

RADIATION EFFECTS ON UNSTEADY MHD FREE CONVECTION WITH HALL CURRENT NEAR AN INFINITE VERTICAL POROUS PLATE

MOHAMED A. SEDDEEK and EMAD M. ABOELDAHAB

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ABSTRACT. Radiation effect on unsteady free convection flow of an electrically conducting, gray gas near equilibrium in the optically thin limit along an infinite vertical porous plate are investigated in the presence of strong transverse magnetic field imposed perpendicularly to the plate, taking Hall currents into account. A similarity parameter length scale (h), as a function of time and the suction velocity are considered to be inversely proportional to this parameter. Similarity equations are then derived and solved numerically using the shooting method. The numerical values of skin friction and the rate of heat transfer are represented in a table. The effects of radiation parameter, Hall parameter, and magnetic field parameters are discussed and shown graphically.

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1. Introduction. Free convection flows past different types of vertical bodies are studied because of their wide applications and hence it has attracted the attention of numerous investigators and scientists. Literature on unsteady MHD convection heat transfer with or without Hall currents are very extensive due to its technical importance in the scientific community. Some of the literature surveys and reviews of pertinent work in this field are documented by Schlichting [8], Soundagekhar et al. [10], Raptis et al. [7], Raptis and Perdakis [6], K. Vajravelu [11], Sacheti et al. [4], and M. A. Al-Nimr and S. Masoud [1]. In all these studies the Hall current effects are not considered.

The unsteady hydromagnetic free convective flow with Hall current is studied by Singh and Raptis [9]. P. C. Ram [5] studied the effects of Hall and ion-slip currents on free convective heat generating flow in a rotating fluid. The Laplace transform technique has been applied to obtain an exact solution in a closed form, when the plate is moving with a velocity which is an arbitrary function of time. In all these studies, the effect of radiation are not considered. In space technology applications and at higher operating temperatures, radiation effects can be quite significant. Since radiation is quite complicated, many aspects of its effect on free convection or combined convection have not been studied in recent years. However, Cogley et al. [2] showed that, in the optically thin limit for a gray gas near equilibrium the following relation holds:

$$\frac{\partial q_r}{\partial y} = 4(T - T_\omega)I, \quad (1.1)$$

where

$$I = \int_0^\infty K_{\lambda\omega} \left(\frac{\partial e_{b\lambda}}{\partial T} \right) d\lambda, \quad (1.2)$$

T is the temperature, q_r is the radiative heat flux, k_λ is the absorption coefficient, $e_{b\lambda}$ is Planck's function, and ω is the properties of the wall.

Greif et al. [3] showed that, for an optically thin limit, the fluid does not absorb its own emitted radiation, this means that there is no self-absorption, but the fluid does absorb radiation emitted by the boundaries.

In space technology and in nuclear engineering applications, such a problem is quite common. But in these fields, the presence of strong magnetic field and Hall current taking effects play an important role and these effects have not been studied in the case of free convective flow of a radiation gas under the condition mentioned above. In this paper, we investigate the solution which the buoyancy, radiation, and Hall currents act simultaneously.

2. Mathematical formulation. Consider unsteady free convection flow of a viscous incompressible and electrically conducting fluid, along an infinite vertical porous plate subjected to time-dependent suction velocity. The flow is assumed to be in the x -direction which is taken along the plate in the upward direction and the y -axis perpendicular to it. A uniform strong magnetic field B_0 is assumed to be applied in the y -direction and the induced magnetic field of the flow is negligible in comparison with the applied one which corresponds to very small magnetic Reynolds number [10]. On neglecting the viscous dissipation effects, the flow under consideration is governed by the following equations:

$$\frac{\partial v}{\partial y} = 0, \quad (2.1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g\beta(T - T_0) + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 B_0^2}{\rho(1+m^2)}(u + mw), \quad (2.2)$$

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \frac{\sigma\mu_e^2 B_0^2}{\rho(1+m^2)}(mu - w), \quad (2.3)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{4(T - T_w)I}{\rho c_p}, \quad (2.4)$$

where (u, w) are the x and z components of velocity, v is the suction velocity, T is the temperature of the fluid, ν is the kinematics viscosity, σ is the electric conductivity, ρ is the density of the fluid, k is the thermal conductivity, c_p is the specific heat at constant pressure, β is the volumetric coefficient of thermal expansion, g is the acceleration due to gravity, and m is the Hall parameter.

The boundary conditions are given by

$$\begin{aligned} u = 0, \quad w = 0, \quad T = T_w, \quad \text{at } y = 0, \\ u \rightarrow 0, \quad w \rightarrow 0, \quad T \rightarrow T_0, \quad \text{at } y \rightarrow \infty, \end{aligned} \quad (2.5)$$

we now define the similarity variables as follows:

$$u = u_0 F(\eta), \quad w = u_0 G(\eta), \quad \theta(\eta) = \frac{T - T_0}{T_w - T_0}, \quad \eta = \frac{y}{h}, \quad (2.6)$$

where h ($= h(t)$) is a similarity parameter length scale and u_0 is the free stream velocity. In terms of $h(t)$, a convenient solution of (2.1) can be given by

$$v = -v_0 \left(\frac{u}{h} \right), \tag{2.7}$$

where v_0 is a nondimensional transpiration parameter, clearly $v_0 > 0$ and $V_0 < 0$ indicates suction or injection, respectively.

Accordingly (2.2), (2.3), and (2.4) take the form

$$\begin{aligned} -\frac{h}{\nu} \frac{\partial h}{\partial t} \eta F' - v_0 F' &= F'' - G_r \theta - \frac{M}{1+m^2} (F+mG), \\ -\frac{h}{\nu} \frac{\partial h}{\partial t} \eta G' - v_0 G' - v_0 G' &= G'' + \frac{M}{1+m^2} (mF-G), \\ -\frac{h}{\nu} \frac{\partial h}{\partial t} \eta \theta' - v_0 \theta' &= \frac{1}{p_r} \theta'' - R(\theta-1), \end{aligned} \tag{2.8}$$

where the Grashof number $G_r = g\beta h^2(T_w - T_\infty)/\nu^2$, the magnetic parameter $M = \sigma \mu_e^2 B_0^2 h^2 / \nu \rho$, Prandtl number $p_r = \rho \nu c_p / K$, and radiation parameter $R = 4Ih^2 / \rho c_p \nu$.

The boundary conditions corresponding to (2.8) are

$$\begin{aligned} F = 0, \quad G = 0, \quad \theta = 1, \quad \text{at } \eta = 0, \\ F \rightarrow 0, \quad G \rightarrow 0, \quad \theta \rightarrow 0, \quad \text{at } \eta \rightarrow \infty. \end{aligned} \tag{2.9}$$

Equations (2.8) are similar except for the term $(h/\nu)(\partial h/\partial t)$, where t appears explicitly. Thus, the similarity condition requires that $(h/\nu)(\partial h/\partial t)$ must be constant. Hence it is assumed that

$$\left(\frac{h}{\nu} \right) \frac{\partial h}{\partial t} = C, \tag{2.10}$$

where C is an arbitrary constant.

At $C = 2$ and by integrating equation (2.10), one obtains $h = 2\sqrt{\nu t}$, which defines the well-established scaling parameter for unsteady boundary layer problems [8]. Hence, the similarity equations are obtained as

$$\begin{aligned} F'' + 2(\eta + a_0)F' &= -G_r \theta - \frac{M}{1+m^2} (F+mG), \\ G'' + 2(\eta + a_0)g' &= -\frac{M}{1+m^2} (mF-G), \\ \theta'' + 2p_r(\eta + a_0)\theta' - Rp_r(\theta-1) &= 0, \end{aligned} \tag{2.11}$$

where $a_0 = v_0/2$.

From the velocity field, we can study the skin friction. It is given by

$$\tau_x = -\mu \frac{\partial u}{\partial y} \Big|_{y=0}, \quad \tau_z = -\mu \frac{\partial w}{\partial y} \Big|_{y=0}, \tag{2.12}$$

and in view of (2.6), we have

$$\tau_x = -\frac{\mu}{h} F'(0), \quad \tau_z = -\frac{\mu}{h} G'(0). \tag{2.13}$$

The values of $F'(0)$ and $G'(0)$ are presented in Table 2.1 for $P_r = 0.7$ and different values of m and M .

TABLE 2.1

m	R	M	$F'(0)$	$G'(0)$
0.5	0.1	5	6.82251	-1.960666
0.7	0.1	5	7.95135	-2.92623
0.5	0.1	7	3.17975	-3.65579
0.5	0.4	5	7.3583	-1.945

3. Results and conclusion. The results of the numerical computations are displayed in Figures 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, and 3.8 for the primary velocity F , secondary velocity G , and temperature θ , respectively for $Pr = 0.73$, $Gr = 5$, $\nu_0 = 0.5$, and for different values of M, m , and R . It is seen, as expected from Figures 3.1, 3.2, and 3.3, that the primary velocity F decreases with increasing the magnetic parameter M , while the secondary velocity profiles G and temperature profiles θ increase when M increases.

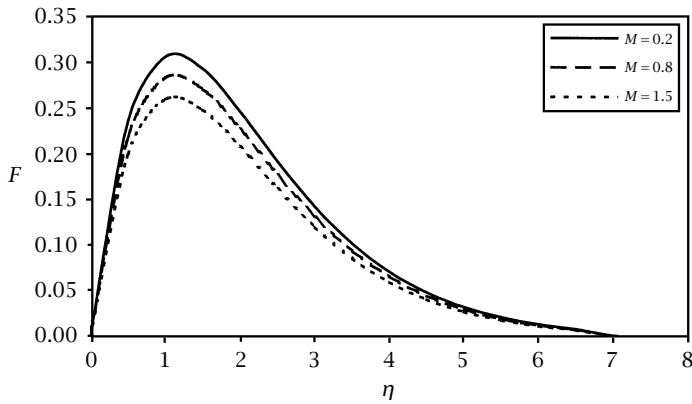


FIGURE 3.1. The variation of F against η for $m = 0.1$ and $R = 1$.

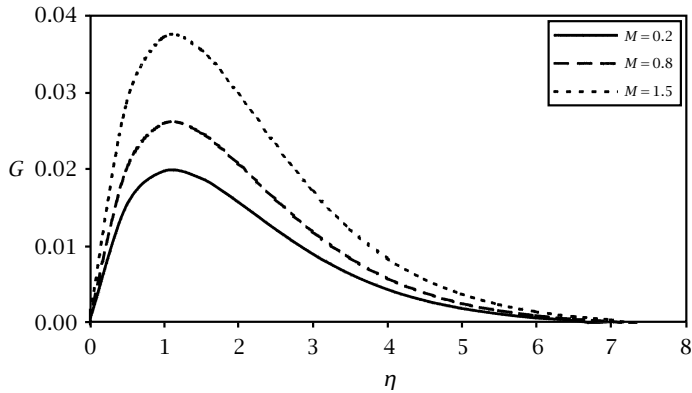


FIGURE 3.2. The variation of G against η for $m = 0.1$ and $R = 1$.

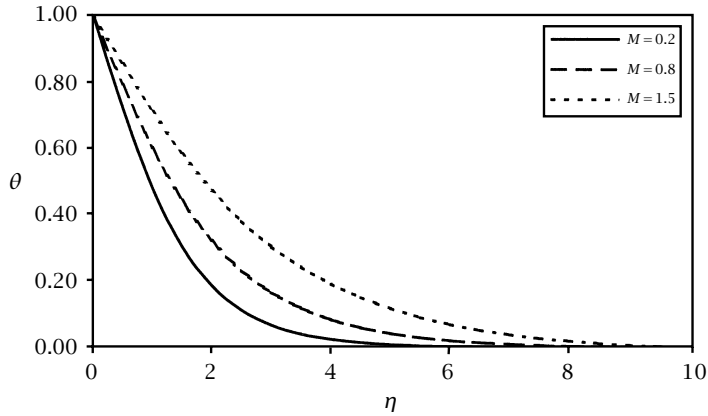


FIGURE 3.3. The variation of θ against η for $m = 0.1$ and $R = 1$.

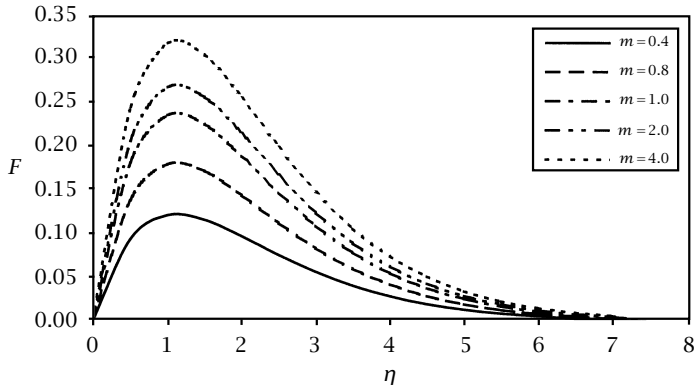


FIGURE 3.4. The variation of F against η for $M = 0.2$ and $R = 1$.

Figures 3.4, 3.5, and 3.6 show the F, G , and θ profiles at different values of m , we see that F increases with increasing the parameter m , but the temperature profiles θ decrease with increasing m . Figure 3.5 shows that the secondary velocity G begins to develop as the Hall parameter m increases in the interval $0 \leq m \leq 1$ and decreases for $m > 1$.

The effects of the thermal radiation parameter R on the primary velocity and temperature profiles in the boundary layer are illustrated in Figures 3.7 and 3.8, respectively. Increasing the thermal radiation parameter R produces significant increase in the thermal condition of the fluid and its thermal boundary layer. This increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase.

From Table 2.1 we observe that an increase of M leads to a decrease in the value of τ_x . But τ_x increases with the increase of m and R . As regards τ_z , we observe that the values τ_z are all negative, and hence the separation in the z -direction may occur, the increase in both M and m leads to the decrease in τ_z , but τ_z increases with the increase of F . We can conclude a set of results corresponding to various special cases:

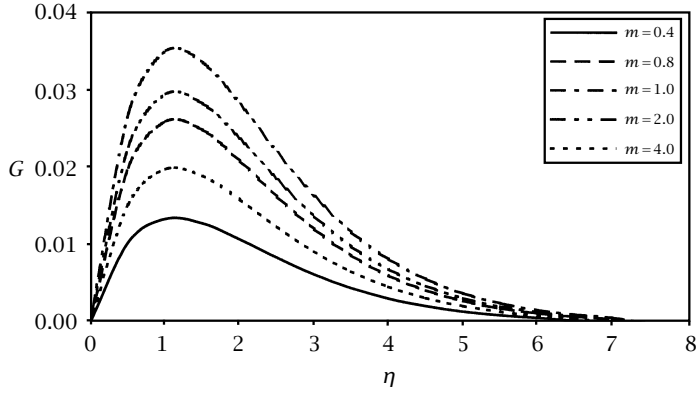


FIGURE 3.5. The variation of G against η for $M = 0.2$ and $R = 1$.

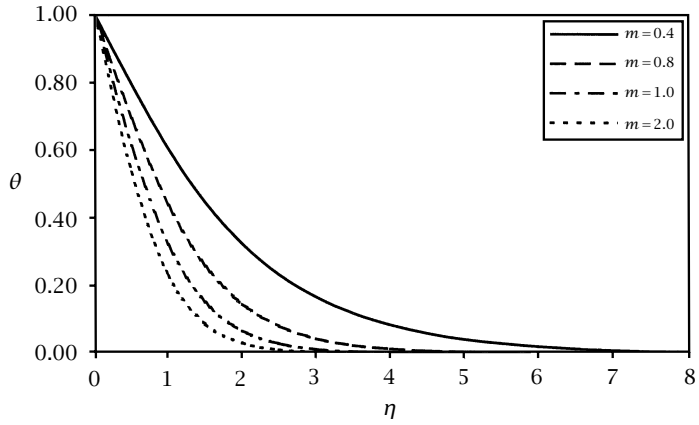


FIGURE 3.6. The variation of θ against η for $M = 0.2$ and $R = 1$.

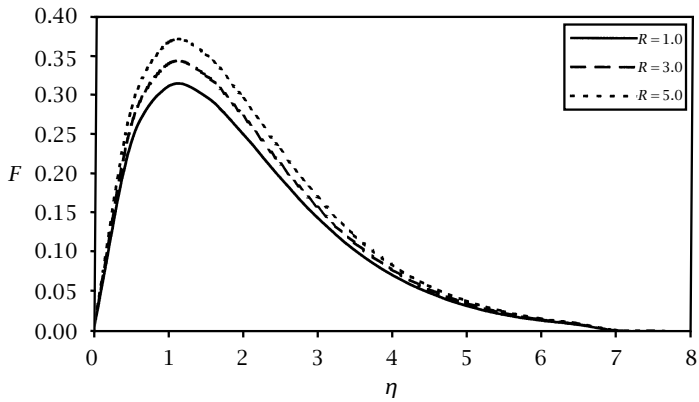


FIGURE 3.7. The variation of F against η for $m = 0.1$ and $M = 0.2$.

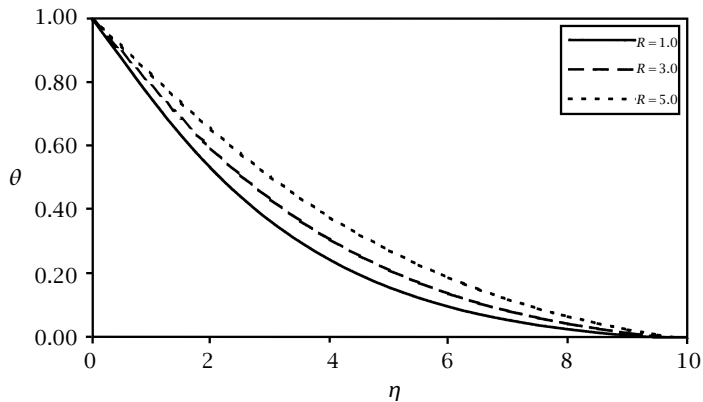


FIGURE 3.8. The variation of θ against η for $m = 0.1$ and $M = 0.2$.

(1) Substitute $M = 0$, $m = 0$, and $R = 0$ in (2.8) yields identical results to those well known in hydrodynamics [8].

(2) Substitute $m = 0$ and $R = 0$ in (2.8) yields identical results to those well known in [4].

REFERENCES

- [1] M. A. Al-Nimr and S. Masoud, *Unsteady free convection flow over a vertical flat plate immersed in a porous medium*, Fluid Dynamics Research **23** (1998), 153-160.
- [2] A. C. Cogley, W. G. Vincenti, and S. E. Gilles, *Differential approximation for radiative transfer in a non-gray gas near equilibrium*, AIAA J. **6** (1968), 551-556.
- [3] R. Grief, I. S. Habib, and J. C. Lin, *Laminar convection of a radiating gas in a vertical channel*, J. Fluid Mech. **46** (1970), 513-520.
- [4] C. S. Nirmal, H. Rallath, and A. K. Singh, *An exact solution for unsteady MHD free convection flow with constant heat flux*, Int. Comm. Heat Mass Transfer **21** (1994), 131-315.
- [5] P. C. Ram, *Effects of Hall and ion-slip currents on free convective heat generating flow in a rotating fluid*, Int. J. of Energy Research **19** (1995), 371-376.
- [6] A. Raptis and C. Perdikis, *Flow of a viscous fluid through a porous medium bounded by a vertical surface*, Int. J. Eng. Sci. **21** (1983), 1327-1330. [Zbl 527.76090](#).
- [7] A. A. Raptis, N. G. Kafousias, C. V. Massalas, and G. J. Tzivanidis, *The effects of variable suction/injection on the free convection flow past an accelerated vertical insulated plate*, Indian J. Pure Appl. Math. **12** (1981), 253-260. [Zbl 491.76079](#).
- [8] H. Schlichting, *Boundary Layer Theory*, McGraw-Hill, New York, 1968.
- [9] A. K. Singh and A. Raptis, *Modelling Simulation and Control*, 1st ed., vol. 8, B. AMSF Press, 1987.
- [10] V. M. Soundalgekar, M. R. Ratia, and I. Pop, *Magneto Hydrodynamic*, 2nd ed., vol. 65, Riga (USSR), 1982.
- [11] K. Vajravelu, *Hydromagnetic flow and heat transfer over a continuous, moving, porous, flat surface*, Acta Mech. **64** (1986), 179-185. [Zbl 614.76114](#).

MOHAMED A. SEDDEEK: DEPARTMENT OF MATHEMATICS, FACULTY OF SCIENCE, HELWAN UNIVERSITY, CAIRO, EGYPT

E-mail address: seddeek_m@hotmail.com

EMAD M. ABOELDAHAB: DEPARTMENT OF MATHEMATICS, FACULTY OF SCIENCE, HELWAN UNIVERSITY, CAIRO, EGYPT