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# FUZZY ASSOCIATIVE $\mathcal{I}$-IDEALS OF IS-ALGEBRAS

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ABSTRACT. We fuzzify the concept of an associative $\mathcal{I}$-ideal in an **IS**-algebra. We give a relation between a fuzzy $\mathcal{I}$-ideal and a fuzzy associative $\mathcal{I}$-ideal, and we investigate some related properties.

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**1. Introduction.** The notion of BCK-algebras was proposed by Imai and Iséki in 1966. In the same year, Iséki [2] introduced the notion of a BCI-algebra which is a generalization of a BCK-algebra. For the general development of BCK/BCI-algebras, the ideal theory plays an important role. In 1993, Jun et al. [4] introduced a new class of algebras related to BCI-algebras and semigroups, called a BCI-semigroup/BCI-monoid/BCI-group. In 1998, for the convenience of study, Jun et al. [7] renamed the BCI-semigroup (respectively, BCI-monoid and BCI-group) as the **IS**-algebra (respectively, **IM**-algebra and **IG**-algebra) and studied further properties of these algebras (see [6, 7]). In [8] Roh et al. introduced the concept of an associative $\mathcal{I}$-ideal and a strong $\mathcal{I}$-ideal in an **IS**-algebra. They gave necessary and sufficient conditions for an $\mathcal{I}$-ideal to be an associative $\mathcal{I}$-ideal and established a characterization of a strong $\mathcal{I}$-ideal of an **IS**-algebras. Jun et al. [3] established the fuzzification of $\mathcal{I}$-ideals in **IS**-algebras.

In this paper, we consider the fuzzification of an associative $\mathcal{I}$-ideal of an **IS**-algebra. We prove that every fuzzy associative $\mathcal{I}$-ideal is a fuzzy $\mathcal{I}$-ideal. By giving an appropriate example, we verify that a fuzzy $\mathcal{I}$-ideal may not be a fuzzy associative $\mathcal{I}$-ideal. We give a condition for a fuzzy $\mathcal{I}$-ideal to be a fuzzy associative $\mathcal{I}$-ideal, and we investigate some related properties.

**2. Preliminaries.** We review some definitions and properties that will be useful in our results.

By a *BCI-algebra* we mean an algebra $(X,*,0)$ of type (2,0) satisfying the following conditions:

(I) $((x*y)*(x*z))*(z*y)=0$,

(II) $(x*(x*y))*y=0$,

(III) $x*x=0$,

(IV) $x*y=0$ and $y*x=0$ imply $x=y$.

A BCI-algebra $X$ satisfying $0\le x$ for all $x\in X$ is called a *BCK-algebra*. In any BCI-algebra $X$ one can define a partial order "$\le$" by putting $x\le y$ if and only if $x*y=0$.

A BCI-algebra $X$ has the following properties for any $x,y,z \in X$:
(1) $x * 0 = x$,
(2) $(x*y)*z = (x*z)*y$,
(3) $x \le y$ implies that $x*z \le y*z$ and $z*y \le z*x$,
(4) $(x*z)*(y*z) \le x*y$,
(5) $x*(x*(x*y)) = x*y$,
(6) $0*(x*y) = (0*x)*(0*y)$,
(7) $0*(0*((x*z)*(y*z))) = (0*y)*(0*x)$.

A nonempty subset $I$ of a BCK/BCI-algebra $X$ is called an *ideal* of $X$ if it satisfies:
(i) $0 \in I$,
(ii) $x*y \in I$ and $y \in I$ imply $x \in I$ for all $x,y \in X$.

Any ideal $I$ has the property: $y \in I$ and $x \le y$ imply $x \in I$.

For a BCI-algebra $X$, the set $X_+ := \{x \in X \mid 0 \le x\}$ is called the *BCK-part* of $X$. If $X_+ = \{0\}$, then we say that $X$ is a *$p$-semisimple* BCI-algebra. Note that a BCI-algebra $X$ is $p$-semisimple if and only if $0*(0*x) = x$ for all $x \in X$.

In [4], Jun et al. introduced a new class of algebras related to BCI-algebras and semigroups, called a *BCI-semigroup*, and in [7] they renamed it as an **IS**-*algebra* for the convenience of study.

By an **IS**-*algebra* [7] we mean a nonempty set $X$ with two binary operations "$*$" and "$\cdot$" and constant 0 satisfying the axioms:
(V) $I(X) := (X,*,0)$ is a BCI-algebra.
(VI) $S(X) := (X,\cdot)$ is a semigroup.
(VII) The operation "$\cdot$" is distributive (on both sides) over the operation "$*$," that is, $x\cdot(y*z) = (x\cdot y)*(x\cdot z)$ and $(x*y)\cdot z = (x\cdot z)*(y\cdot z)$ for all $x,y,z \in X$.

Especially, if $I(X) := (X,*,0)$ is a $p$-semisimple BCI-algebra in the definition of **IS**-algebras, we say that $X$ is a **PS**-*algebra*. We write the multiplication $x\cdot y$ by $xy$, for convenience.

**Example 2.1** (see [8]). Let $X = \{0,a,b,c\}$ be a set with Cayley tables:

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $c$ | $b$ |
| $a$ | $a$ | 0 | $c$ | $b$ |
| $b$ | $b$ | $b$ | 0 | $c$ |
| $c$ | $c$ | $c$ | $b$ | 0 |

| $\cdot$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | 0 | 0 | 0 |
| $b$ | 0 | 0 | $b$ | $c$ |
| $c$ | 0 | 0 | $c$ | $b$ |

Then $X$ is an **IS**-algebra.

Every $p$-semisimple BCI-algebra gives an abelian group by defining $x+y := x*(0*y)$, and hence a **PS**-algebra leads to the ring structure. On the while, every ring gives a BCI-algebra by defining $x*y := x-y$ and so we can construct an **IS**-algebra. This means that *the category of* **PS**-*algebras is equivalent to the category of rings*. In Example 2.1, we can see that $(a+b)+c = 0 \ne a = a+(b+c)$ if we define $x+y := x*(0*y)$. Hence the **IS**-*algebra is a generalization of the ring.*

**Lemma 2.2** [4, Proposition 1]. *Let $X$ be an* **IS**-*algebra. Then for any $x, y, z \in X$, we have*

(i) $0x = x0 = 0$,

(ii) $x \leq y$ *implies that* $xz \leq yz$ *and* $zx \leq zy$.

A nonempty subset $A$ of a semigroup $S(X) := (X, \cdot)$ is said to be *left* (respectively, *right*) *stable* [1] if $xa \in A$ (respectively, $ax \in A$) whenever $x \in S(X)$ and $a \in A$.

A nonempty subset $A$ of an **IS**-algebra $X$ is called a *left* (respectively, *right*) *$\mathcal{I}$-ideal* of $X$ [7] if

($a_1$) $A$ is a left (respectively, right) stable subset of $S(X)$,

($a_2$) for any $x, y \in I(X)$, $x * y \in A$ and $y \in A$ imply that $x \in A$.

Note that $\{0\}$ and $X$ are left (respectively, right) $\mathcal{I}$-ideals. If $A$ is a left (respectively, right) $\mathcal{I}$-ideal of an **IS**-algebra $X$, then $0 \in A$. Thus $A$ is an ideal of $I(X)$.

We now review some fuzzy logic concepts.

A *fuzzy set* in a set $X$ is a function $\mu : X \to [0,1]$. For $t \in [0,1]$ the set $U(\mu; t) := \{x \in X \mid \mu(x) \geq t\}$ is called a *level subset* of $\mu$.

A fuzzy set $\mu$ in a BCI-algebra $X$ is called a *fuzzy ideal* of $X$ if

($b_1$) $\mu(0) \geq \mu(x)$ for all $x \in X$,

($b_2$) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$ for all $x, y \in X$.

A fuzzy set $\mu$ in a semigroup $S(X) := (X, \cdot)$ is said to be *fuzzy left* (respectively, *fuzzy right*) *stable* [5] if $\mu(xy) \geq \mu(y)$ (respectively, $\mu(xy) \geq \mu(x)$) for all $x, y \in X$.

A fuzzy set $\mu$ in an **IS**-algebra $X$ is called a *fuzzy left* (respectively, *fuzzy right*) *$\mathcal{I}$-ideal* of $X$ [3] if

($b_3$) $\mu$ is a fuzzy left (respectively, fuzzy right) stable set in $S(X)$,

($b_4$) $\mu$ is a fuzzy ideal of a BCI-algebra $X$.

From now on, a (fuzzy) $\mathcal{I}$-ideal shall mean a (fuzzy) left $\mathcal{I}$-ideal.

## 3. Fuzzy associative $\mathcal{I}$-ideals

**Definition 3.1** (see [8]). A nonempty subset $A$ of an **IS**-algebra $X$ is called a *left* (respectively, *right*) *associative $\mathcal{I}$-ideal* of $X$ if

($a_1$) $A$ is a left (respectively, right) stable subset of $S(X)$,

($a_3$) for any $x, y, z \in I(X)$, $(x * y) * z \in A$ and $y * z \in A$ imply that $x \in A$.

We start with the fuzzification of a left (respectively, right) associative $\mathcal{I}$-ideal.

**Definition 3.2.** A fuzzy set $\mu$ in an **IS**-algebra $X$ is called a *fuzzy left* (respectively, *fuzzy right*) *associative $\mathcal{I}$-ideal* of $X$ if

($b_3$) $\mu$ is a fuzzy left (respectively, fuzzy right) stable set in $S(X)$,

($b_5$) $\mu(x) \geq \min\{\mu((x * y) * z), \mu(y * z)\}$ for all $x, y, z \in X$.

In what follows, a (fuzzy) associative $\mathcal{I}$-ideal shall mean a (fuzzy) left associative $\mathcal{I}$-ideal.

**Example 3.3.** Consider an **IS**-algebra $X = \{0, a, b, c\}$ with the following Cayley tables:

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | $b$ | $b$ |
| $a$ | $a$ | 0 | $c$ | $b$ |
| $b$ | $b$ | $b$ | 0 | 0 |
| $c$ | $c$ | $b$ | $a$ | 0 |

| $\cdot$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | $a$ | 0 | $a$ |
| $b$ | 0 | 0 | $b$ | $b$ |
| $c$ | 0 | $a$ | $b$ | $c$ |

Define a fuzzy set $\mu$ in $X$ by $\mu(0) = \mu(a) = 0.7$ and $\mu(b) = \mu(c) = 0.5$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$.

**Example 3.4.** Consider an **IS**-algebra $X = \{0, a, b, c\}$ with Cayley tables as follows:

| $*$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | $a$ | $b$ | $c$ |
| $a$ | $a$ | 0 | $c$ | $b$ |
| $b$ | $b$ | $c$ | 0 | $a$ |
| $c$ | $c$ | $b$ | $a$ | 0 |

| $\cdot$ | 0 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | $a$ | $b$ | $c$ |
| $b$ | 0 | $a$ | $b$ | $c$ |
| $c$ | 0 | 0 | 0 | 0 |

Let $\mu$ be a fuzzy set in $X$ defined by $\mu(0) = t_0$, $\mu(a) = t_1$, $\mu(b) = \mu(c) = t_2$, where $t_0 > t_1 > t_2$ in $[0, 1]$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$.

We give a relation between a fuzzy associative $\mathcal{I}$-ideal and a fuzzy $\mathcal{I}$-ideal. To do this, we need the following lemma.

**Lemma 3.5** (see [3]). *A fuzzy set $\mu$ in an* **IS***-algebra $X$ is a fuzzy $\mathcal{I}$-ideal of $X$ if and only if it satisfies:*

(i) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$ *for all* $x, y \in X$,

(ii) $\mu(xy) \geq \mu(y)$ *for all* $x, y \in X$.

**Theorem 3.6.** *Every fuzzy associative $\mathcal{I}$-ideal is a fuzzy $\mathcal{I}$-ideal.*

**Proof.** Let $\mu$ be a fuzzy associative $\mathcal{I}$-ideal of an **IS**-algebra $X$ and let $x, y \in X$. Then

$$\begin{aligned} \mu(x) &\geq \min\{\mu((x * y) * 0), \mu(y * 0)\} \quad \text{(by (b}_5\text{))} \\ &= \min\{\mu(x * y), \mu(y)\} \quad \text{(by (1))}. \end{aligned} \tag{3.1}$$

It follows from Lemma 3.5 that $\mu$ is a fuzzy $\mathcal{I}$-ideal of $X$. □

The following example shows that the converse of Theorem 3.6 may not be true.

**Example 3.7.** Let $X$ be an **IS**-algebra in Example 3.3 and let $\mu$ be a fuzzy set in $X$ defined by $\mu(0) = \mu(b) = 0.6$ and $\mu(a) = \mu(c) = 0.2$. It is routine to check that $\mu$ is a fuzzy $\mathcal{I}$-ideal of $X$. But $\mu$ is not a fuzzy associative $\mathcal{I}$-ideal of $X$, since

$$\mu(a) < \min\{\mu((a * b) * c), \mu(b * c)\}. \tag{3.2}$$

Now we find a condition for a fuzzy $\mathcal{I}$-ideal to be a fuzzy associative $\mathcal{I}$-ideal. Let $\mu$ be a fuzzy set in an **IS**-algebra $X$ and consider the following inequality:

($b_6$) $\mu(x) \geq \mu((x * y) * y)$ for all $x, y \in X$.

We know that, in general, a fuzzy $\mathcal{I}$-ideal of an **IS**-algebra $X$ may not satisfy the condition ($b_6$). In fact, if we take the fuzzy $\mathcal{I}$-ideal $\mu$ in Example 3.7, then $\mu(a) = 0.2 < 0.6 = \mu((a * c) * c)$. But we have the following theorem.

**Theorem 3.8.** *Every fuzzy associative $\mathcal{I}$-ideal of an* **IS**-*algebra satisfies inequality ($b_6$).*

**Proof.** Let $\mu$ be a fuzzy associative $\mathcal{I}$-ideal of an **IS**-algebra $X$ and let $x, y \in X$. Using (III) and ($b_5$), we get

$$\begin{aligned}\mu(x) &\geq \min\{\mu((x * y) * y), \mu(y * y)\} \\ &= \min\{\mu((x * y) * y), \mu(0)\} \\ &= \mu((x * y) * y).\end{aligned} \tag{3.3}$$

This completes the proof. □

It is natural to have the question: is a fuzzy set satisfying ($b_6$) a fuzzy $\mathcal{I}$-ideal? The following example provides a negative answer, and hence we know that the converse of Theorem 3.8 may not be true.

**Example 3.9.** In Example 3.4, define a fuzzy set $\mu$ in $X$ by $\mu(0) = \mu(a) = \mu(b) = 0.8$ and $\mu(c) = 0.5$. Then $\mu$ satisfies the condition ($b_6$), but $\mu$ is not a fuzzy $\mathcal{I}$-ideal and hence not a fuzzy associative $\mathcal{I}$-ideal of $X$.

**Theorem 3.10.** *If a fuzzy $\mathcal{I}$-ideal of an* **IS**-*algebra satisfies condition ($b_6$), then it is a fuzzy associative $\mathcal{I}$-ideal.*

**Proof.** Let $\mu$ be a fuzzy $\mathcal{I}$-ideal of an **IS**-algebra $X$ satisfying condition ($b_6$). It is sufficient to show that $\mu$ satisfies condition ($b_5$). Notice that

$$((x * z) * z) * (y * z) = ((x * z) * (y * z)) * z \leq (x * y) * z \tag{3.4}$$

for all $x, y, z \in X$. It follows from ($b_6$) and Lemma 3.5(i) that

$$\begin{aligned}\mu(x) &\geq \mu((x * z) * z) \\ &\geq \min\{\mu(((x * z) * z) * (y * z)), \mu(y * z)\} \\ &\geq \min\{\mu((x * y) * z), \mu(y * z)\}\end{aligned} \tag{3.5}$$

for all $x, y, z \in X$. This completes the proof. □

By Theorems 3.8 and 3.10, we have the following corollary.

**Corollary 3.11.** *Let $\mu$ be a fuzzy $\mathcal{I}$-ideal of an* **IS**-*algebra $X$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ if and only if it satisfies condition ($b_6$).*

**Proposition 3.12.** *Let $\mu$ be a fuzzy set in an* **IS**-*algebra. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ if and only if the nonempty level set $U(\mu; t)$ of $\mu$ is an associative $\mathcal{I}$-ideal of $X$ for every $t \in [0, 1]$.*

We then call $U(\mu; t)$ a *level associative $\mathcal{I}$-ideal* of $\mu$.

**Proof.** Suppose that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$. Let $x \in S(X)$ and $y \in U(\mu;t)$. Then $\mu(y) \geq t$ and so $\mu(xy) \geq \mu(y) \geq t$, which implies that $xy \in U(\mu;t)$. Hence $U(\mu;t)$ is a stable subset of $S(X)$. Let $x,y,z \in I(X)$ be such that $(x*y)*z \in U(\mu;t)$ and $y*z \in U(\mu;t)$. Then $\mu((x*y)*z) \geq t$ and $\mu(y*z) \geq t$. It follows from ($b_5$) that

$$\mu(x) \geq \min\{\mu((x*y)*z), \mu(y*z)\} \geq t \tag{3.6}$$

so that $x \in U(\mu;t)$. Hence $U(\mu;t)$ is an associative $\mathcal{I}$-ideal of $X$. Conversely, assume that the nonempty level set $U(\mu;t)$ of $\mu$ is an associative $\mathcal{I}$-ideal of $X$ for every $t \in [0,1]$. If there are $x_0, y_0 \in S(X)$ such that $\mu(x_0y_0) < \mu(y_0)$, then by taking $t_0 := (1/2)(\mu(x_0y_0)+\mu(y_0))$ we have $\mu(x_0y_0) < t_0 < \mu(y_0)$. It follows that $y_0 \in U(\mu;t_0)$ and $x_0y_0 \notin U(\mu;t_0)$. This is a contradiction. Therefore $\mu$ is a fuzzy stable set in $S(X)$. Suppose that $\mu(x_0) < \min\{\mu((x_0*y_0)*z_0), \mu(y_0*z_0)\}$ for some $x_0,y_0,z_0 \in X$. Putting $s_0 := (1/2)(\mu(x_0)+\min\{\mu((x_0*y_0)*z_0), \mu(y_0*z_0)\})$, then $\mu(x_0) < s_0 < \min\{\mu((x_0*y_0)*z_0), \mu(y_0*z_0)\}$, which shows that $(x_0*y_0)*z_0 \in U(\mu;s_0)$, $y_0*z_0 \in U(\mu;s_0)$ and $x_0 \notin U(\mu;s_0)$. This is impossible. Thus $\mu$ satisfies the condition ($b_5$). This completes the proof. □

Using Proposition 3.12, we can consider a generalization of Example 3.3 as follows.

**Proposition 3.13.** *Let $A$ be an associative $\mathcal{I}$-ideal of an* **IS**-*algebra $X$ and let $\mu$ be a fuzzy set in $X$ defined by*

$$\mu(x) := \begin{cases} t_0 & \text{if } x \in A, \\ t_1 & \text{otherwise}, \end{cases} \tag{3.7}$$

*where $t_0 > t_1$ in $[0,1]$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$, and $U(\mu;t_0) = A$.*

**Proof.** Notice that

$$U(\mu;t) = \begin{cases} \varnothing & \text{if } t_0 < t, \\ A & \text{if } t_1 < t \leq t_0, \\ X & \text{if } t \leq t_1. \end{cases} \tag{3.8}$$

It follows from Proposition 3.12 that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$. Clearly, we have $U(\mu;t_0) = A$. □

Proposition 3.13 suggests that any associative $\mathcal{I}$-ideal of an **IS**-algebra $X$ can be realized as a level associative $\mathcal{I}$-ideal of some fuzzy associative $\mathcal{I}$-ideal of $X$.

We now consider the converse of Proposition 3.13.

**Proposition 3.14.** *For a nonempty subset $A$ of an* **IS**-*algebra $X$, let $\mu$ be a fuzzy set in $X$ which is given in Proposition 3.13. If $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$, then $A$ is an associative $\mathcal{I}$-ideal of $X$.*

**Proof.** Assume that $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ and let $x \in S(X)$ and $y \in A$. Then $\mu(xy) \geq \mu(y) = t_0$ and so $xy \in U(\mu;t_0) = A$. Hence $A$ is a stable subset of $S(X)$. Let $x,y,z \in I(X)$ be such that $(x*y)*z \in A$ and $y*z \in A$. From ($b_5$) it follows that

$$\mu(x) \geq \min\{\mu((x*y)*z), \mu(y*z)\} = t_0 \tag{3.9}$$

so that $x \in U(\mu; t_0) = A$. This completes the proof. □

The following theorem shows that the concept of a fuzzy associative $\mathcal{I}$-ideal of an **IS**-algebra is a generalization of an associative $\mathcal{I}$-ideal. The proof is straightforward by using Propositions 3.13 and 3.14.

**Theorem 3.15.** *Let $A$ be a nonempty subset of an* **IS***-algebra $X$ and let $\mu$ be a fuzzy set in $X$ such that $\mu$ is into $\{0,1\}$, so that $\mu$ is the characteristic function of $A$. Then $\mu$ is a fuzzy associative $\mathcal{I}$-ideal of $X$ if and only if $A$ is an associative $\mathcal{I}$-ideal of $X$.*

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