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ON CERTAIN CLASSES OF GALOIS EXTENSIONS OF RINGS

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ABSTRACT. Relations between the following classes of Galois extensions are given: (1) centrally projective Galois extensions (CP-Galois extensions), (2) faithfully Galois extensions, and (3) H-separable Galois extensions. Moreover, it is shown that the intersection of the class of CP-Galois extensions and the class of faithfully Galois extensions is the class of Azumaya Galois extensions.

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1. Introduction. The following classes of Galois extensions of noncommutative rings have been investigated:

(1) central Galois extensions and Galois algebras,
(2) the center of a ring which is Galois with the Galois group induced by and isomorphic to the Galois group of the ring (see [4, 6, 9]),
(3) H-separable Galois extensions (see [9]).

Recently, a class broader than classes (1) and (2) has been studied in [1, 2], that is, the class of the Azumaya Galois extensions, where S is called an Azumaya G-Galois extension if it is a G-Galois extension of S^G which is an Azumaya C^G-algebra, where C is the center of S and G is a finite automorphism group of S. Moreover, properties of a Galois skew group ring of G over S and $S*G$ were studied in [10]. Now, we continue the above investigation into the different types of Galois extensions of rings and also further study the properties of $S*G$. We define another two classes of Galois extensions containing all the Azumaya Galois extensions:

(i) centrally projective Galois extension S, that is, S is G-Galois extension and centrally projective over S^G (S is a direct summand of a finite direct sum of S^G as a S^G-bimodule),
(ii) faithfully Galois extension S, that is, S is faithful as a left $S*G$-module and $S*G$ is an Azumaya Z-algebra, where $S*G$ is the skew group ring of G over S with center Z.

The purpose of the present paper is to show some relations between the above two classes (i) and (ii) and the H-separable Galois extensions as studied in [9]. Moreover, we show that the intersection of the class of CP-Galois extensions and the class of faithfully Galois extensions is the class of Azumaya Galois extensions.

2. Preliminaries. Throughout, we keep all notations and facts as given in [10]. Let S be a ring with 1, G a finite automorphism group of S of order n for some integer n invertible in S, S^G the subring of the elements fixed under each element in G, $S*G$

a skew group ring of G over S, Z the center of $S*G$, and G' the inner automorphism group of $S*G$ induced by the elements in G.

Following [3, 4, 9, 11], we call S a G-Galois extension of S^G if there exist elements $\{c_i, d_i$ in S, $i=1,2,\dots,k$ for some integer $k\}$ such that $\sum c_i g(d_i)=\delta_{1,g}$ for each $g\in G$. The set $\{c_i, d_i\}$ is called a G-Galois system for S. Let B be a subring of a ring A with 1. We denote by $V_A(B)$ the commutator subring of B in A, and by A° the opposite ring of A. We call A a separable extension of B if there exist $\{a_i, b_i$ in A, $i=1,2,\dots,m$ for some integer $m\}$ such that $\sum a_i b_i=1$, and $\sum s a_i\otimes b_i=\sum a_i\otimes b_i s$ for all s in A, where $\otimes$ is over B, and $\{a_i, b_i\}$ is called a separable system for A over B. An Azumaya algebra is a separable extension of its center. A ring A is called an H-separable extension of B if $A\otimes_B A$ is isomorphic to a direct summand of a finite direct sum of A as an A-bimodule (i.e., $A\otimes_B A$ is a centrally projective module over A). A ring S is called an Azumaya G-Galois extension of S^G if it is a G-Galois extension of S^G which is a C^G-Azumaya algebra, where C is the center of S. S is called a centrally projective Galois extension (CP-Galois extension) if S is G-Galois and centrally projective over S^G (i.e., S is a direct summand of a finite direct sum of S^G as an S^G-bimodule). We call S a faithfully Galois extension if S is faithful as a left $S*G$-module and $S*G$ is an Azumaya Z-algebra, and S is an H-separable G-Galois extension of S^G if it is a G-Galois and an H-separable extension of S^G. We employ several important facts about a Galois extension, an H-separable extension, and an Azumaya algebra. For convenience, we list them in the following.

(1) If S is an H-separable and G-Galois extension of S^G, then
   (i) $V_S(V_S(S^G))=S^G$,
   (ii) $V_S(S^G)$ is C-finitely generated projective of rank n, where $n=|G|$,
   (iii) $n^{-1}\in S^G$ if and only if $V_S(S^G)$ is a separable C-algebra [9, Proposition 4].

(2) If S is an H-separable extension of S^G, then $\mathrm{Hom}_{S^G}(S,S)\cong S\otimes_C (V_S(S^G))^\circ$ (see [9, second paragraph of Section 3, page 124]).

(3) If S is centrally projective over S^G, then $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S [8, Proposition 11].

(4) In case S is a progenerator over S^G, $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S if and only if S is centrally projective over S^G [8, Proposition 11].

(5) Let S be an H-separable extension of S^G. Then,
   (i) $\mathrm{Hom}_{S^G}(S,S)$ is a separable extension of S if and only if $V_S(S^G)$ is separable over C,
   (ii) $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S if and only if $V_S(S^G)$ is an Azumaya C-algebra [8, Proposition 12].

(6) Let S be a G-Galois extension of S^G. If all the elements of G are inner-automorphisms of S, then S is an H-separable extension of S^G [9, Corollary 3].

(7) Let A be H-separable over B and M a left A-module. Then M is a generator over A if M is a generator over B [5, lemma, page 17].

(8) S is a G-Galois extension of S^G if and only if $S*G\cong \mathrm{Hom}_{S^G}(S,S)$ and S is a finitely generated and projective right S^G-module [4, Theorem 1].

(9) Let S be a finitely generated and projective right S^G-module. Then $\mathrm{Hom}_{S^G}(S,S)$ is centrally projective over S if and only if S is an H-separable extension of S^G [8, Corollary 3, page 202].

(10) If S is centrally projective over S^G, then $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S^G [8, Theorem 6].

(11) Let A be a projective H-separable extension of B. Then A is an Azumaya algebra if B is an Azumaya algebra [7, Theorem 1].

(12) Let A be an Azumaya algebra over R. Suppose that B is a separable subalgebra of A. Set $C = V_A(B)$. Then C is a separable subalgebra of A and $V_A(C) = B$. If B is also central, so is C and the R-algebra map $B \otimes C \to A$, given by $b \otimes c \to bc$, is an isomorphism [3, Theorem 4.3, page 57].

3. H-separable Galois extensions.

In [9], the class of H-separable G-Galois extension was investigated. In this section, we characterize an H-separable and a CP-Galois extension, and an H-separable and a faithfully Galois extension. Consequently, the expressions of $S * G$ are derived.

Theorem 3.1. *Let S be an H-separable Galois extension of S^G. Then, the center of $V_S(S^G)$ is C if and only if S is a CP-Galois extension of S^G.*

Proof. Since S is an H-separable and Galois extension of S^G, $S * G \cong \mathrm{Hom}_{S^G}(S,S) \cong S \otimes_C (V_S(S^G))^\circ$ such that $V_S(S^G)$ is a separable C-algebra (see [9, second paragraph of Section 3 and Proposition 4]). Also, by hypothesis, the center of $V_S(S^G)$ is C, so it is an Azumaya C-algebra. Hence, $S * G\ (\cong \mathrm{Hom}_{S^G}(S,S))$ is an H-separable extension of S [8, Proposition 12]. Noting that n is a unit in S and that $S * G \cong \mathrm{Hom}_{S^G}(S,S)$, we get that S is a progenerator over S^G; and so S is centrally projective over S^G [8, Proposition 11]. Thus, S is a CP-Galois extension of S^G.

Conversely, since S is centrally projective over S^G, $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S [8, Proposition 11]. Therefore, $V_S(S^G)$ is an Azumaya C-algebra [8, Proposition 12]. □

Theorem 3.2. *If S is a G-Galois extension, then*

(1) *$S * G$ is an H-separable G'-Galois extension,*

(2) *$S * G$ is not a CP-Galois extension of $(S * G)^{G'}$ with Galois group G'.*

Proof. (1) Since S is G-Galois extension, $S * G$ is a G'-Galois extension with the same Galois system as S (for G' restricted to S is G). But G' is inner, so $S * G$ is an H-separable extension of $(S * G)^{G'}$ [9, Corollary 3].

(2) Assume that $S * G$ is a CP-Galois extension over $(S * G)^{G'}$. Then the center of $V_{S*G}((S * G)^{G'})$ is Z by Theorem 3.1. Clearly, $\sum g_i$ is in the center of $V_{S*G}((S * G)^{G'})$, but not in Z. This is a contradiction. Thus, $S * G$ is not a CP-Galois extension of $(S * G)^{G'}$. □

We note that Theorem 3.2 provides an example of an H-separable Galois extension, but not of a CP-Galois extension.

Next, we characterize an H-separable and CP-Galois extension S, and then derive an expression of the skew group ring $S * G$ of G over S.

Theorem 3.3. *S is an H-separable and CP-Galois extension of S^G with Galois group G if and only if $S * G \cong S \otimes_C (V_S(S^G))^\circ$ such that $V_S(S^G)$ is an Azumaya C-algebra, where $(V_S(S^G))^\circ$ is the opposite ring of $V_S(S^G)$.*

Proof. (⇒) Since S is an H-separable and G-Galois extension of S^G, $S*G \cong \mathrm{Hom}_{S^G}(S,S) \cong S\otimes_C (V_S(S^G))^{\circ}$, where S is a finitely generated and projective module over S^G. Moreover, n is a unit in S, so S is a progenerator over S^G. Therefore, $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S because S is centrally projective over S^G by hypothesis [8, Proposition 11]. But then $V_S(S^G)$ is an Azumaya C-algebra [8, Proposition 12].

(⇐) Since $S*G \cong S\otimes_C (V_S(S^G))^{\circ}$ such that $V_S(S^G)$ is an Azumaya C-algebra, $V_S(S^G)$ is H-separable over C; and so $S*G$ is H-separable over S. Now, S is a generator over S, so S is a generator over $S*G$ [5, lemma, page 17], where S is considered as a left $S*G$-module by $(tg)(s) = t(g(s))$ for all $t,s \in S$ and $g \in G$. On the other hand, n is a unit in S, so $S*G$ is a separable extension of S with a separable system $\{(1/n)g_i,\ g_i^{-1} \mid i = 1,2,\dots,n\}$. Then S can be shown to be a finitely generated and projective left $S*G$-module by using the above separable system since S is a free module over itself (see [3, proof of Proposition 2.3]). But then S is a progenerator over $S*G$. Noting that $S^G \cong \mathrm{Hom}_{S*G}(S,S)$, we see that $S*G \cong \mathrm{Hom}_{S^G}(S,S)$ and S is a finitely generated and projective right S^G-module by Morita theorem. This implies that S is a G-Galois extension of S^G [4, Theorem 1]. Again, since $S*G$ is an H-separable extension of S, $\mathrm{Hom}_{S^G}(S,S)$ is an H-separable extension of S. Hence, S is a centrally projective S^G-module [8, Proposition 11]. Moreover, since $V_S(S^G)$ is an Azumaya C-algebra by hypothesis, it is centrally projective over C. This implies that $S\otimes_C (V_S(S^G))^{\circ}$ is centrally projective over S; and so S is an H-separable extension of S^G (see [8, Corollary 3, page 202]). Consequently, S is an H-separable and CP-Galois extension over S^G. □

Theorem 3.4. *S is an H-separable and faithfully G-Galois extension of S^G if and only if $S*G \cong S\otimes_C (V_S(S^G))^{\circ}$ such that $V_S(S^G)$ is a projective separable C-algebra and S is an Azumaya C-algebra.*

Proof. (⇒) Since S is an H-separable G-Galois extension of S^G, $S*G \cong S\otimes_C (V_S(S^G))^{\circ}$, where $V_S(S^G)$ is a projective separable C-algebra [9, Proposition 4]. Also, S is a faithfully G-Galois extension of S^G, so $S*G$ is an Azumaya Z-algebra. Hence, $S\otimes_C (V_S(S^G))^{\circ}$ is an Azumaya C'-algebra, where C' is the center of $V_S(S^G)$. But $S\otimes_C (V_S(S^G))^{\circ} \cong (S\otimes_C C')\otimes_{C'} (V_S(S^G))^{\circ}$, so both $S\otimes_C C'$ and $V_S(S^G)$ are Azumaya C'-algebras by the commutator theorem for Azumaya algebras (see [3, Theorem 4.3, page 57]). Since $V_S(S^G)$ is a projective separable C-algebra, C is a direct summand of $V_S(S^G)$. Hence, C is a direct summand of C'. This implies that S is an Azumaya C-algebra [3, Corollary 1.10].

(⇐) Since $S*G \cong S\otimes_C (V_S(S^G))^{\circ} \cong (S\otimes_C C')\otimes_{C'} (V_S(S^G))^{\circ}$ as Azumaya C'-algebras, $S*G$ is an Azumaya Z-algebra and S is a faithful left $S*G$-module. Thus, S is a faithfully G-Galois extension. Moreover, $V_S(S^G)$ is a projective separable C-algebra, so it is finitely generated [3, Proposition 2.1]. Hence, $V_S(S^G)$ is a centrally projective C-module, and so $S\otimes (V_S(S^G))^{\circ}$ is centrally projective over S. Thus, $S*G$ is centrally projective over S. Noting that $S*G \cong \mathrm{Hom}_{S^G}(S,S)$, we conclude that S is an H-separable extension of S^G [8, Corollary 3]. □

Corollary 3.5. *If S is an H-separable and faithfully G-Galois extension of S^G, then S^G is a projective separable C^G-algebra.*

Proof. Since S is an H-separable and G-Galois extension of S^G, $V_S(V_S(S^G)) = S^G$ [9, Proposition 4]. Therefore, $C(= V_S(S))$ is contained in $V_S(V_S(S^G)) = S^G$. Hence, $C = C^G$. Thus, by Theorem 3.4, S is an Azumaya C^G-algebra. Noting that S is finitely generated and projective over S^G and S^G is an S^G-direct summand of S (for n is a unit in S), we see that S^G is a projective separable C^G-algebra [3, Proposition 1.13]. □

4. CP and faithfully Galois extensions. It can be shown that Azumaya Galois extensions are included in the class of CP and faithfully Galois extensions, respectively, and that the later two classes are noncomparable. In this section, we give sufficient conditions under which a CP-Galois extension is an Azumaya Galois extension, and a faithfully Galois extension is an Azumaya Galois extension. Consequently, the class of Azumaya Galois extensions is the intersection of the class of CP-Galois extensions and the class of faithfully Galois extensions.

Theorem 4.1. *S is a CP-Galois extension of S^G which is an Azumaya algebra if and only if S is an Azumaya Galois extension of S^G.*

Proof. Since S is a CP-Galois extension, $S*G$ is an H-separable over S by the argument in the proof of Theorem 3.3. Hence, $Z = C^G$ [2, Theorem 3.1]. Again, since S is a CP-Galois extension, S is a progenerator over S^G. Therefore, $S*G$ is a projective H-separable extension of S^G [8, Theorem 6]. Since S^G is an Azumaya algebra, $S*G$ is an Azumaya algebra [7, Theorem 1]. Thus, $S*G$ is an Azumaya C^G-algebra. Now, consider $S*G \cong \mathrm{Hom}_{S^G}(S,S)$ as a C^G-subalgebra of $\mathrm{Hom}_{C^G}(S,S)$ which is an Azumaya C^G-algebra since S is finitely generated and projective over C^G. By the commutator theorem for Azumaya algebras, $V_{\mathrm{Hom}_{C^G}(S,S)}(S*G)$ is an Azumaya C^G-algebra. Noting that $S^G \cong \mathrm{Hom}_{S*G}(S,S) \cong V_{\mathrm{Hom}_{C^G}(S,S)}(S*G)$ and $Z = C^G$, we have that S^G is an Azumaya C^G-algebra. So, S is an Azumaya Galois extension of S^G.

Conversely, suppose that S is an Azumaya Galois extension of S^G. Then, $S \cong S^G \otimes_{C^G} V_S(S^G)$ such that $V_S(S^G)$ is a G-Galois algebra over C^G [1, Theorem 2]. Hence, S is a CP-Galois extension of S^G. This completes the proof. □

Theorem 4.2. *S is a CP and faithfully Galois extension of S^G if and only if S is an Azumaya Galois extension.*

Proof. (⇐) Let S be an Azumaya Galois extension of S^G. Then, by Theorem 4.1, S is a CP-Galois extension of S^G. Moreover, by the proof of Theorem 4.1, $S*G$ is an Azumaya Z-algebra. Since S is a G-Galois extension, S is a faithful left $\mathrm{Hom}_{S^G}(S,S)$-module. Therefore, S is a faithful left $S*G$-module since $S*G \cong \mathrm{Hom}_{S^G}(S,S)$. Hence, S is also a faithfully Galois extension of S^G.

(⇒) By Theorem 4.1, it suffices to show that S^G is an Azumaya algebra. In fact, $S^G \cong \mathrm{Hom}_{S*G}(S,S) \cong V_{\mathrm{Hom}_Z(S,S)}(S*G)$. Since S is faithfully Galois, $S*G$ is Z-Azumaya and S is a progenerator over Z. Therefore, $\mathrm{Hom}_Z(S,S)$ is an Azumaya Z-algebra. Thus, $V_{\mathrm{Hom}_Z(S,S)}(S*G)$ is an Azumaya algebra. Hence, S^G is an Azumaya algebra. □

Corollary 4.3. *The class of Azumaya Galois extensions is the intersection of the class of CP-Galois extensions and the class of faithfully Galois extensions.*

Proof. The Corollary is immediate by Theorem 4.2. □

Next, we give four examples of Galois extensions S to demonstrate the relations among different classes of Galois extensions:

(1) S is an H-separable G-Galois extension, but not a faithfully Galois extension,
(2) S is an H-separable G-Galois extension, but not a CP-Galois extension,
(3) S is an Azumaya Galois extension, but not an H-separable Galois extension,
(4) S is an H-separable Galois extension, but not an Azumaya Galois extension.

Example 4.4. S is an H-separable G-Galois extension, but not a faithfully Galois extension.

Let T be the tensor product of infinite copies of the 2×2 matrix algebra $M_2(\mathbb{Q})$ over the rational field $\mathbb{Q}$, that is, $T = M_2(\mathbb{Q})\otimes M_2(\mathbb{Q})\otimes\cdots\otimes M_2(\mathbb{Q})\otimes\cdots$. Then it is easy to check that the center of T is a tensor product of infinite copies of $\mathbb{Q}$ and contains a proper subalgebra that is isomorphic to $\mathbb{Q}$ by the map $a \to a\otimes a\otimes\cdots\otimes a\otimes\cdots$ for all $a\in\mathbb{Q}$. For convenience, we identity a and $a\otimes a\otimes\cdots\otimes a\otimes\cdots$. Since $M_2(\mathbb{Q})$ has rank 4 over $\mathbb{Q}$, T is infinitely generated over its center $\mathbb{Q}\otimes\mathbb{Q}\otimes\cdots\otimes\mathbb{Q}\otimes\cdots$. Hence, T is non-Azumaya algebra such that $2^{-1}\in T$. Let $S = T[i,j,k]$ be the quaternion algebra over T and $G = \{1, g_i, g_j, g_k\}$, where $g_i(x) = ixi^{-1}$, $g_j(x) = jxj^{-1}$, and $g_k(x) = kxk^{-1}$ for all x in S. Then

(1) S is a G-Galois extension of S^G with a Galois system $\{1/2, i/2, j/2, k/2; 1/2, -i/2, -j/2, -k/2\}$.

(2) S is an H-separable extension of S^G by [9, Corollary 3.3]. Hence, S is an H-separable G-Galois extension of S^G.

(3) $S*G$ is an H-separable G'-Galois extension of $(S*G)^{G'}$ such that $S*G\cong S\otimes_C (V_S(S^G))^\circ$, where $V_S(S^G)$ is a projective separable C-algebra [9, Proposition 4]. Noting that S is not an Azumaya C-algebra for T is not Azumaya, $S*G$ is not an Azumaya Z-algebra by Theorem 3.4. Thus, S is not a faithfully Galois extension, but it is an H-separable G-Galois extension.

Example 4.5. S is an H-separable G-Galois extension, but not a CP-Galois extension.

Let S be any G-Galois extension with Galois group G. Then $S*G$ is an H-separable Galois extension, but not a CP-Galois extension by Theorem 3.2.

Example 4.6. S is an Azumaya Galois extension, but not an H-separable Galois extension.

Let $T = M_{2\times 2}(\mathbb{Q})$ be the full matrix ring of order 2 over the rational field $\mathbb{Q}$ and $S = T\times T = \{(a,b)\mid a,b\in T\}$, and $G = \{1, g \mid g(a,b) = (b,a)$ for all $(a,b)\in S\}$. Then,

(1) $g^2 = 1$.
(2) $S^G = \{(a,a)\mid a\in T\}\cong T$.
(3) T is an Azumaya D-algebra, where $D = \mathbb{Q}I\approx\mathbb{Q}$ and I is the identity of T.
(4) $C = D\times D$.
(5) $C^G = \{(d,d)\mid d\in D\}\approx\mathbb{Q}$.
(6) S is a G-Galois extension of S^G with the G-Galois system: $\{(0,1),(1,0);(0,1),(1,0)\}$.
(7) S^G is an Azumaya C^G-algebra.
(8) S is an Azumaya Galois extension by (6) and (7).
(9) $V_S(S^G) = D\times D = C$.

(10) S is not an H-separable Galois extension. Suppose that S is an H-separable Galois extension. Since $n(=2)$ is a unit in S, $V_S(S^G)(=D\times D=C)$ is finitely generated projective over C of rank 2 [9, Proposition 4]. This is a contradiction.

Remark 4.7. By Corollary 4.3, Example 4.6 can be considered as an example of (i) S is a CP-Galois extension, but not an H-separable Galois extension, or (ii) S is a faithfully Galois extension, but not an H-separable Galois extension.

Example 4.8. S is an H-separable Galois extension, but not an Azumaya Galois extension.

Let $S=\mathbb{Q}[i,j,k]$ be the quaternion algebra over the rational field $\mathbb{Q}$ and $G=\{1,g_i \mid g_i(x)=ixi^{-1}$ for all x in $\mathbb{Q}[i,j,k]\}$. Then S is a G-Galois extension of S^G with the G-Galois system $\{1/2,-i/2,-j/2,-k/2;1/2,i/2,j/2,k/2\}$. It is easy to check that $S^G=\mathbb{Q}[i]$ which is a commutative separable $\mathbb{Q}$-algebra. But the center of S is $\mathbb{Q}(=C^G)$, so S^G is not an Azumaya C^G-algebra. Thus, S is not an Azumaya Galois extension. But S is projective over S^G and $S=\mathbb{Q}[i,j,k]$ is an Azumaya $\mathbb{Q}$-algebra, so S is an H-separable extension of S^G [5, Theorem 1].

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