fuzzy bi-ideal of an intraregular AG-groupoid S with left identity. Now by Theorem 3.3, f and g become $(\in, \in \vee q)$-fuzzy ideals of S. Then by Theorem 3.16, Lemmas 8, and 9, we get the following:

$$\begin{aligned} f \circ_{0.5} g &= f \wedge_{0.5} g = g \wedge_{0.5} f = S \wedge_{0.5} (g \wedge_{0.5} f) \\ &= S \wedge_{0.5} (g \circ_{0.5} f) = S \circ_{0.5} (g \circ_{0.5} f) \leq g \circ_{0.5} f. \end{aligned} \tag{3.34}$$

This implies that $f\circ_{0.5} g \leq g\circ_{0.5} f$. Similarly we can prove that $g\circ_{0.5} f \leq f\circ_{0.5} g$.

Hence $f\circ_{0.5} g = g\circ_{0.5} f$. Moreover,

$$f(a) = f\left(\left(xa^2\right)y\right) \geq f\left(a^2\right). \tag{3.35}$$

Thus, $f(a) \geq f(a^2)$. Similarly $g(a) \geq g(a^2)$.

(ii)⇒(i) Let A be a bi-ideal and B be a quasi-ideal of S, then by Lemma 3.6, $(C_A)_{0.5}$, and $(C_B)_{0.5}$ are $(\in, \in \vee q)$-fuzzy bi and $(\in, \in \vee q)$-fuzzy quasi-ideals; therefore, by using Lemma 3.6 and (ii),

$$(C_{AB})_{0.5} = C_A\circ_{0.5} C_B = C_B\circ_{0.5} C_A = (C_{BA})_{0.5}. \tag{3.36}$$

Therefore $AB = BA$. Now since $(C_A)_{0.5}$ and $(C_B)_{0.5}$ are semiprime so by Lemma 3.6, A and B are semiprime. Hence by Theorem 3.20, S is semiprime. □

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