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RESEARCH NOTES

PIS FOR n-COUPLED NONLINEAR SYSTEMS

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ABSTRACT. A numerical algorithm dealing with solutions of equations with one variable may not be extended to solve nonlinear systems with n unknowns. Even when such extensions are possible, properties of these two similar algorithms are, in general, different. In [2] a perturbed iterative scheme (PIS) has been developed to solve nonlinear equations with one variable. Its properties with regard to nonlinear systems were analyzed in [1]. Here these properties were extended to n-coupled nonlinear systems.

KEY WORDS AND PHRASES. *Perturbed numerical iterations, nonlinear equations.*
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1. THE ALGORITHM.

In [2] a simple functional iterative scheme has been developed to solve nonlinear equations with one variable by adding a unique perturbation parameter to Picard's iterations. The algorithm was directly extended to nonlinear systems

[1] and convergence properties were analyzed using a special mapping called D-mapping [1]. We will study here that for n-coupled nonlinear systems analysis of convergence may be done in a similar way.

Let us consider a nonlinear system having n-coupled equations:

$$x^i = F^i(x^1, x^2, \ldots x^n) \tag{1.1}$$

where, $x^i = (x^i_1\ x^i_2\ \ldots\ x^i_n)^T \in D^i \subset R^n$

$F^i = (F^i_1\ F^i_2\ \ldots\ F^i_n)^T \in D^i \subset R^n,$

$i = 1, 2, \ldots n$ and R^n = real n-dimensional space. Thus, each F^i: $D^1 \times D^2 \times \ldots \times D^n \subset R^n \times R^n \times \ldots \times R^n \to D^i$. Let $R = R^n \times R^n \times \ldots \times R^n$ and $D = D^1 \times D^2 \times \ldots \times D^n$. As before [1] we assume that (1.1) has a solution in D given by $x^* = (x^{1,*}\ x^{2,*}\ \ldots\ x^{n,*})^T \in D$ where $x^{i,*} = (x^{i,*}_1\ x^{i,*}_2\ \ldots\ x^{i,*}_n)^T \in D^i$. Hence,

$$x^{i,*} = F^i(x^{1,*}, x^{2,*}, \ldots x^{n,*}) \tag{1.2}$$

In the element form, PIS is:

$$x^{i,k}_j = w^{i,k}_j + F^i_j(x^{1,k}, \ldots x^{i-1,k}, x^{i,k}_1 \ldots x^{i,k}_{j-1},$$
$$x^{i,k-1}_j \ldots x^{i,k-1}_n, x^{i+1,k-1}, \ldots x^{n,k-1}) \tag{1.3}$$

$i = 1, 2, \ldots n$ and $j = 1, 2, \ldots n$.

To compute the perturbation parameters $w^{i,k}_j$ we assume that they are small and their squares may be neglected; also the functionals F^i_j are $\partial F^i_j / \partial x^i_j \neq 1$, and $\partial^2 F^i_j / \partial x^{i2}_j$ are bounded $\forall\ x^i \in D^i$. Then assuming convergence after (k-1) iterations we have:

$$w^{i,k}_j + F^{i,k}_j = F^i_j(x^{1,k}, \ldots x^{i-1,k}, x^{1,k}_1 \ldots x^{i,k}_{j-1},$$
$$w^{i,k}_j + F^{i,k}_j, x^{i,k-1}_{j+1} \ldots x^{i,k-1}_n, x^{i+1,k-1} \ldots, x^{n,k-1}) \tag{1.4}$$

where $F^{i,k}_j$ is given by the second term of the right side of (1.3). Expanding the right hand side of (1.4) by Taylor's theorem and using the above assumptions we have:

$$w_j^{i,k} = (\overline{F}_j^{i,k} - F_j^{i,k}) \ / \ (1 - \partial_i F_j^{i,k}) \tag{1.5}$$

where $\overline{F}_j^{i,k} = F_j^i(x^{1,k} \ldots x^{i-1,k}, x_1^{i,k} \ldots x_{j-1}^{i,k}, F_j^{i,k}, x_{j+1}^{i,k-1} \ldots$

$x_n^{i,k-1}, x^{i+1,k-1}, \ldots, x^{n,k-1})$

and

$$\partial_j F_j^{i,k} = \left[\frac{\partial F_j^i}{\partial x_j^i}\right]_{x^{1,k} \ldots x^{i-1,k}, x_1^{i,k} \ldots x_{j-1}^{i,k}, F_j^{i,k}, x_{j+1}^{i,k-1} \ldots x_n^{i,k-1}, x^{i+1,k-1}, \ldots x^{n,k-1}}$$

Once $w_j^{i,k}$ is known from (1.5) we use PIS to get:

$$x_j^{i,k} = w_j^{i,k} + F_j^{i,k} \tag{1.6}$$

for $i = 1, 2, \ldots n$; $j = 1, 2, \ldots n$. Comparing this equation with (1.2) it is clear that a necessary condition for convergence is:

$$\lim_{k \to \infty} |w_j^{i,k}| = 0, \ \forall \ i,j \tag{1.7}$$

If we write, $X^k = (x^{1,k} \ldots x^{n,k})^T \varepsilon \ D$ and $W^k = (w^{1,k} \ldots w^{n,k})^T \varepsilon \ R$ then (1.6) may be expressed as:

$$X^k = W^k + F(X^k, X^{k-1}) \tag{1.8}$$

where $F : D \times D \subset R \times R \to D$

Now it is clear that $X^* = F(X^*, X^*)$ giving X^* as the fixed image of F on $D \times D$. Also, if F is a D-mapping [1] on $D \times D$, (1.7) will be both necessary and sufficient condition for convergence of PIS.

Hence, after studying the convergence analysis of PIS in [1], it is now easy to see the same concept being extended for n-coupled nonlinear systems.

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