Internat. J. Math. & Math. Sci.
Vol. 7 No. 2 (1984) 303-310

PRECONVERGENCE COMPACTNESS AND P-CLOSED SPACES

ROBERT A. HERRMANN

Mathematics Department
U. S. Naval Academy
Annapolis, Maryland 21402 U.S.A.

(Received February 11, 1983 and in revised form June 11, 1983)

ABSTRACT. In this article the major result characterizes preconvergence compactness in terms of the preconvergence closedness of second projections. Applying this result to a topological space (X,T) yields similar characterizations for H-closed, nearly compact, completely Hausdorff-closed, extremely disconnected Hausdorff-closed, Urysohn-closed, S-closed and R-closed spaces, among others. Moreover, it is established that the s-convergence of Thompson (i.e. rc-convergence) is equivalent to topological convergence where the topology has as a subbase the set of all regular-closed elements of T.

KEY WORDS AND PHRASES. Convergence spaces, preconvergence spaces, H-closed, nearly compact, completely Hausdorff-closed, EDH-closed, Urysohn-closed, S-closed, R-closed.
1980 MATHEMATICS SUBJECT CLASSIFICATION CODE. 54A05, 54D30

1. INTRODUCTION.

In 1959, S. Mrówka [1] established the following major characterization for compact tological spaces. A topological space X is compact iff the second projection $P_2:X\times Y \to Y$ is a closed map for every topological space Y. Since that time numerous researchers have either generalized Mrówka's result to obtain propositions of the form: a topological space X has a specific property iff the map $P_2:X\times Y \to Y$ is a closed-like map for every topological space Y or they have improved upon Mrówka's basic result.

Recently it has been observed by Joseph [2] that the class of all topological spaces mentioned in Mrówks's theorem may be reduced to the class S of all topological Hausdorff, completely normal, fully normal door spaces. The topological characterizations extremely disconnected and zero-dimensional may also be added to this list. Moreover, Joseph has shown that a similar theorem holds true when compactness is replaced by H-closedness [resp. near-compactness] and the closed projection concept is replaced by a θ-closed [resp. δ-closed] projection [Joseph [3].

The fact that Mrówka's important characterization can be generalized to topologically nearly-compact spaces is not very surprising since near-compactness and δ-convergence are determined by the topology known as the semiregularization (Herrmann [4]). On the other hand, the basic topological θ-concepts are all equivalent to similar concepts for the pretopological θ-convergence structure. Furthermore, as shown in Herrmann [5] the θ-convergence structure is topological iff the ground topological space is almost-regular. Consequently Joseph's characterization for H-closedness is a pure extension of Mrówka's fundamental conclusion and indicates the importance of the class S of all Hausdorff completely normal, fully normal, door extremely disconnected and zero-dimensional topological spaces when such projection characterizations are being considered.

The main and rather surprising result of this present investigation shows that the class S and closed-like second projections characterize a wide variety of compact-like spaces. Many new results as well as those of Mrówka, Joseph and other researchers will follow as nontrivial corollaries. The main result also adjoins an additional proposition to the growing list of results which continue to establish that the convergence function as introduced by Kent [6] is one of the most fundamentally important and useful generalizations for the topological space yet devised.

2. PRELIMINARIES.

Let X be nonempty and $P(X)$ denotes the power set of X. If $\mathcal{F} \subset P(X)$ has the finite intersection property, then let $[\mathcal{F}]$ [resp. $<\mathcal{F}>$] denote the filter base [resp. filter] generated by $\mathcal{F}$. In this article only proper filters are considered. Let F(X) [resp. U(X)] denote the set of all filters [resp. ultrafilters] on X.

A map $q: F(X) \to P(X)$ determines a preconvergence space (X,q) if

(1) for each $x \in X$, $x \in q(<\{x\}>)$ and

(2) for each $\mathcal{F}$, $\mathcal{G} \in F(X)$ such that $\mathcal{F} \subset \mathcal{G}$, it follows that $q(\mathcal{F}) \subset q(\mathcal{G})$.

If (X,q) is a preconvergence space and $x \in q(\mathcal{F})$, where $\mathcal{F} \in F(X)$, then $\mathcal{F}$ is said to q-converge to x. This is often denoted by $\mathcal{F} \to x$. For a preconvergence space (X,q), if $x \in q(\mathcal{F})$ implies that $x \in q(\mathcal{F} \cap <\{x\}>)$, then (X,q) is a <u>convergence space</u> in the sense of Kent.

Throughout the remainder of this paper (X,q) and (Y,p) and the like denote preconvergence spaces. Let $A \subset X$. Then $cl_q(A) = \{x \mid \exists\, \mathcal{U}\,[[\mathcal{U} \in U(X)] \wedge [A \in \mathcal{U}] \wedge [\mathcal{U} \to x]]\}$ is the q-<u>closure</u> of A. For $\mathcal{F} \in F(X)$, $a_q(\mathcal{F}) = \{x \mid \exists\, \mathcal{U}\,[[\mathcal{U} \in U(X)] \wedge [\mathcal{F} \subset \mathcal{U}] \wedge [\mathcal{U} \to x]]\}$ is the set of all q-<u>adherence</u> points for $\mathcal{F}$. As usual if $f: (X,q) \to (Y,p)$ (i.e. $f: X \to Y$ and X and Y are preconvergence spaces) and $\mathcal{F} \in F(X)$, then $f(\mathcal{F}) = <\{f[F] \mid F \in \mathcal{F}\}>$. Also is $\mathcal{G} \in F(X)$ and $f[X] \cap G \neq \emptyset$ for each $G \in \mathcal{G}$, then $f^{-1}(\mathcal{G}) = \{f^{-1}[G] \mid G \in \mathcal{G}\} \in F(X)$. The product preconvergence space $(X \times Y, \pi)$ where $\pi = q \times p$

is defined in the usual manner. A filter $\mathcal{F} \varepsilon F(X \times Y)$ π-converges to $(x,y) \varepsilon X \times Y$ if the first projection $P_1(\mathcal{F})$ q-converges to x and the second projection $P_2(\mathcal{F})$ p-converges to y. If every $\mathcal{U} \varepsilon U(X)$ q-converges to some $x \varepsilon X$, then (X,q) is q-compact. A set $A \subset X$ is q-closed if $A = cl_q(A)$ and a map $f:(X,q) \to (Y,p)$ is a (q,p)-closed (or simple closed) map if f maps q-closed subsets in X onto p-closed subsets of Y. In convergence space literature it is often the case that the "q" and "p" notation which appear in such terms as "compact", "closed", etc. are dropped when no confusion will occur. Finally we always assume the axiom of choice.

3. MAIN RESULTS.

THEOREM 1. Let (X,q) be q-compact. Then the projection $P_2:(X \times Y, \pi) \to (Y,p)$ has the property that for each $A \subset X \times Y$

$$P_2[cl_\pi(A)] = cl_p(P_2[A]). \tag{1.1}$$

PROOF. Since the projections are preconvergence continuous, then for each $A \subset X \times Y$, it follows that $P_2[cl_\pi(A)] \subset cl_p(P_2[A])$ for continuity preserves convergence of the filters. Let (X,q) be q-compact and $y \varepsilon cl_p(P_2[A])$. Then there exists some $\mathcal{U} \varepsilon U(Y)$ such that $\mathcal{U} \to y$ and $P_2[A] \varepsilon \mathcal{U}$. Since for each $U \varepsilon \mathcal{U}$, $P_2[A] \cap U \neq \emptyset$, then it follows that $<\{P_2^{-1}[U] \cap A | U \varepsilon \mathcal{U}\}> = \mathcal{G} \varepsilon F(X \times Y)$. Now $P_2(\mathcal{G}) = \mathcal{U}$ implies that $P_2(\mathcal{G}) \to y$. The compactness of (X,q) implies that there exists some $x \varepsilon a_q(P_1(\mathcal{G}))$. Since $\{P_1(\mathcal{U}) | [\mathcal{U} \varepsilon U(X \times Y)] \wedge [\mathcal{G} \subset \mathcal{U}]\} = \{\mathcal{V} | [\mathcal{V} \varepsilon U(X)] \wedge [P_1(\mathcal{G}) \subset \mathcal{V}]\}$, then it follows that there exists some $\mathcal{U}_0 \varepsilon U(X \times Y)$ such that $P_1(\mathcal{U}_0) \to x$ and $\mathcal{G} \subset \mathcal{U}_0$. Observe that $P_2(\mathcal{U}_0) = P_2(\mathcal{G}) = \mathcal{U}$. From the definition of $\pi = q \times p$, it follows that $\mathcal{U}_0 \to (x,y)$. Moreover, since $A \varepsilon \mathcal{G}$, then $A \varepsilon \mathcal{U}_0$. Consequently, $(x,y) \varepsilon cl_\pi(A)$ implies that $y \varepsilon P_2[cl_\pi(A)]$ and this completes the proof.

COROLLARY 1. Let (X,q) be q-compact. Then $P_2:(X \times Y), \pi) \to (Y,p)$ is a closure preserving and closed mapping.

Assume that $z \notin X$ and $\mathcal{U} \varepsilon FU(X)$, the set of all nonprincipal ultrafilters on X. Let $\mathcal{B}_\mathcal{U} = \{U \cup \{z\} | U \varepsilon \mathcal{U}\} \cup \{\{x\} | x \varepsilon X\}$. The set $\mathcal{B}_\mathcal{U}$ is a base for a topology $T_\mathcal{U}$ on $Z = X \cup \{z\}$. It is not difficult to show that $T_\mathcal{U} \varepsilon \mathcal{S}$. Let $\mathcal{S}_z = \{T_\mathcal{U} | \mathcal{U} \varepsilon FU(X)\}$. Recall that a preconvergence space is T_1 if each principal ultrafilter converges to one and only one point. Obviously a finite preconvergence space is compact.

THEOREM 2. Let (X,q) be a T_1 preconvergence space and assume that $z \notin X$. If for each $T_\mathcal{U} \varepsilon \mathcal{S}_z$ the second projection $P_2:(X \times Z, q \times T_\mathcal{U}) \to (Z, T_\mathcal{U})$ is a $(q \times T_\mathcal{U}, T_\mathcal{U})$-closed map, then X is q-compact.

PROOF. Assume that X is infinite, T_1 and not q-compact. Then there exists a non-q-converging $\mathcal{U} \varepsilon U(X)$. Since every principal ultrafilter is q-convegent, then $\mathcal{U}$ is nonprincipal. Assume that $z \notin X$ and consider the $T_\mathcal{U}$ topology on $Z = X \cup \{z\}$. Let $D = \{(x,x) | x \varepsilon X\}$. We show that D is $\pi = q \times T_\mathcal{U}$ - closed. Suppose that (a,b) ε X×Z - D. Then $a \neq b$.

For the first case, assume that $b \neq z$. Let $\mathcal{V} \varepsilon U(X \times Z)$, $\mathcal{V} \to (a,b)$ and $D \varepsilon \mathcal{V}$. The first projection $P_1[D] \varepsilon P_1(\mathcal{V})$, $P_1(\mathcal{V}) \to a$ and the second projection

$P_2[D] \in P_2(\mathcal{V}) \to b$. From the construction of $T_{\mathcal{U}}$ it is obvious that $P_2(\mathcal{V}) = <\{b\}>$. Hence since $b \in X$, then $\{P_2[V] \cap X | V \in \mathcal{V}\} = P_2(\mathcal{V})_X \in U(X)$. Let $U \in \mathcal{V}$. Then $\emptyset \neq U \cap D = V \in \mathcal{V}$. Since for any $(c,d) \in V$, $c = d$, then it is clear that $P_1[V] = P_2[V] \in P_1(\mathcal{V})$. Thus for each $U \in \mathcal{V}$ there exists some $V \in \mathcal{V}$ such that $V \subset X\times Z$, $V \subset U$, $X \subset P_1[V] = P_2[V] \in P_2(\mathcal{V})$ and $P_2[V] \in P_1(\mathcal{V})$. Consequently, for each $U \in \mathcal{V}$ and each $V \in \mathcal{V}$, $P_1[V] \cap P_2[U] \cap X \neq \emptyset$. Hence $P_1(\mathcal{V}) = P_2(\mathcal{V})_X$. Thus $P_2(\mathcal{V})_X \to a$. Since $P_2(\mathcal{V}) = <\{b\}>$, then $P_2(\mathcal{V})_X = <\{b\}>_X \to b$. The T_1 property for q implies the contradiction that $a = b$. Consequently for the case that $b \neq z$ the result that $(a,b) \notin cl_\pi(D)$ implies that $(a,b) \in X\times Y - cl_\pi(D)$.

For the second case assume that $b = z$, $\mathcal{V} \in U(X\times Z)$, $\mathcal{V} \to (a,z)$ and $D \in \mathcal{V}$. Thus $P_1(\mathcal{V}) \to a$, $P_1(\mathcal{V}) \neq \mathcal{U}$ since $\mathcal{U}$ is non-q-convergent and $P_2[D] \in P_2(\mathcal{V}) \to z$. Consequently, there exists some $V \in \mathcal{V}$ and $U \in \mathcal{U}$ such that $P_1(V) \cap U = \emptyset$. Letting $V_1 = V \cap D$, then it follows that $P_1[V_1] = P_2[V_1]$, $P_1[V_1] \in P_1(\mathcal{V})$, $P_1[V_1] \in P_2(\mathcal{V})$, $P_1[V_1] \subset X$ and $P_1[V_1] \cap U = \emptyset$. From the construction of T the result that $P_2[V_1] \cap U = \emptyset$ implies that $z \in P_2[V_1]$. Since $z \notin X$ and $P_2[V_1] \subset X$ we have a contradiction. Hence $(a,z) \in X\times Z - cl_\pi(D)$ implies that $X\times Z - D \subset X\times Z - cl_\pi(D)$.

Observing that in general $D \subset cl_\pi(D)$, then application of the above two cases yields $D = cl_\pi(D)$ and that $P_2[D] = cl_{T_{\mathcal{U}}}(P_2[D])$. However, since $P(_2[D]) = X$, then X is $T_{\mathcal{U}}$-closed in Z. The construction of $T_{\mathcal{U}}$ implies that $z \in cl_{T_{\mathcal{U}}}(X) = X$. This final contradiction completes the proof.

COROLLARY 2. Let (X,q) be T_1. Then $P_2:(X\times Z, q\times T_{\mathcal{U}}) \to (Z, T_{\mathcal{U}})$ is a $(q\times T_{\mathcal{U}}, T_{\mathcal{U}})$-closed map for each $T_{\mathcal{U}} \in \mathcal{S}_Z$ iff X is q-compact.

COROLLARY 3. Let (X,q) be T_1. Then $P_2:(X\times Z, q\times T_{\mathcal{U}}) \to (Z, T_{\mathcal{U}})$ is a $(q\times T_{\mathcal{U}}, T_{\mathcal{U}})$-closed map for each $(Z, T_{\mathcal{U}}) \in \mathcal{S}$ iff (X,q) is q-compact.

4. TOPOLOGICAL IMPLICATIONS.

In what follows we shall need to compare topological and preconvergence space concepts. When a term denotes a topological concept it will be preceded by the symbol "top."

Recently many special preconvergence structures have been of interest to the general topologist. Assume that (X,T) is a top space and for each $x \in X$ let $\mathcal{C}(x) = \{cl_T(G) | x \in G \in T\}$. Then a filter or filter base $\mathcal{F}$ on X θ-converges to $x \in X$ iff $<\mathcal{C}(x)> \subset <\mathcal{F}>$ iff for each $C \in \mathcal{C}(x)$ there exists some $F \in \mathcal{F}$ such that $F \subset C$. Let (X,T_*) denote the top space generated by the set of all top regular-open members of T. This is called the <u>semiregularization</u>. A filter or filter base $\mathcal{F}$ on X δ-converges to $x \in X$ if it T_*-converges. Herrington [7] introduces f-convergence which is implicitly shown in Herrmann [8] to be equivalent to w-convergence. Let (X,T_w) be the weak topological space generated by $C(X)$. A filter or filter base <u>w-converges</u> to $x \in X$ if it T_w-converges. Let $RC(X)$ denote the set of all top regular-closed subsets of X. Then $RC(X) = \{cl_T(G) | G \in T\}$. For each $x \in X$, let $\mathcal{C}_r(x) = \{R | [R \in RC(X)] \wedge [x \in R]\}$. A filter or filter base $\mathcal{F}$ <u>rc-converges</u> to $x \in X$ iff $<\mathcal{C}_r(x)> \subset <\mathcal{F}>$ iff for each $R \in \mathcal{C}_r(x)$ there exists some $F \in \mathcal{F}$ such that $F \subset R$. It is known that rc-convergence is equivalent to s-convergence of Thmopson [9]. Let $x \in X$ and $x \in G \subset cl_T(G) \subset H$, where $G,H \in T$, be denoted by (x,G,H). A filter or filter base F u-converges to $x \in X$ if for each (x,G,H) there exists some $F \in F$ such that $F \subset cl_T(H)$ (Herrington [10]). For each $x \in X$ let $S(x)$ be a set of top open neighborhoods of x. The set $S(x)$ is said to be shrinkable if for each $G \in S(x)$ there exists some $H \in S(x)$ such that $cl_T(G) \subset H$. A filter or filter base F on X sh-converges to $x \in X$ if for each shrinkable $S(x)$ there exists some $G \in S(x)$ and some $F \in F$ such that $F \subset G$ (Herrington [11]). (Note: In the literature sh is denoted by s.) It is not difficult to show that all of the convergence concepts defined in this paragraph are at least preconvergence structures. Moreover, the rc and the θ-convergence structures are pretopological, and the w and δ are topological. The concept of the "accumulation" point for a filter or filter base is defined set-theoretically in the above references. These accumulation point definitions are all equivalent to the preconvergence adherence concept applied to these special topologically dependent preconvergence structures. In some of these special cases, the authors have also set-theoretically defined closure-type operators. In particular, the θ-closure is equivalent to the q-closure, where $q = \theta$; the s-closure (Dickman and Krystak [12]) and the θ-semiclosure (Joseph and Kwack[13]) are the same and are equivalent to the q-closure, where q = rc and the δ-closure is equivalent to the q-closure, where $q = T_*$.

The major interest in the above special preconvergence structures is that they characterize various compact-like top spaces. A top T_2 space is top T_2-closed iff it is θ-compact. A top almost normal Hausdorff space is top almost normal Hausdorff-closed if it is δ-compact (Singel and Mathur [14]). If P is any top property which implies top almost completely regular and which is possessed by every top nearly-compact space, then a top P-space is top P-closed iff it is δ-compact (Herrmann [15]). A top completely Hausdorff space is completely Hausdorff-closed iff it is w-compact. A top Urysohn space is top Urysohn-closed iff it is u-compact. A top T_3 space is top regular-closed iff it is sh-compact. A top space iss-closed iff it is rc-compact. A top extremely disconnected Hausdorff space is top extremely disconnect Hausdorff-closed iff it is rc-compact (Mathur [16]).

Prior to applying theorem 2 and its corollaries to elements of $C = \{\theta,\delta,w,rc,,sh\}$ it is necessary to find useful topological conditions which imply that (X,q), $q \in C$, is T_1. The following are known. The θ[resp. δ, w]-structure is T_1 iff (X,T) is top Hausdorff [res. weakly-Hausdorff, completely Hausdorff].

THEOREM 3. Let (X,T) be a top space.

(i) If (X,T) is top Urysohn, then (X,u) is T_1.

(ii) If (X,T) is top T_3, then (X,sh) is T_1.

PROOF. (i) Assume that (X,T) is top Urysohn and (X,u) is not T_1. Then there exists a principal ultrafilter $<\{x\}>$, $x \in X$, such that $<\{x\}> \rightarrow x$ and $<\{x\}> \rightarrow y$, $y \neq x$.

Consider any triple (y,G,H). Then there exists some $A \subset X$ such that $y \in A$ and $A \subset cl_T(H)$. However $x \in cl_T(H)$ implies that $x \in cl_T(G)$. Now (X,T) being top Urysohn implies that there exists $G_1, G_2 \in T$ such that $x \in G_1 \subset cl_T(G\ )$, $y \in G_2 \subset cl_T(G_2)$ and $cl_T(G_1) \cap cl_T(G_2) = \emptyset$. Thus $\{G_2, X - cl_T(G_1)\}$ has the property that $y \in G_2 \subset cl_T(G_2) \subset X - cl_T(G_1)$. From the above argument this yields that $x \in cl_T(G_2)$. The result follows from this contradiction.

(ii) Assume that (X,T) is top T_3 and (X,sh) is not T_1. Then there exist $x,y \in X$ such that $x \neq y$ and $<\{x\}> \to y$. Now the top open filter $G(x) = \{G | x \in G \in T\}$ is top regular-open on X and $< G(x)> \to x$. Hence $ad_T(< G(x)>) = \{x\}$ since (X,T) is top T_2. However $<\{x\}> \to y$ implies that $y \in ad_{sh}(< G(x)>)$. This contradicts theorem 2.3 part (a) in Herrington [11] and the result follows.

If the top space (X,T) is allowed to carry the appropriate property which assures that (X,q) is T_1 where $q \in \{\theta, \delta, w, u, sh\}$, then theorem 2 and its corollaries characterize a wide variety of top P-closed spaces.

It is well known that a pretopological convergence space (X,q) is topological iff the q-closure is idempotent. Let $C \subset P(X)$ be a cover for X. For each $x \in X$, let $C_x = \{C | [C \in C] \wedge [x \in C]\}$ and $C' = \{C_x | \ x \in C\}$. The next result is straightforward and the proof is omitted. Recall that for (X,q), $\lambda(q)$ is the finest top coarser than q and (X,q) is topological iff $q = \lambda(q)$.

THEOREM 4. A preconvergence space (X,q) is topological iff there exists a cover C of X such that for each $C_x \in C'$, $<C_x> \to x$ and whenever a filter $F \to y \in X$, then $<C_y> \subset F$.

COROLLARY 4. The rc-convergence structure is topological. The set RC(X) is a subbase for the rc-topology, T_{rc}.

COROLLARY 5. The rc-convergence structure is T_1 iff for each pair of distinct points $x,y \in X$, there exists a set E which is a finite intersection of top regular-closed subsets in X and $x \in E$, $y \notin E$.

REMARK: The fact that the rc-convergence structure is topological implies that many of the results relative to S-closed spaces which have recently appeared in the literature are simple re-statements of well-known topological propositions.

The results thus far obtained raise a number of interesting problems. It is known that $\theta \leq rc$ and that $rc \leq T$ iff (X,T) (the top space which generates the rc-structure) is top extremely disconnected (top T_2 not assumed). Moreover, it follows easily that rc = T-convergence iff (X,T) is top extremely disconnected and regular. It is known that θ is topological iff (X,T_*) is top regular. Now since it is always the case that $\theta \leq T_*$ -convergence $\leq$ T-convergence, then it follows that $\theta = T$ iff (X,T) is top regular. Observe that in general $u \leq T$ and $sh \leq T$ for any (X,T). If follows immediately from the definition of these two preconvergence structures, that if (X,T) is top T_3 [resp. to T_4], then u = T-convergence [resp. sh = T-convergence].

THEOREM 5. Let (X,T) be top Hausdorff [resp. weakly-Hausdorff (i.e. T_* is

Hausdorff), completely Hausdorff, T_1 in T_{rc}, Urysohn, T_3]. Let q = θ [resp. δ, w,rc,u,sh] be the definied modification of T and $q_U = \theta_U$ [resp. δ_U, w_U, rc_U, u_U, sh_U] be the corresponding modification of T_U. Then $P_2:(X\times Z, q\times q_U) \to (Z, q_U)$ is $(q \times q_U, q_U)$-closed for each $T_U \in S_Z$ iff (X,q) is compact.

PROOF. Simply observe that for each $T_U \in S_Z$, $T_U = \theta_U = \delta_U = w_U = rc_U = u_U = sh_U$.

Problem (1). Characterize those top spaces for which sh [resp. u] is pseudotopological, pretopological or topological.

Problem (2). Characterize those top spaces (X,T) such that sh = T [resp. u = T].

Problem (3). Characterize those top spaces (X,T) such that rc × rc = $rc(T\times T_U)$ (the rc-structure generated by $T \times T_U$) [resp. u × u = $u(T \times T_U)$, sh × sh = $sh(T \times T_U)$].

REFERENCES.

1. MROWKA, S. Compactness and product spaces, Colloq. Math. 7(1959), 125-133.
2. JOSEPH, J. On H-closed spaces, Proc. Amer. Math Soc. 55(1976), 223-226.
3. JOSEPH, J. Characterizations of nearly-compact spaces, Boll.U.M.I., B(5) 13 (1976), 311-321
4. HERRMANN, R. A. RC-convergence, Proc. Amer. Math. Soc. 75(1979), 311-317.
5. HERRMANN, R. A. θ-rigidity and the idempotent θ-closure, Math. Seminar Notes, 6(1978), 217-220.
6. KENT, D. Convergence functions and their related topologies, Fund. Math. 54(1967), 213-222.
7. HERRINGTON, L. L. Characterizations of completely Hausdorff-closed spaces, Proc. Amer. Math. Soc. 55(1976), 140-144.
8. HERRMANN, R. A. Point monads and P-closed spaces, Notre Dame J. Formal Logic, 20(1979), 395-400.
9. THOMPSON, T. S-closed spaces, Proc. Amer. Math. Soc. 60(1976), 335-338.
10. HERRINGTON, L. L. Characterizations of Urysohn-closed spaces, Proc. Amer. Math. Soc. 55(1976), 435-439.
11. HERRINGTON, L. L. Characterizations of regular-closed spaces, Math. Chronicle, 5(1976/77), 168-178.
12. DICKMAN, R. F. and KRYSTOCK, R. L. S-sets and s-perfect mappings, Proc. Amer. Math. Soc. 80(1980), 687-692.
13. JOSEPH, J. and KWACK, M. H. On S-closed spaces, Proc. Amer. Math. Soc. 80(1980), 341-348.
14. SINGAL, M. K. and MATHUR, A. On minimial almost normal spaces, Glasnik Mat. 10(30)(1975), 145-149.

15. HERRMANN, R. A. Nearly-compact Hausdorff extensions, Glasnik Mat. 12(32) (1977), 125-132.

16. MATHUR, A. A note in S-closed spaces, Proc. Amer. Math Soc. 74(1979), 350-352.