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WEAKLY COMPATIBLE MAPS IN 2-NON-ARCHIMEDEAN MENGER PM-SPACES

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The aim of this paper is to introduce the concept of weakly compatible maps in 2-non-Archimedean Menger probabilistic metric (PM) spaces and to prove a theorem for these mappings without appeal to continuity. We also provide an application.

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1. Introduction. In 1999, Chugh and Sumitra [2] introduced the concept of 2-N.A. Menger PM-space as follows.

Definition 1.1. Let X be any nonempty set and L the set of all left continuous distribution functions. An ordered pair (X,F) is said to be a 2-non-Archimedean probabilistic metric space (briefly 2-N.A. PM-space) if F is a mapping from $X\times X\times X$ into L satisfying the following conditions (where the value of F at $x,y,z\in X\times X\times X$ is represented by $F_{x,y,z}$ or $F(x,y,z)$ for all $x,y,z\in X$):

(i) $F_{x,y,z}(t)=1$ for all $t>0$ if and only if at least two of the three points are equal,
(ii) $F_{x,y,z}=F_{x,z,y}=F_{z,y,x}$,
(iii) $F_{x,y,z}(0)=0$,
(iv) if $F_{x,y,s}(t_1)=F_{x,s,z}(t_2)=F_{s,y,z}(t_3)=1$, then $F_{x,y,z}(\max\{t_1,t_2,t_3\})=1$.

Definition 1.2. A t-norm is a function $\Delta:[0,1]\times[0,1]\times[0,1]\to[0,1]$ which is associative, commutative, nondecreasing in each coordinate and $\Delta(a,1,1)=a$ for every $a\in[0,1]$.

Definition 1.3. A 2-N.A. Menger PM-space is an order triplet (X,F,Δ) where Δ is a t-norm and (X,F) is 2-N.A. PM-space satisfying the following condition:

(v) $F_{x,y,z}(\max\{t_1,t_2,t_3\})\geq\Delta(F_{x,y,s}(t_1),F_{x,s,z}(t_2),F_{s,y,z}(t_3))$ for all $x,y,z,s\in X$ and $t_1,t_2,t_3\geq 0$.

Definition 1.4. Let (X,F,Δ) be a 2-N.A. Menger PM-space and Δ a continuous t-norm, then (X,F,Δ) is a Hausdorff in the topology induced by the family of neighbourhoods of x

$$\{U_x(\epsilon,\lambda,a_1,a_2,\ldots,a_n),\ x,\ a_i\in X,\ \epsilon>0,\ i=1,2,\ldots,n,\ n\in\mathbb{Z}^+\},\tag{1.1}$$

where $\mathbb{Z}^+$ is the set of all positive integers and

$$\begin{aligned}U_x(\epsilon,\lambda,a_1,a_2,\ldots,a_n)&=\{y\in X;\ F_{x,y,a_i}(\epsilon)>1-\lambda,\ 1\leq i\leq n\}\\&=\bigcap_{i=1}^{n}\{y\in X;\ F_{x,y,a_i}(\epsilon)>1-\lambda,\ 1\leq i\leq n\}.\end{aligned}\tag{1.2}$$

Definition 1.5. A 2-N.A. Menger PM-space (X,F,Δ) is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F_{x,y,z}(t)) \le g(F_{x,y,a}(t)) + g(F_{x,a,z}(t)) + g(F_{a,y,z}(t)) \tag{1.3}$$

for all $x,y,z,a \in X$ and $t \ge 0$, where $\Omega = \{g; g : [0,1] \to [0,\infty)\}$ is continuous, strictly decreasing, $g(1) = 0$ and $g(0) < \infty$.

Definition 1.6. A 2-N.A. Menger PM-space (X,F,Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that

$$g(\Delta(t_1,t_2,t_3)) \le g(t_1) + g(t_2) + g(t_3) \quad \forall t_1,t_2,t_3 \in [0,1]. \tag{1.4}$$

Definition 1.7. Let (X,F,Δ) be a 2-N.A. Menger PM-space where Δ is a continuous t-norm and $A,S : X \to X$ be mappings. The mappings A and S are said to be weakly compatible if they commute at the coincidence point, that is, the mappings A and S are weakly compatible if and only if $Ax = Sx$ implies $ASx = SAx$.

Remark 1.8. (1) If 2-N.A. PM-space (X,F,Δ) is of type $(D)_g$, then (X,F,Δ) is of type $(C)_g$.

(2) If (X,F,Δ) is a 2-N.A. PM-space and $\Delta \ge \Delta_m$, where $\Delta_m(r,s,t) = \max\{r+s+t-1,0,0\}$, then (X,F,Δ) is of type $(D)_g$ for $g \in \Omega$ defined by $g(t) = 1-t$.

Throughout this paper, let (X,F,Δ) be a complete 2-N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing t-norm Δ.

Let $\phi : [0,\infty) \to [0,\infty)$ be a function satisfying the condition (Φ):

(Φ) ϕ is upper semi-continuous from right and $\phi(t) < t$ for all $t > 0$.

Lemma 1.9 (see [1]). *If a function $\phi : [0,\infty) \to [0,\infty)$ satisfies the condition (Φ), then*

(1) *for all $t \ge 0$, $\lim_{n\to\infty} \phi^n(t) = 0$ where $\phi^n(t)$ is the nth iteration of $\phi(t)$;*

(2) *if $\{t_n\}$ is a nondecreasing sequence of real numbers and $t_{n+1} \le \phi(t_n)$, $n = 1,2,\dots$, then $\lim_{n\to\infty} t_n = 0$. In particular, if $t \le \phi(t)$ for all $t \ge 0$, then $t = 0$.*

Lemma 1.10 (see [1]). *Let $\{y_n\}$ be a sequence in X such that $\lim_{n\to\infty} F_{y_n,y_{n+1},a}(t) = 1$ for all $t > 0$. If the sequence $\{y_n\}$ is not Cauchy sequence in X, then there exist $\epsilon_0 > 0$, $t_0 > 0$, and two sequences $\{m_i\}$ and $\{n_i\}$ of positive integers such that*

(i) *$m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$,*

(ii) *$F_{y_{m_i},y_{n_i},a}(t_0) < 1-\epsilon_0$ and $F_{y_{m_i-1},y_{n_i},a}(t_0) > 1-\epsilon_0$, $i = 1,2,\dots$.*

Chugh and Sumitra [2] proved the following theorem.

Theorem 1.11. *Let A, B, S, $T : X \to X$ be mappings satisfying the following conditions:*

(i) *$A(X) \subset T(X)$ and $B(X) \subset S(X)$;*

(ii) *the pairs A, S and B, T are weak compatible of type (A);*

(iii) *S and T are continuous;*

(iv) *for all* $a \in X$ *and* $t > 0$,

$$g(F_{Ax,By,a}(t)) \le \phi\Big(\max\Big\{g(F_{Sx,Ty,a}(t)), g(F_{Sx,Ax,a}(t)), g(F_{Ty,By,a}(t)), \frac{1}{2}\big(g(F_{Sx,By,a}(t)) + g(F_{Ty,Ax,a}(t))\big)\Big\}\Big), \tag{1.5}$$

where a function $\phi : [0,\infty) \to [0,\infty)$ *satisfies the condition* (Φ).

Then A, B, S, and T have a unique common fixed points in X.

Now we prove the following theorem.

Theorem 1.12. *Let* $A, B, S, T : X \to X$ *be mappings satisfying*

$$A(X) \subset T(X), \qquad B(X) \subset S(X), \tag{1.6}$$

$$\text{the pairs } A,\ S \text{ and } B,\ T \text{ are weakly compatible}, \tag{1.7}$$

$$g(F_{Ax,By,a}(t)) \le \phi\Big(\max\Big\{g(F_{Sx,Ty,a}(t)), g(F_{Sx,Ax,a}(t)), g(F_{Ty,By,a}(t)), \frac{1}{2}\big(g(F_{Sx,By,a}(t)) + g(F_{Ty,Ax,a}(t))\big)\Big\}\Big) \tag{1.8}$$

for all $t > 0$, $a \in X$ *where a function* $\phi : [0,\infty) \to (0,\infty)$ *satisfies the condition* (Φ). *Then A, B, S, and T have a unique common fixed point in X.*

Proof. By (1.6) since $A(X) \subset T(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this x_1, we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on, inductively, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}, \quad \text{for } n = 0, 1, 2, \dots. \tag{1.9}$$

□

First we prove the following lemma.

Lemma 1.13. *Let* $A, B, S, T : X \to X$ *be mappings satisfying conditions (1.6) and (1.8), then the sequence* $\{y_n\}$ *defined by (1.9), such that* $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ *for all* $t > 0$, $a \in X$, *is a Cauchy sequence in X.*

Proof. Since $g \in \Omega$, it follows that $\lim_{n\to\infty}(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$ if and only if $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$. By Lemma 1.10, if $\{y_n\}$ is not a Cauchy sequence in X, there exist $\epsilon_0 > 0$, $t_0 > 0$, and two sequences $\{m_i\}$, $\{n_i\}$ of positive integers such that

(A) $m_i > n_i + 1$ and $n_i \to \infty$ as $i \to \infty$,

(B) $g(F_{y_{m_i},y_{n_i},a}(t_0)) > g(1-\epsilon_0)$ and $g(F_{y_{m_i-1},y_{n_i},a}(t_0)) \le g(1-\epsilon_0)$, $i = 1, 2, \dots$.

Thus we have

$$\begin{aligned} g(1-\epsilon_0) < g(F_{y_{m_i},y_{n_i},a}(t_0)) &\le g(F_{y_{m_i},y_{n_i},y_{m_i-1}}(t_0)) \\ &\quad + g(F_{y_{m_i},y_{m_i-1},a}(t_0)) + g(F_{y_{m_i-1},y_{n_i},a}(t_0)) \\ &\le g(F_{y_{m_i},y_{n_i},y_{m_i-1}}(t_0)) + g(F_{y_{m_i},y_{m_i-1},a}(t_0)) + g(1-\epsilon_0). \end{aligned} \tag{1.10}$$

Letting $i \to \infty$ in (1.10), we have

$$\lim_{n\to\infty} g(F_{y_{m_i},y_{n_i},a}(t_0)) = g(1-\epsilon_0). \tag{1.11}$$

On the other hand, we have

$$g(1-\epsilon_0) < g(F_{y_{m_i},y_{n_i},a}(t_0)) \le g(F_{y_{m_i},y_{n_i},y_{n_i}+1}(t_0)) + g(F_{y_{m_i},y_{n_i}+1,a}(t_0)) + g(F_{y_{n_i}+1,y_{n_i},a}(t_0)). \tag{1.12}$$

Now, consider $g(F_{y_{m_i},y_{n_i}+1,a}(t_0))$ in (1.12), without loss of generality, assume that both n_i and m_i are even.

Then by (1.8), we have

$$\begin{aligned} g(F_{y_{m_i},y_{n_i}+1,a}(t_0)) &= g(F_{Axm_i,Bxn_i+1,a}(t_0)) \\ &\le \phi\Big(\max\Big\{g(F_{Sxm_i,Txn_i+1,a}(t_0)), \\ &\qquad g(F_{Sxm_i,Axm_i,a}(t_0)), g(F_{Txn_i+1,Bxn_i+1,a}(t_0)), \\ &\qquad \frac{1}{2}(g(F_{Sxm_i,Bxn_i+1,a}(t_0)) + g(F_{Txn_i+1,Axm_i+1,a}(t_0)))\Big\}\Big) \\ &= \phi\Big(\max\Big\{g(F_{y_{m_i},-1,y_{n_i},a}(t_0)), \\ &\qquad g(F_{y_{m_i},-1,y_{m_i},a}(t_0)), g(F_{y_{n_i},y_{n_i}+1,a}(t_0)), \\ &\qquad \frac{1}{2}(g(F_{y_{m_i},-1,y_{n_i}+1,a}(t_0)) + g(F_{y_{n_i},y_{m_i},a}(t_0)))\Big\}\Big). \end{aligned} \tag{1.13}$$

By (1.11), (1.12), and (1.13), letting $i \to \infty$ in (1.13), we have

$$g(1-\epsilon_0) \le \phi(\max\{g(1-\epsilon_0), 0, 0, g(1-\epsilon_0)\}) = \phi(g(1-\epsilon_0)) < g(1-\epsilon_0) \tag{1.14}$$

which is a contradiction. Therefore, $\{y_n\}$ is a Cauchy sequence in X. □

Now, we are ready to prove our main theorem.

If we prove $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$, then by Lemma 1.13, the sequence $\{y_n\}$ defined by (1.9) is a Cauchy sequence in X. First we prove that $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$. In fact, by (1.8) and (1.9), we have

$$\begin{aligned} g(F_{y_{2n},Y_{2n+1},a}(t)) &= g(F_{Ax_{2n},Bx_{2n+1},a}(t)) \\ &\le \phi\Big(\max\Big\{g(F_{Sx_{2n},Tx_{2n+1},a}(t)), \\ &\qquad g(F_{Sx_{2n},Ax_{2n},a}(t)), g(F_{Tx_{2n+1},Bx_{2n+1},a}(t)), \\ &\qquad \frac{1}{2}(g(F_{Sx_{2n},Bx_{2n+1},a}(t)) + g(F_{Tx_{2n+1},Ax_{2n},a}(t)))\Big\}\Big) \\ &= \phi\Big(\max\Big\{g(F_{y_{2n-1},y_{2n},a}(t)), g(F_{y_{2n-1},y_{2n},a}(t)), \\ &\qquad g(F_{y_{2n},y_{2n+1},a}(t)), \frac{1}{2}(g(F_{y_{2n-1},y_{2n+1},a}(t)) + g(1))\Big\}\Big) \\ &\le \phi\Big(\max\Big\{g(F_{y_{2n-1},y_{2n},a}(t)), g(F_{y_{2n},y_{2n+1},a}(t)), \\ &\qquad \frac{1}{2}(g(F_{y_{2n-1},y_{2n},a}(t)) + g(F_{y_{2n},y_{2n+1},a}(t)))\Big\}\Big). \end{aligned} \tag{1.15}$$

If $g(F_{y_{2n-1},y_{2n},a}(t)) \le g(F_{y_{2n},y_{2n+1},a}(t))$ for all $t > 0$, then by (1.8),

$$g(F_{y_{2n},y_{2n+1},a}(t)) \le \phi(g(F_{y_{2n},y_{2n+1},a}(t))) \tag{1.16}$$

and thus, by Lemma 1.9, $g(F_{y_{2n},y_{2n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$. Similarly, we have $g(F_{y_{2n+1},\, y_{2n+2},\, a}(t)) = 0$, thus we have $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$. On the other hand, if $g(F_{y_{2n-1},y_{2n},a}(t)) \ge g(F_{y_{2n},y_{2n+1},a}(t))$, then by (1.8), we have

$$g(F_{y_{2n},y_{2n+1},a}(t)) \le \phi(g(F_{y_{2n-1},y_{2n},a}(t))) \quad \forall a \in X,\ t > 0. \tag{1.17}$$

Similarly, $g(F_{y_{2n+1},y_{2n+2},a}(t)) \le \phi(g(F_{y_{2n},y_{2n+1},a}(t)))$ for all $a \in X$ and $t > 0$. Thus we have $g(F_{y_n,y_{n+1},a}(t)) \le \phi(g(F_{y_{n-1},y_n,a}(t)))$ for all $a \in X$ and $t > 0$ and $n = 1,2,3,\ldots$, therefore by Lemma 1.9, $\lim_{n\to\infty} g(F_{y_n,y_{n+1},a}(t)) = 0$ for all $a \in X$ and $t > 0$, which implies that $\{y_n\}$ is a Cauchy sequence in X by Lemma 1.13. Since (X,F,Δ) is complete, the sequence $\{y_n\}$ converges to a point $z \in X$ and so the subsequences $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$, $\{Tx_{2n+1}\}$ of $\{y_n\}$ also converge to the limit z. Since $B(X) \subset S(X)$, there exists a point $u \in X$ such that $z = Su$.

Now

$$g(F_{Au,z,a}(t)) \le g(F_{Au,Bx_{2n+1},Z}(t)) + g(F_{Bx_{2n+1},z,a}(t)) + g(F_{Au,Bx_{2n+1},a}(t)). \tag{1.18}$$

From (1.8), we have

$$\begin{aligned} g(F_{Au,Bx_{2n+1},a}(t)) \le \phi\Big(\max\Big\{&g(F_{Su,Tx_{2n+1},a}(t)), g(F_{Su,Au,a}(t)), g(F_{Tx_{2n+1},Bx_{2n+1},a}(t)),\\ &\frac{1}{2}(g(F_{Su,Bx_{2n+1},a}(t)) + g(F_{Tx_{2n+1},Au,a}(t)))\Big\}\Big). \end{aligned} \tag{1.19}$$

From (1.18) and (1.19), letting $n \to \infty$, we have

$$\begin{aligned} g(F_{Au,z,a}(t)) &\le \phi\Big(\max\Big\{g(F_{Su,z,a}(t)), g(F_{Su,Au,a}(t)), g(F_{z,z,a}(t)),\\ &\qquad \frac{1}{2}(g(F_{Su,z,a}(t)) + g(F_{z,Au,a}(t)))\Big\}\Big)\\ &= \phi(g(F_{z,Au,a}(t))) \quad \forall a \in X,\ t > 0, \end{aligned} \tag{1.20}$$

which means $z = Au = Su$. Since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $z = Tv$. Then, again using (1.8), we have

$$\begin{aligned} g(F_{z,Bv,a}(t)) &= g(F_{Au,Bv,a}(t))\\ &\le \phi\Big(\max\Big\{g(F_{Su,Tv,a}(t)), g(F_{Su,Au,a}(t)), g(F_{Tv,Bv,a}(t)),\\ &\qquad \frac{1}{2}(g(F_{Su,Bv,a}(t)) + g(F_{Tv,Au,a}(t)))\Big\}\Big)\\ &= \phi(g(Fz,Bv,a(t))), \quad \forall a \in X,\ t > 0, \end{aligned} \tag{1.21}$$

which implies that $Bv = z = Tv$.

Since pairs of maps A and S are weakly compatible, then $ASu = SAu$, that is, $Az = Sz$. Now we show that z is a fixed point of A. If $Az \neq z$, then by (1.8),

$$\begin{aligned} g(F_{Az,z,a}(t)) &= g(F_{Az,Bv,a}(t)) \\ &\leq \phi\Big(\max\Big\{g(F_{Sz,Tv,a}(t)), g(F_{Sz,Az,a}(t)), g(F_{Tv,Bv,a}(t)), \\ &\qquad \frac{1}{2}(g(F_{Sz,Bv,a}(t)) + g(F_{Tv,Az,a}(t)))\Big\}\Big) \\ &= \phi(\max\{g(F_{Az,z,a}(t))\}), \quad \text{implies } Az = z. \end{aligned} \tag{1.22}$$

Similarly, pairs of maps B and T are weakly compatible, we have $Bz = Tz$. Therefore,

$$\begin{aligned} g(F_{Az,z,a}(t)) &= g(F_{Az,Bz,a}(t)) \\ &\leq \phi\Big(\max\Big\{g(F_{Sz,Tz,a}(t)), g(F_{Sz,Az,a}(t)), g(F_{Tz,Bz,a}(t)), \\ &\qquad \frac{1}{2}(g(F_{Sz,Bz,a}(t)) + g(F_{Tz,Az,a}(t)))\Big\}\Big) \\ &= \phi(\max\{g(F_{z,Tz,a}(t))\}). \end{aligned} \tag{1.23}$$

Thus we have $Bz = Tz = z$.

Therefore, $Az = Bz = Sz = Tz$ and z is a common fixed point of A, B, S, and T. The uniqueness follows from (1.8).

2. Application

Theorem 2.1. *Let (X, F, Δ) be a complete 2-N.A. Menger PM-space and A, B, S, and T be the mappings from the product $X \times X$ to X such that*

$$\begin{gathered} A(X \times \{y\}) \subseteq T(X \times \{y\}), \qquad B(X \times \{y\}) \subseteq (X \times \{y\}), \\ g(F_{A(T(x,y),y),T(A(x,y),y),a}(t)) \leq g(F_{A(x,y),T(x,y),a}(t)), \\ g(F_{B(S(x,y),y),S(B(x,y),y),a}(t)) \leq g(F_{B(x,y),S(x,y),a}(t)) \end{gathered} \tag{2.1}$$

for all $a \in X$ and $t > 0$ and

$$\begin{aligned} & g(F_{A(x,y),B(x',y'),a}(t)) \\ & \leq \phi\Big(\max\Big\{g(F_{S(x,y),T(x',y'),a}(t)), g(F_{S(x,y),A(x,y),a}(t)), g(F_{T(x',y'),B(x',y'),a}(t)), \\ & \qquad \frac{1}{2}(g(F_{S(x,y),B(x',y'),a}(t)) + g(F_{T(x',y'),A(x,y),a}(t)))\Big\}\Big) \end{aligned} \tag{2.2}$$

for all $a \in X$, $t > 0$, and x, y, x', y' in X, then there exists only one point b in X such that

$$A(b,y) = S(b,y) = B(b,y) = T(b,y) \quad \forall y \text{ in } X. \tag{2.3}$$

Proof. By (2.2),

$$\begin{aligned} & g(F_{A(x,y),B(x',y')}(t)) \\ & \leq \phi\Big(\max\Big\{g(F_{S(x,y),T(x',y'),a}(t)), g(F_{S(x,y),A(x,y),a}(t)), g(F_{T(x',y'),B(x',y'),a}(t)), \\ & \qquad \frac{1}{2}(g(F_{S(x,y),B(x',y'),a(t)}) + g(F_{T(x',y'),A(x,y),a}(t)))\Big\}\Big) \end{aligned} \tag{2.4}$$

for all $a \in X$ and $t > 0$; therefore by Theorem 1.12, for each y in X, there exists only one $x(y)$ in X such that

$$A(x(y), y) = S(x(y), y) = B(x(y), y) = T(x(y), y) = x(y) \tag{2.5}$$

for every y, y' in X,

$$\begin{aligned} &g(F_{x(y),x(y'),a}(t)) \\ &\quad = g(F_{A(x(y),y),A(x(y'),y'),a}(t)) \\ &\quad \le \phi\Big(\max\Big\{g(F_{A(x,y),A(x',y'),a}(t)), g(F_{A(x,y),A(x,y),a}(t)), g(F_{T(x',y'),A(x',y'),a}(t)), \\ &\qquad\qquad \frac{1}{2}(g(F_{A(x,y),A(x',y'),a}(t)) + g(F_{A(x',y'),A(x,y),a}(t)))\Big\}\Big) \\ &\quad = g(F_{x(y),x(y'),a}(t)). \end{aligned} \tag{2.6}$$

This implies $x(y) = x(y')$ and hence $x(y)$ is some constant $b \in X$ so that

$$A(b, y) = b = T(b, y) = S(b, y) = B(b, y) \quad \forall y \text{ in } X. \tag{2.7}$$

□

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