Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 528932, 13 pages
doi:10.1155/2012/528932

Research Article

LMI-Based Sliding Mode Observers for Incipient Faults Detection in Nonlinear System

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Received 26 August 2012; Accepted 11 November 2012

Academic Editor: Xinzhi Liu

This paper presents a diagnosis scheme based on a linear matrix inequality (LMI) algorithm for incipient faults in a nonlinear system class with unknown input disturbances. First, the nonlinear system is transformed into two subsystems, one of which is unrelated to the disturbances. Second, for the subsystem that is free from disturbances, a Luenberger observer is constructed; a sliding mode observer is then constructed for the subsystem which is subjected to disturbances, so that the effect of the unknown input disturbances is eliminated. Together, the entire system achieves both robustness to disturbances and sensitivity to incipient faults. Finally, the effectiveness and feasibility of the proposed method are verified through a numerical example using a single-link robotic arm.

1. Introduction

An electronic system is structurally complex, involving a number of electronic components. It is therefore difficult to accurately analyze the relationship between the input and output for each component and the cause of any faults within, as each fault may exhibit diverse manifestations. Incipient faults are important in electronic systems, and some common manifestations include zero drift, reduced precision, delayed response, and equipment aging. If these potential incipient faults cannot be detected in time, the long-term stable operation of the devices will be affected and, more importantly, they will cause a decline in production capacity and an increase in the cost of production, or even accidents. Therefore, the study of incipient faults in electronic devices has practical significance.

The sliding mode variable structure control algorithm has the advantages of simplicity, high robustness, and high reliability and is extensively used in motion control. The sliding mode observer has attracted attention for fault diagnosis since the sliding mode variable structure control enables the system to be robust against unknown input disturbances, especially against external disturbances [1, 2]. Observer technology is one of the most important methods for fault diagnosis and detection, estimation and fault-tolerant control [3–5]. A robust fault diagnosis method for the sliding mode observer has been proposed [6]. This scheme adopts the Walcoot-Zak sliding mode observer, which makes system state error robust against disturbances, and is maintained on the sliding mode surface. Much progress has been made in measuring the deviation in the trajectory of the motion of the system by using a sliding mode surface [5, 7, 8]. However, the current fault diagnosis based on sliding mode observers is mostly used for mutant or intermittent faults that have significant numerical changes. There have been few reports on slow faults with a small initial value. This is because the sliding mode control is essentially a type of continuous nonlinear control, and buffeting is inevitable for the system output, which causes the incipient fault signals to be submerged in the buffeting signals for a long period of time after their generation, thus they cannot be detected.

Chen and Chowdhury [9] have proposed a fault detection scheme for actuator's incipient faults in linear systems. Building on the work of Chen and Chowdhury [9], this paper presents an LMI-based fault diagnosis scheme, which is designed for incipient fault detection in nonlinear systems with unknown input disturbances. The adoption of the LMI algorithm relaxes the selection criteria for key parameters in the design, making it easier to obtain these parameters.

2. System Descriptions

For a nonlinear systems with disturbances,

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + f(x,u,t) + Dd(x,u,t), \\ y(t) &= Cx(t), \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is unmeasurable state vector; $u(t) \in R^m$ is measurable input vector; $y(t) \in R^p$ is the measurable output vector; $f(x,u,t) \in R^n$ is the known nonlinear function; $d(x,u,t) \in R^q$ is unknown bounded nonlinear function, representing the unknown input disturbances and modeling errors, which are collectively referred to as input disturbances; $D \in R^{n\times q}$ is known disturbance distribution matrix. A, B, and C are known matrices, where $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $n > p > q$.

Assumption 2.1. (A, C) is observable.

Assumption 2.2. D is full column rank and $\operatorname{rank}(CD) = \operatorname{rank}(D)$.

Assumption 2.3. d is a bounded disturbance such that $\|d\| \le \gamma_1$, where γ_1 is a known function greater than 0.

3. Design of Fault Diagnosis Scheme

3.1. Coordinate Transformation

Coordinate transformation is used, under certain geometric conditions, to decouple the unknown input disturbance and faults [10]. Assumption 2.2 implies that there are two transformation matrixes, T and S [9]. The system (2.1) can be decomposed into the following two subsystems (the details of the transformation process are given elsewhere [9]):

$$\begin{aligned} \dot{z}_1 &= \overline{A}_{11} z_1 + \overline{A}_{12} z_2 + \overline{B}_1 u + \overline{f}_1, \\ v_1 &= \overline{C}_{11} z_1, \end{aligned} \tag{3.1}$$

$$\begin{aligned} \dot{z}_2 &= \overline{A}_{21} z_1 + \overline{A}_{22} z_2 + \overline{B}_2 u + \overline{f}_2 + D_2 d, \\ v_2 &= \overline{C}_{22} z_2, \end{aligned} \tag{3.2}$$

where $z_1 \in R^{n-q}$, $z_2 \in R^q$, $\overline{A}_{11} \in R^{(n-q)\times(n-q)}$, $\overline{A}_{12} \in R^{(n-q)\times q}$, $\overline{A}_{21} \in R^{q\times(n-q)}$, $\overline{A}_{22} \in R^{q\times q}$, $v_1 \in R^{p-q}$, $v_2 \in R^q$, $\overline{C}_{11} \in R^{(p-q)\times(n-q)}$, $\overline{C}_{22} \in R^{q\times q}$.

The matrix transformation and definition in systems (3.1) and (3.2) are as follows:

$$\begin{gathered} TAT^{-1} = \overline{A} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix}, \qquad TB = \overline{B} = \begin{bmatrix} \overline{B}_1 \\ \overline{B}_2 \end{bmatrix}, \qquad Tf = \overline{f} = \begin{bmatrix} \overline{f}_1 \\ \overline{f}_2 \end{bmatrix}, \\ TD = \overline{D} = \begin{bmatrix} 0 \\ \overline{D}_2 \end{bmatrix}, \qquad SCT^{-1} = \begin{bmatrix} \overline{C}_{11} & 0 \\ 0 & \overline{C}_{22} \end{bmatrix}, \end{gathered} \tag{3.3}$$

where $\overline{C}_{22}$ is an invertible matrix.

The transformation matrix T is

$$T = \begin{bmatrix} I_{n-q} & -D_1 D_2^{-1} \\ 0 & I_q \end{bmatrix}. \tag{3.4}$$

3.2. Observer Design

The following assumptions are made for transformed systems (3.1) and (3.2).

Assumption 3.1. $(\overline{A}_{11}, \overline{C}_{11})$ and $(\overline{A}_{22}, \overline{C}_{22})$ are observable.

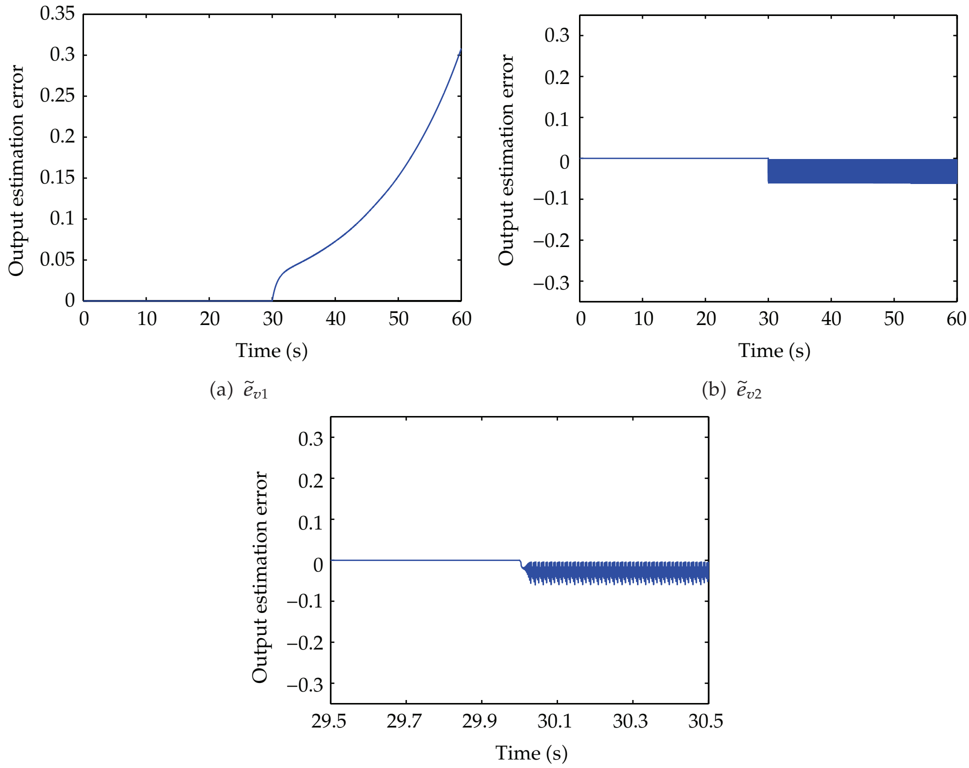
Assumption 3.1 implies that there are matrixes L_1 and L_2, which enable A_{01} and A_{02} to be stable matrices:

$$A_{01} = \overline{A}_{11} - L_1 \overline{C}_{11}, \qquad A_{02} = \overline{A}_{22} - L_2 \overline{C}_{22}. \tag{3.5}$$

There are also two Lyapunov equations:

$$A_{01}^{\mathrm{T}} P_1 + P_1 A_{01} = -Q_1, \qquad A_{02}^{\mathrm{T}} P_2 + P_2 A_{02} = -Q_2, \tag{3.6}$$

where P_1, Q_1, P_2, and Q_2 are all symmetric positive definite (SPD) matrices.



(a) $\tilde{e}_{v1}$

(b) $\tilde{e}_{v2}$

(c) $\tilde{e}_{v2}$ at 29.5–30.5 s

Figure 1: Output estimation error in case 1 using a high frequency disturbance.

Assumption 3.2. The function $\overline{f}$ satisfies the Lipschitz condition:

$$\left\|\overline{f}(x,u,t)-\overline{f}(\hat{x},u,t)\right\| \le \overline{\gamma}\|x-\hat{x}\|, \tag{3.7}$$

where $\overline{\gamma}\|x-\hat{x}\| = \overline{\gamma}\|T^{-1}\|\|z-\hat{z}\| = \gamma\|z-\hat{z}\|$, and γ is the Lipschitz constant.

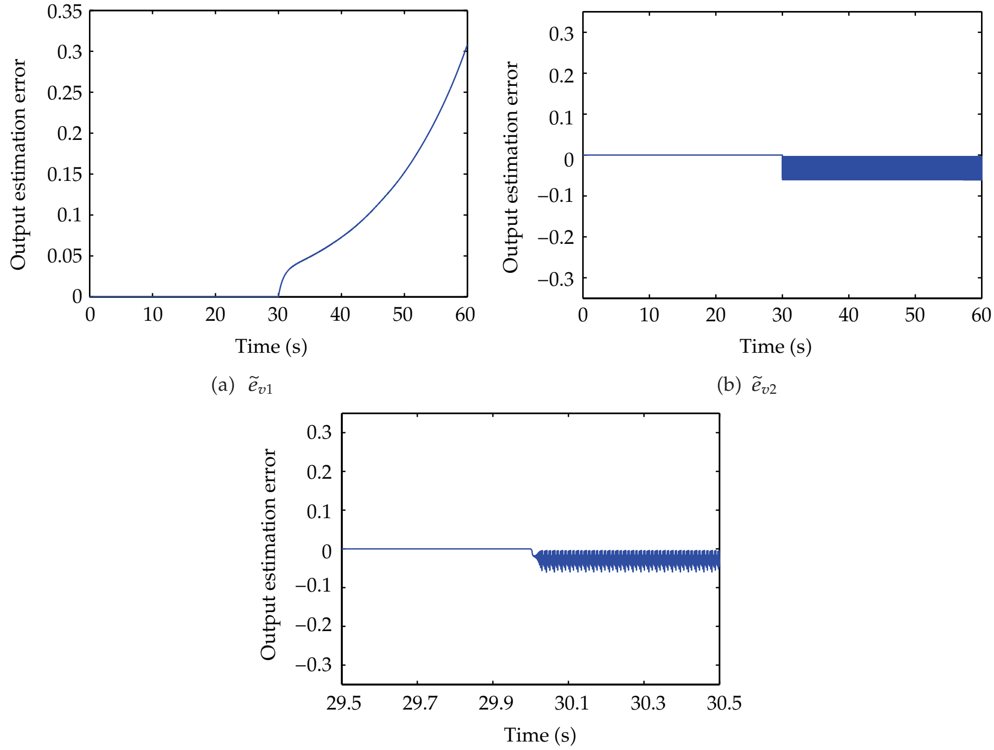
A Luenberger observer is constructed for subsystem (3.1):

$$\begin{aligned}\dot{\hat{z}}_1 &= \overline{A}_{11}\hat{z}_1+\overline{A}_{12}\hat{z}_2+\overline{B}_1 u+\overline{f}_1\left(T^{-1}\hat{z},u,t\right)+L_1(v_1-\hat{v}_1),\\ \hat{v}_1 &= \overline{C}_{11}\hat{z}_1,\end{aligned} \tag{3.8}$$

where superscript "Λ" indicates estimate value.

A sliding mode variable structure observer is constructed for subsystem (3.2):

$$\begin{aligned}\dot{\hat{z}}_2 &= \overline{A}_{21}\hat{z}_1+\overline{A}_{22}\hat{z}_2+B_2 u+\overline{f}_2\left(T^{-1}\hat{z},u,t\right)+\overline{D}_2 w+L_2(v_2-\hat{v}_2),\\ \hat{v}_2 &= \overline{C}_{22}\hat{z}_2,\end{aligned} \tag{3.9}$$



(a) $\tilde{e}_{v1}$ (b) $\tilde{e}_{v2}$ (c) $\tilde{e}_{v2}$ at 29.5–30.5 s

Figure 2: Output estimation error in case 1 using a low frequency disturbance.

where w is the input signal of sliding mode variable structure, expressed as

$$w = \begin{cases} -\rho \dfrac{F(\hat{v}_2 - v_2)}{\|F(\hat{v}_2 - v_2)\| + \delta} & \text{if } \hat{v}_2 - v_2 \neq 0 \\ 0 & \text{if } \hat{v}_2 - v_2 = 0, \end{cases} \tag{3.10}$$

where $F \in R^{q\times q}$ is the matrix to be designed; ρ is the scalar function to be designed, $\rho \geq \gamma_1$; δ is a positive constant of small value.

Assumption 3.3. The matrices P_2 and F have to be chosen such that $P_2\overline{D}_2 = \overline{C}_{22}^{\mathrm{T}} F^{\mathrm{T}}$.

Define $e_1 = z_1 - \hat{z}_1$, $e_2 = z_2 - \hat{z}_2$, $e = \left[\begin{smallmatrix} e_1 \\ e_2 \end{smallmatrix}\right]$ as the state estimation errors and $e_{v1} = v_1 - \hat{v}_1 = \overline{C}_{11}e_1$, $e_{v2} = v_2 - \hat{v}_2 = \overline{C}_{22}e_2$ as the output estimation errors.

Based on (3.1), (3.2), (3.8), and (3.9), the corresponding observation-error dynamic equations are given by:

$$\begin{aligned} \dot{e}_1 &= \left(\overline{A}_{11} - L_1\overline{C}_{11}\right)e_1 + \overline{A}_{12}e_2 + \overline{f}_1\left(T^{-1}z, u, t\right) - \overline{f}_1\left(T^{-1}\hat{z}, u, t\right), \\ \dot{e}_2 &= \left(\overline{A}_{22} - L_2C_{22}\right)e_2 + \overline{A}_{21}e_1 + \overline{f}_2\left(T^{-1}z, u, t\right) - \overline{f}_2\left(T^{-1}\hat{z}, u, t\right) + \overline{D}_2(d - w_2). \end{aligned} \tag{3.11}$$

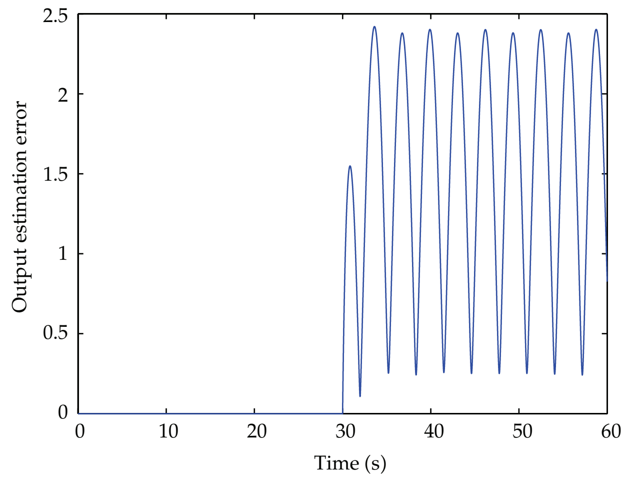


Figure 3: Output estimation error in case 2 using a high frequency disturbance.

Lemma 3.4 (see [11]). *If $g(x,u,t)$ satisfies the Lipschitz condition, then there will be an SPD matrix P that satisfies the following equation:*

$$2\varepsilon^T P\big(g(x_1,u,t)-g(x_2,u,t)\big) \le k^2\varepsilon^T PP\varepsilon + \varepsilon^T\varepsilon, \tag{3.12}$$

where $\varepsilon = x_1 - x_2$ and k is the Lipschitz constant.

Lemma 3.5 (Schur Complement [12]). *For a given symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{bmatrix}$, where $S_{11} \in R^{r\times r}$. The following three conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Theorem 3.6. *Under Assumptions 2.1–2.3 and 3.1–3.3, if the following LMI holds,*

$$\begin{bmatrix} H_1 & \overline{A}_{21}^T P_2 + P_1\overline{A}_{12} & P_1 & 0 \\ \overline{A}_{12}^T P_1 + P_2\overline{A}_{21} & H_2 & 0 & P_2 \\ P_1 & 0 & -\dfrac{1}{\gamma^2}I & 0 \\ 0 & P_2 & 0 & -\dfrac{1}{\gamma^2}I \end{bmatrix} < 0, \tag{3.13}$$

where $H_1 = \overline{A}_{11}^T P_1 + P_1\overline{A}_{11} - Y_1^T - Y_1 + I$, $H_2 = \overline{A}_{22}^T P_2 + P_2\overline{A}_{22} - Y_2^T - Y_2 + I$, $Y_1 = P_1 L_1 \overline{C}_{11}$ and $Y_2 = P_2 L_2 \overline{C}_{22}$, then e_1 and e_2 will converge to the zero point.

Proof. Consider the following Lyapunov function:

$$V = e_1^{\mathrm{T}} P_1 e_1 + e_2^{\mathrm{T}} P_2 e_2. \tag{3.14}$$

The derivative of the Lyapunov function with respect to time is

$$\begin{aligned}\dot{V} &= e_1^{\mathrm{T}}\left(A_{01}^{\mathrm{T}} P_1 + P_1 A_{01}\right)e_1 + 2e_1^{\mathrm{T}} P_1\left(\overline{A}_{12} e_2 + \overline{f}_1(z,u,t) - \overline{f}_1(\hat{z},u,t)\right) \\ &\quad + e_2^{\mathrm{T}}\left(A_{02}^{\mathrm{T}} P_2 + P_2 A_{02}\right)e_2 + 2e_2^{\mathrm{T}} P_2\left(\overline{A}_{21} e_1 + \overline{f}_2(z,u,t) - \overline{f}_2(\hat{z},u,t) + \overline{D}_2(d-w)\right).\end{aligned} \tag{3.15}$$

Let

$$\widetilde{A} = \begin{bmatrix} \overline{A}_{11} - L_1\overline{C}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} - L_2\overline{C}_{22} \end{bmatrix}. \tag{3.16}$$

then

$$\begin{aligned}\dot{V} &= e^{\mathrm{T}}\left(\widetilde{A}^{\mathrm{T}} P + P\widetilde{A}\right)e + 2e^{\mathrm{T}} P\left(\overline{f}(z,u,t) - \overline{f}(\hat{z},u,t)\right) + 2e_2^{\mathrm{T}}\overline{C}_{22}^{\mathrm{T}} F^{\mathrm{T}}\left(d - \rho\frac{Fe_{v2}}{\|Fe_{v2}\|}\right) \\ &\le e^{\mathrm{T}}\left(\widetilde{A}^{\mathrm{T}} P + P\widetilde{A}\right)e + 2e^{\mathrm{T}} P\left(\overline{f}(z,u,t) - \overline{f}(\hat{z},u,t)\right) - 2\|Fe_{v2}\|(\rho - \gamma_1).\end{aligned} \tag{3.17}$$

From Lemma 3.4 we find that

$$\dot{V} \le e^{\mathrm{T}}\left(\widetilde{A}^{\mathrm{T}} P + P\widetilde{A} + \gamma^2 PP + I\right)e. \tag{3.18}$$

According to Lemma 3.5, when the following inequality is satisfied,

$$\begin{bmatrix} \widetilde{A}^{\mathrm{T}} P + P\widetilde{A} + I & P \\ P & -\dfrac{1}{\gamma^2} I \end{bmatrix} < 0 \tag{3.19}$$

then $\dot{V} < 0$.

Since $\widetilde{A} = \begin{bmatrix} \overline{A}_{11} - L_1\overline{C}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} - L_2\overline{C}_{22} \end{bmatrix}$, $P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$, $L = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}$, the above LMI (3.19) can be transform into inequality (3.13), which is listed in Theorem 3.6.

This completes the proof. □

The main advantage of the method presented in this paper is that the complex equations for obtaining parameters given by Chen and Chowdhury [9] are transformed into inequality (3.13), a standard LMI. Then the solution becomes much easier when the LMI toolbox is used in MATLAB.

3.3. Fault Diagnosis

When a fault occurs, the systems (3.1) and (3.2) are transformed into the following equations:

$$\begin{aligned}
\dot{z}_1 &= \overline{A}_{11}z_1 + \overline{A}_{12}z_2 + \overline{B}_1u + \overline{f}_1 + \overline{E}_1 f_a(t)\\
v_1 &= \overline{C}_{11}z_1\\
\dot{z}_2 &= \overline{A}_{21}z_1 + \overline{A}_{22}z_2 + \overline{B}_2u + \overline{f}_2 + \overline{D}_2 d + \overline{E}_2 f_a(t),\\
v_2 &= \overline{C}_{22}z_2,
\end{aligned} \tag{3.20}$$

where $\overline{E}_1$ is a nonzero matrix, $\overline{E}_1 \in R^{n-q}$, $TE = \begin{bmatrix}\overline{E}_1\\ \overline{E}_2\end{bmatrix}$, $E \in R^{n\times r}$ is a known fault distribution matrix. $f_a(t) \in R^r$ is an unknown bounded nonlinear function, which represents incipient faults of the system.

The dynamic equations of e_1 and e_2 of observer can be obtained from (3.8), (3.9), and(3.20), thus errors e_{v1}, e_{v2} are written in the following forms:

$$\begin{aligned}
\dot{\tilde{e}}_1 &= \left(\overline{A}_{11} - L_1\overline{C}_{11}\right)e_1 + \overline{A}_{12}e_2 + \overline{f}_1\left(T^{-1}z, u, t\right) - \overline{f}_1\left(T^{-1}\hat{z}, u, t\right) + \overline{E}_1 f_a(t),\\
\tilde{e}_{v1} &= \overline{C}_{11}\tilde{e}_1,
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
\dot{\tilde{e}}_2 &= \left(\overline{A}_{22} - L_2C_{22}\right)e_2 + \overline{A}_{21}e_1 + \overline{f}_2\left(T^{-1}z, u, t\right) - \overline{f}_2\left(T^{-1}\hat{z}, u, t\right) + \overline{D}_2(d - w_2) + \overline{E}_2 f_a(t),\\
\tilde{e}_{v2} &= \overline{C}_{11}\tilde{e}_2.
\end{aligned} \tag{3.22}$$

It can be concluded from Theorem 3.6 that, in spite of the unknown input disturbance, e_{v1} and e_{v2} still converge to zero field when there is no fault, or deviate from zero when the faults occur. In (3.22), which is an explicit of both faults and the unknown input disturbance, a sliding mode observer is designed for weakening the impact resulting from the disturbance. However, the inherent chattering phenomenon of slide mode variable structure control makes it difficult to distinguish the early incipient faults signals from the chattering signals until the faults develop to be serious enough. Thus, $R_2(t) = \tilde{e}_{v2}$ is unsuitable to be the faults detecting residual. By contrast, the $\tilde{e}_{v1}$ in (3.21) is designed with no sliding mode observer in order to avoid the impact from chattering; therefore, $R_1(t)$ can be used as the residual for incipient fault detecting. The simulation results in Section 4 will verify the above statements.

4. Simulation Study

Consider a nonlinear system, a single-link robotic arm with a revolute elastic joint rotating in a vertical plane whose motion equations are [13]

$$\begin{aligned}
J_l\ddot{q}_1 + F_l\dot{q}_1 + k(q_1 - q_2) + mgl\sin q_1 &= 0,\\
J_m\ddot{q}_2 + F_m\dot{q}_2 - k(q_1 - q_2) &= u,
\end{aligned} \tag{4.1}$$

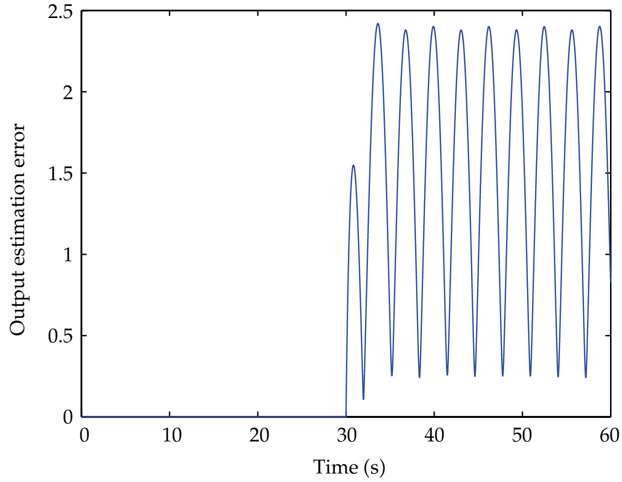


Figure 4: Output estimation error in case 2 using a low frequency disturbance.

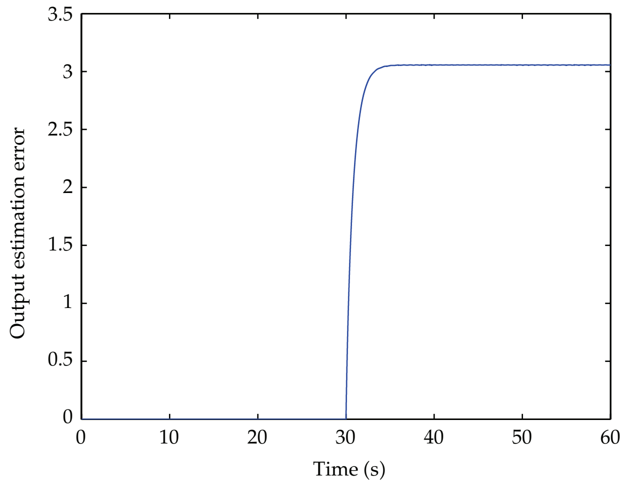


Figure 5: Output estimation error in case 3 using a high frequency disturbance.

where q_1 and q_2 are the link displacement and the rotor displacement, respectively. The link inertia J_l, the motor rotor inertia J_m, the elastic constant k, the link mass m, the gravity constant g, the center of mass l, and the viscous friction coefficients F_l, F_m are all positive constant parameters. The control u is the torque delivered by the motor.

The state variables are chosen as $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$.

When handling different objects, the load was carried by the manipulator changes. In addition, the friction coefficient of the joint and other parameters also varies over time. All these factors are uniformly classified as unknown input disturbances, denoted by d. Suppose a fault f_a occurs to the manipulator, then the single-link robotic arm model with unknown

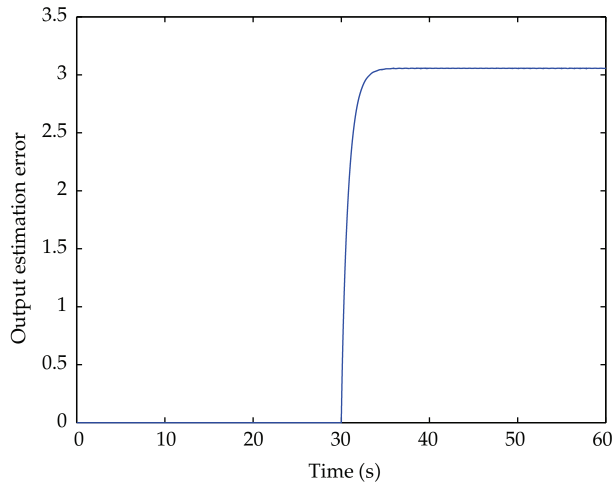


Figure 6: Output estimation error in case 3 using a low frequency disturbance.

input disturbances and faults can be expressed as the following fourth-order nonlinear state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k}{J_l} & \dfrac{-F_l}{J_l} & \dfrac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{J_m} & 0 & \dfrac{-k}{J_m} & \dfrac{-F_m}{J_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{-mgl}{J_l} \sin x_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{J_m} \end{bmatrix} u + Ef_a + Dd,$$
$$y = C\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T. \tag{4.2}$$

The selected manipulator parameters are $k = 2\,\text{Nm/rad}$, $F_m = 1$, $F_l = 0.5\,\text{Nm/(rad/s)}$, $J_m = 1\,\text{Nm}^2$, $J_l = 2\,\text{Nm}^2$, $m = 0.15\,\text{kg}$, $g = 9.8$, $l = 0.3\,\text{m}$.

Corresponding to system (2.1), the parameter matrices for each equation are, respectively,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.25 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -2 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$f(x,u,t) = \begin{bmatrix} 0 \\ -0.2205 \sin x_1 \\ 0 \\ 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 1 & 0 & 0.5 \end{bmatrix}^{\mathrm{T}}, \qquad E = \begin{bmatrix} 2.929 & 3.814 & 4 & 1 \end{bmatrix}^{\mathrm{T}}. \tag{4.3}$$

The selected transformation matrices T and S are, respectively,

$$T = \begin{bmatrix} I_{n-q} & -D_1 D_2^{-1} \\ 0 & I_q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4.4}$$

Using the transformation matrices described above, system (4.2) is decomposed into the following two subsystems:

$$\dot{z}_1 = \begin{bmatrix} \dot{z}_{11} \\ \dot{z}_{12} \\ \dot{z}_{13} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -5 & -0.25 & 5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{11} \\ z_{12} \\ z_{13} \end{bmatrix} + \begin{bmatrix} 2 \\ 1.5 \\ 1 \end{bmatrix} z_2 + \begin{bmatrix} 0 \\ -0.2205 \sin(z_{11}) \\ 0 \end{bmatrix} + \begin{bmatrix} 2.929 \\ 1.814 \\ 4 \end{bmatrix} f_a + \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix} u,$$
$$v_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_{11} \\ z_{12} \\ z_{13} \end{bmatrix}, \tag{4.5}$$

$$\dot{z}_2 = \begin{bmatrix} 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} z_{11} \\ z_{12} \\ z_{13} \end{bmatrix} - z_2 + 0 + f_a + 0.5d + u,$$
$$v_2 = z_2. \tag{4.6}$$

After the coordinate transformation, a reduced-order subsystem, as shown in (4.5), is decomposed from the system (4.2). The subsystem only contains fault f_a; it does not contain the unknown input disturbances d. The impact of d on subsystem (4.5) is delivered by the state variables z_1 and z_2. Using the above observer, the impact of d on the system is eliminated, and the unknown input disturbance and the fault are decoupled.

The LMI toolbox in MATLAB is used to solve the inequalities. L_1, L_2, and F are selected to satisfy Assumptions 3.1 and 3.3.

$$L_1 = \begin{bmatrix} 6.6074 & 0.5546 \\ 0.3804 & 2.6950 \\ 0.0483 & 1.4015 \end{bmatrix}, \qquad L_2 = [2.7052], \qquad F = [0.3428]. \tag{4.7}$$

It is assumed that the incipient fault $f_a(t)$ begins at time instant 30 s, and $f_a(t) = 0$, when $t < 30$ s. The input signal is $u = \sin(t/3)$ and the simulation parameters are $\rho = 220$, $\delta = 0.01$.

With the above simulation parameters, we use three kinds of faults to verify the effectiveness of the proposed method. In the first case, an incipient fault is selected as $f_a(t) = 0.002 \exp(0.0667t)$, and a high frequency disturbance $d(t) = \sin(200t)$ is used. As shown in Figure 1, the output estimation error from the sliding mode observer $\tilde{e}_{v2}$ vibrates at a high frequency after the fault occurs, though its amplitude is small. This indicates that it is insensitive to the fault. However, the output estimation error $\tilde{e}_{v1}$ increases rapidly after the fault occurs. Thus, the error $\tilde{e}_{v1}$ is defined as the fault diagnosis residual $R_1(t)$, and a fault alarm can be sent immediately after a time of 30 s.

Another low frequency disturbance $d(t) = \sin(0.1t)$ is used to verify the proposed method. The simulation results are shown in Figure 2. It can be seen that output estimation error from the Luenberger observer is sensitive to the fault, but that from the sliding mode observer is still insensitive.

In the second case, a low-frequency sinusoidal signal is selected to illustrate that the fault detection is sensitive to incipient faults, that is, $f_a(t) = \sin(t)$. A high frequency disturbance $d(t) = \sin(200t)$ and a low frequency disturbance $d(t) = \sin(0.1t)$ are used, respectively. The associated simulation results in Figures 3 and 4 verify that the proposed approach can be applied to detect incipient fault rapidly.

In the third case, $f_a(t) = \tanh(t)$. A high frequency disturbance $d(t) = \sin(200t)$ and a low frequency disturbance $d(t) = \sin(0.1t)$ are used, respectively. The associated simulations are shown in Figures 5 and 6. Through the above verifications, we can see that the proposed method can effectively detect incipient faults.

5. Conclusion

This paper has introduced a fault diagnosis scheme which combines a sliding mode observer and a Luenberger observer for nonlinear systems. The proposed method makes full use of the complementarity between the sliding mode observer and the Luenberger observer and transforms the system, using a coordinate transformation, into two subsystems. The input estimation error from the Luenberger observer, which is constructed for the unrelated subsystem with unknown input disturbances, is defined as the fault detection residual in incipient fault diagnosis. A sliding mode observer is constructed to eliminate the impact of faults and disturbances in the other subsystem. An LMI-based approach is employed to design the observer, which makes it easy to obtain its parameters. The simulation results verify the validity of the proposed method.

Acknowledgments

This work is supported by the Natural Science Foundation of China (61273157 and 61104024), Hunan Province Education Department (12A040), and Construct Program of the Key Discipline in Hunan Province.

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