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WEAK AND STRONG FORMS OF IRRESOLUTE MAPS

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Abstract. We consider new weak and stronger forms of irresolute and semi-closure via the concept sg-closed sets which we call ap-irresolute maps, ap-semi-closed maps and contra-irresolute and use it to obtain a characterization of semi-$T_{1/2}$ spaces.

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1. Introduction. The concept of a semi-generalized closed set (written in short as sg-closed set) of a topological space was introduced by Bhattacharyya and Lahiri [2]. These sets were also considered by various authors (e.g., Sundaram, Maki and Balachandran [15], Caldas [4] and Dontchev and Maki [9]).

In this paper, we introduce the concept of irresoluteness called ap-irresolute maps and ap-semi-closed maps by using sg-closed sets and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve sg-closed sets. Also, in this paper, we present a new generalization of irresoluteness called contra-irresolute. We define this last class of map by the requirement that the inverse image of each semi-open set in the codomain is semi-closed in the domain. This notion is a stronger form of ap-irresoluteness. Finally, we also characterize the class of semi-$T_{1/2}$ spaces in terms of ap-irresolute and ap-semi-closed maps.

Throughout this paper, (X,τ), (Y,σ), and (Z,γ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space (X,τ), $\mathrm{Cl}(A)$, and $\mathrm{Int}(A)$ denote the closure of A and the interior of A, respectively.

2. Preliminaries. Since we require the following known definitions, notations, and some properties, we recall them in this section.

Definition 2.1. A subset A of a space (X,τ) is said to be semi-open [11] if there exists $O \in \tau$ such that $O \subseteq A \subseteq \mathrm{Cl}(O)$. The semi-interior [6] of A denoted by $\mathrm{sInt}(A)$, is defined by the union of all semi-open sets of (X,τ) contained in A.

Remark 2.2. (i) A subset A is semi-open [6] if and only if $\mathrm{sInt}(A) = A$.
(ii) $\mathrm{sInt}(A) = A \cap \mathrm{Cl}(\mathrm{Int}(A))$ [10].
By $\mathrm{SO}(X,\tau)$ we mean the collection of all semi-open sets in (X,τ).

Definition 2.3. A subset B of (X,τ) is said to be semi-closed [3] if its complement B^c is semi-open in (X,τ). The semi-closure [3] of a set B of (X,τ) denoted by

$\mathrm{sCl}_X(B)$, briefly $\mathrm{sCl}(B)$, is defined to be the intersection of all semi-closed sets of (X,τ) containing B.

Remark 2.4. (i) A subset B is semi-closed [13] if and only if $\mathrm{sCl}(B) = B$.
(ii) $\mathrm{sCl}(B) = B \cup \mathrm{Int}(\mathrm{Cl}(B))$ [10].

Definition 2.5. A map $f : (X,\tau) \to (Y,\sigma)$ is called irresolute [7] if $f^{-1}(O)$ is semi-open in (X,τ) for every $O \in \mathrm{SO}(Y,\sigma)$.

Definition 2.6. A map $f : (X,\tau) \to (Y,\sigma)$ is called pre-semi-closed (resp., pre-semi-open) [7] if for every semi-closed (resp., semi-open) set B of (X,τ), $f(B)$ is semi-closed (resp., semi-open) in (Y,σ).

Definition 2.7. A subset F of (X,τ) is said to be semi-generalized closed (written in short as sg-closed) in (X,τ) [2] if $\mathrm{sCl}(F) \subseteq O$ whenever $F \subseteq O$ and O is semi-open in (X,τ). A subset B is said to be semi-generalized open (written as sg-open) in (X,τ) [2] if its complement $B^c = X - B$ is sg-closed in (X,τ).

3. Ap-irresolute, ap-semi-closed and contra-irresolute maps.

Let $f : (X,\tau) \to (Y,\sigma)$ be a map from a topological space (X,τ) into a topological space (Y,σ).

Definition 3.1. A map $f : (X,\tau) \to (Y,\sigma)$ is said to be approximately irresolute (or ap-irresolute) if $\mathrm{sCl}(F) \subseteq f^{-1}(O)$ whenever O is a semi-open subset of (Y,σ), F is a sg-closed subset of (X,τ), and $F \subseteq f^{-1}(O)$.

Definition 3.2. A map $f : (X,\tau) \to (Y,\sigma)$ is said to be approximately semi-closed (or ap-semi-closed) if $f(B) \subseteq \mathrm{sInt}(A)$ whenever A is a sg-open subset of (Y,σ), B is a semi-closed subset of (X,τ), and $f(B) \subseteq A$.

Clearly irresolute maps are ap-irresolute and pre-semi-closed maps are ap-semi-closed, but not conversely.

The proof follows from Definition 3.1 and [2, Def. 1] (resp., Definition 3.2 and [2, Thm. 6]).

The following example shows the converse implications do not hold.

Example 3.3. Let $X = \{a,b\}$ be the Sierpinski space with the topology, $\tau = \{\emptyset, \{a\}, X\}$. Let $f : X \to X$ be defined by $f(a) = b$ and $f(b) = a$. Since the image of every semi-closed set is semi-open, then f is ap-semi-closed (similarly, since the inverse image of every semi-open set is semi-closed, then f is ap-irresolute). However $\{b\}$ is semi-closed in (X,τ) (resp., $\{a\}$ is semi-open) but $f(\{b\})$ is not semi-closed (resp., $f^{-1}(\{a\})$ is not semi-open in (X,τ)). Therefore f is not pre-semi-closed (resp., f is not irresolute).

Theorem 3.4. (i) *$f : (X,\tau) \to (Y,\sigma)$ is ap-irresolute if $f^{-1}(O)$ is semi-closed in (X,τ) for every $O \in \mathrm{SO}(Y,\sigma)$.*
(ii) *$f : (X,\tau) \to (Y,\sigma)$ is ap-semi-closed if $f(B) \in \mathrm{SO}(Y,\sigma)$ for every semi-closed subset B of (X,τ).*

Proof. (i) Let $F \subseteq f^{-1}(O)$, where $O \in \mathrm{SO}(Y,\sigma)$ and F is a sg-closed subset of (X,τ). Therefore $\mathrm{sCl}(F) \subseteq \mathrm{sCl}(f^{-1}(O)) = f^{-1}(O)$. Thus f is ap-irresolute.

(ii) Let $f(B) \subseteq A$, where B is a semi-closed subset of (X,τ) and A is a sg-open subset of (Y,σ). Therefore $\mathrm{sInt}(f(B)) \subseteq \mathrm{sInt}(A)$. Then $f(B) \subseteq \mathrm{sInt}(A)$. Thus f is ap-semi-closed. □

This theorem was used in Example 3.3.

Remark 3.5. Let (X,τ) denote the topological space defined in Example 3.3. Then the identity map on (X,τ) is both ap-irresolute and ap-semi-closed, it is clear that the converses of Theorem 3.4 do not hold.

In the following theorem, we get under certain conditions that the converse of Theorem 3.4 is true.

Theorem 3.6. *Let $f:(X,\tau)\to(Y,\sigma)$ be a map from a topological space (X,τ) in a topological space (Y,σ).*

(i) *If the semi-open and semi-closed sets of (X,τ) coincide, then f is ap-irresolute if and only if $f^{-1}(O)$ is semi-closed in (X,τ) for every $O \in \mathrm{SO}(Y,\sigma)$.*

(ii) *If the semi-open and semi-closed sets of (Y,σ) coincide, then f is ap-semi-closed if and only if $f(B) \in \mathrm{SO}(Y,\sigma)$ for every semi-closed subset B of (X,τ).*

Proof. (i) Assume f is ap-irresolute. Let A be an arbitrary subset of (X,τ) such that $A \subseteq Q$, where $Q \in \mathrm{SO}(X,\tau)$. Then by hypothesis $\mathrm{sCl}(A) \subseteq \mathrm{sCl}(Q) = Q$. Therefore all subsets of (X,τ) are sg-closed (and hence all are sg-open). So, for any $O \in \mathrm{SO}(Y,\sigma)$, $f^{-1}(O)$ is sg-closed in (X,τ). Since f is ap-irresolute $\mathrm{sCl}(f^{-1}(O)) \subseteq f^{-1}(O)$. Therefore $\mathrm{sCl}(f^{-1}(O)) = f^{-1}(O)$, i.e., $f^{-1}(O)$ is semi-closed in (X,τ).

The converse is clear by Theorem 3.4.

(ii) Assume f is ap-semi-closed. Reasoning as in (i), we obtain that all subsets of (Y,σ) are sg-open. Therefore for any semi-closed subset of B of (X,τ), $f(B)$ is sg-open in Y. Since f is ap-semi-closed $f(B) \subseteq \mathrm{sInt}(f(B))$. Therefore $f(B) = \mathrm{sInt}(f(B))$, i.e, $f(B)$ is semi-open. The converse is clear by Theorem 3.4. □

As immediate consequence of Theorem 3.6, we have the following.

Corollary 3.7. *Let $f:(X,\tau)\to(Y,\sigma)$ be a map from a topological space (X,τ) in a topological space (Y,σ).*

(i) *If the semi-open and semi-closed sets of (X,τ) coincide, then f is ap-irresolute if and only if f is irresolute.*

(ii) *If the semi-open and semi-closed sets of (Y,σ) coincide, then f is ap-semi-closed if and only if f is pre-semi-closed.*

A map $f:(X,\tau)\to(Y,\sigma)$ is called contra-irresolute if $f^{-1}(O)$ is semi-closed in (X,τ) for each $O \in \mathrm{SO}(Y,\sigma)$, and contra-pre-semi-closed if $f(B) \in \mathrm{SO}(Y,\sigma)$ for each semi-closed set B of (X,τ).

Remark 3.8. In fact, contra-irresoluteness and irresoluteness are independent notions. Example 3.3 shows that contra-irresoluteness does not imply irresoluteness while the reverse is shown in the following example.

Example 3.9. An irresolute map need not be contra-irresolute. The identity map on the topological space (X,τ) where $\tau = \{\emptyset,\{a\},X\}$ is an example of an irresolute map which is not contra-irresolute.

In the same manner, we can prove that contra-pre-semi-closed maps and pre-semi-closed are independent notions.

The following result can be easily verified. Its proof is straightforward.

Theorem 3.10. *Let $f:(X,\tau)\to(Y,\sigma)$ be a map. Then the following conditions are equivalent:*

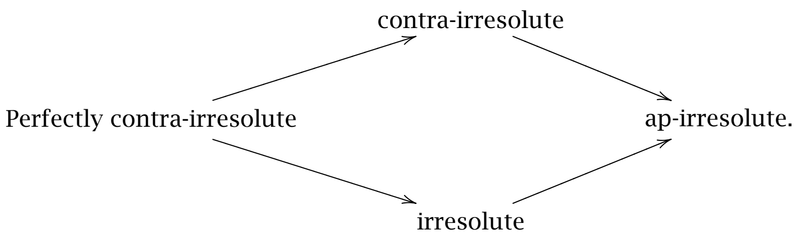
(i) *f is contra-irresolute.*

(ii) *The inverse image of each semi-closed set in Y is semi-open in X.*

Remark 3.11. By Theorem 3.4, we have that every contra-irresolute map is ap-irresolute and every contra-pre-semi-closed is ap-semi-closed, the converse implication do not hold.

A map $f:(X,\tau)\to(Y,\sigma)$ is called perfectly contra-irresolute if the inverse of every semi-open set in Y is semi-clopen in X. Hence, every perfectly contra-irresolute map is contra-irresolute and irresolute.

Clearly, the following diagram holds and none of its implications is reversible:



(3.1)

The next two theorems establish conditions under which maps and inverse maps preserve sg-closed sets.

Sundaram, Maki and Balachandran in [15, Thm. 3.7] showed that the irresolute pre-semi-closed inverse image of a sg-closed set is sg-closed. We strengthen this result slightly by replacing the pre-semi-closed requirement with ap-semi-closed.

Theorem 3.12. *If a map $f:(X,\tau)\to(Y,\sigma)$ is irresolute and ap-semi-closed, then $f^{-1}(A)$ is sg-closed (resp., sg-open) whenever A is sg-closed (resp., sg-open) subset of (Y,σ).*

Proof. Let A be a sg-closed subset of (Y,σ). Suppose that $f^{-1}(A)\subseteq O$ where $O\in \mathrm{SO}(X,\tau)$. Taking complements we obtain $O^c\subseteq f^{-1}(A^c)$ or $f(O^c)\subseteq A^c$. Since f is an ap-semi-closed and $\mathrm{sInt}(A)=A\cap \mathrm{Cl}(\mathrm{Int}(A))$ and $\mathrm{sCl}(A)=A\cup \mathrm{Int}(\mathrm{Cl}(A))$, then $f(O^c)\subseteq \mathrm{sInt}(A^c)=(\mathrm{sCl}(A))^c$. It follows that $O^c\subseteq (f^{-1}(\mathrm{sCl}(A)))^c$ and hence $f^{-1}(\mathrm{sCl}(A))\subseteq O$. Since f is irresolute $f^{-1}(\mathrm{sCl}(A))$ is semi-closed. Thus we have $\mathrm{sCl}(f^{-1}(A))\subseteq \mathrm{sCl}(f^{-1}(\mathrm{sCl}(A)))=f^{-1}(\mathrm{sCl}(A))\subseteq O$. This implies that $f^{-1}(A)$ is sg-closed in (X,τ). A similar argument shows that inverse images of sg-open are sg-open. □

This is known (see [15]) that the semi-continuous pre-semi-closed image of a sg-closed set is sg-closed. The following theorem test this result replacing the semi-continuous requirement with ap-irresolute.

Theorem 3.13. *If a map $f : (X,\tau) \to (Y,\sigma)$ is ap-semi-irresolute and pre-semi-closed, then for every sg-closed F of (X,τ), $f(F)$ is sg-closed set of (Y,σ).*

Proof. Let F be a sg-closed subset of (X,τ). Let $f(F) \subseteq O$ where $O \in \mathrm{SO}(Y,\sigma)$. Then $F \subseteq f^{-1}(O)$ holds. Since f is ap-irresolute $\mathrm{sCl}(F) \subseteq f^{-1}(O)$ and hence $f(\mathrm{sCl}(F)) \subseteq O$. Therefore, we have $\mathrm{sCl}(f(F)) \subseteq \mathrm{sCl}(f(\mathrm{sCl}(F))) = f(\mathrm{sCl}(F)) \subseteq O$. Hence $f(F)$ is sg-closed in (Y,σ). □

Now, reasoning as in [9], we obtain that the composition of two contra-irresolute maps need not be contra-irresolute. Really, Let $X = \{a,b\}$ be the Sierpinski space and set $\tau = \{\emptyset,\{a\},X\}$ and $\sigma = \{\emptyset,\{b\},X\}$. The identity maps $f : (X,\tau) \to (X,\sigma)$ and $g : (X,\sigma) \to (X,\tau)$ are both contra-irresolute but their composition $g \circ f : (X,\tau) \to (X,\tau)$ is not contra-irresolute.

However the following theorem holds. The proof is easy and hence omitted.

Theorem 3.14. *Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\gamma)$ be two maps such that $g \circ f : (X,\tau) \to (Z,\gamma)$. Then,*

(i) *$g \circ f$ is contra-irresolute, if g is irresolute and f is contra-irresolute.*

(ii) *$g \circ f$ is contra-irresolute, if g is contra-irresolute and f is irresolute.*

In an analogous way, we have the following.

Theorem 3.15. *Let $f : (X,\tau) \to (Y,\sigma)$, $g : (Y,\sigma) \to (Z,\gamma)$ be two maps such that $g \circ f : (X,\tau) \to (Z,\gamma)$. Then,*

(i) *$g \circ f$ is ap-semi-closed, if f is pre-semi-closed and g is ap-semi-closed.*

(ii) *$g \circ f$ is ap-semi-closed, if f is ap-semi-closed and g is pre-semi-open and g^{-1} preserves sg-open sets.*

(iii) *$g \circ f$ is ap-irresolute, if f is ap-irresolute and g is irresolute.*

Proof. To prove statement (i), suppose B is an arbitrary semi-closed subset in (X,τ) and A is a sg-open subset of (Z,γ) for which $g \circ f(B) \subseteq A$. Then $f(B)$ is semi-closed in (Y,σ) because f is pre-semi-closed. Since g is ap-semi-closed, $g(f(B)) \subseteq \mathrm{sInt}(A)$. This implies that $g \circ f$ is ap-semi-closed.

To prove statement (ii), suppose B is an arbitrary semi-closed subset of (X,τ) and A is a sg-open subset of (Z,γ) for which $g \circ f(B) \subseteq A$. Hence $f(B) \subseteq g^{-1}(A)$. Then $f(B) \subseteq \mathrm{sInt}(g^{-1}(A))$ because $g^{-1}(A)$ is sg-open and f is ap-semi-closed. Thus,

$$(g \circ f)(B) = g\big(f(B)\big) \subseteq g\big(\mathrm{sInt}\big(g^{-1}(A)\big)\big) \subseteq \mathrm{sInt}\big(gg^{-1}(A)\big) \subseteq \mathrm{sInt}(A). \tag{3.2}$$

This implies that $g \circ f$ is ap-semi-closed.

To prove statement (iii), suppose F is an arbitrary sg-closed subset of (X,τ) and $O \in \mathrm{SO}(Z,\gamma)$ for which $F \subseteq (g \circ f)^{-1}(O)$. Then $g^{-1}(O) \in \mathrm{SO}(Y,\sigma)$ because g is irresolute. Since f is ap-irresolute, $\mathrm{sCl}(F) \subseteq f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$. This proves that $g \circ f$ is ap-irresolute. □

As a consequence of Theorem 3.15, we have the following.

Corollary 3.16. *Let $f_\alpha : X \to Y_\alpha$ be a map for each $\alpha \in \Omega$ and $f : X \to \prod Y_\alpha$ the product map given by $f(x) = (f_\alpha(x))$. If f is ap-irresolute, then f_α is ap-irresolute for each α.*

Proof. For each β let $P_\beta : \prod Y_\alpha \to Y_\beta$ be the projection map. Then $f_\beta = P_\beta \circ f$, where P_β is irresolute. By Theorem 3.15(iii), f_β is ap-irresolute. □

Regarding the restriction f_A of a map $f : (X,\tau) \to (Y,\sigma)$ to a subset A of X, we have the following.

Theorem 3.17. (i) *If $f : (X,\tau) \to (Y,\sigma)$ is ap-semi-closed and A is a semi-closed set of (X,τ), then its restriction $f_A : (A,\tau_A) \to (Y,\sigma)$ is ap-semi-closed.*

(ii) *If $f : (X,\tau) \to (Y,\sigma)$ is ap-irresolute and A is an open, sg-closed subset of (X,τ), then $f_A : (A,\tau_A) \to (Y,\sigma)$ is ap-irresolute.*

Proof. (i) Suppose B is an arbitrary semi-closed subset of (A,τ_A) and O a sg-open subset of (Y,σ) for which $f_A(B) \subseteq O$. By [12, Thm. 2.6] B is semi-closed of (X,τ) because A is semi-closed of (X,τ). Then $f_A(B) = f(B) \subseteq O$. Using Definition 3.2, we have $f_A(B) \subseteq \mathrm{sInt}(O)$. Thus f_A is an ap-semi-closed map.

(ii) Assume that F is a sg-closed subset relative to A, i.e., sg-closed in (A,τ_A), and G is a semi-open subset of (Y,σ) for which $F \subseteq (f_A)^{-1}(G)$. Then $F \subseteq f^{-1}(G) \cap A$. By [2, Thm. 3] F is sg-closed in X. Since f is ap-irresolute $sC(F) \subseteq f^{-1}(G)$. Then $\mathrm{sCl}(F) \cap A \subseteq f^{-1}(G) \cap A$. Using the fact that $\mathrm{sCl}(F) \cap A = \mathrm{sCl}_A(F)$ for every pre-open subset [14, Thm. 2.4], we have $\mathrm{sCl}_A(F) \subseteq (f_A)^{-1}(G)$. Thus $f_A : (A,\tau_A) \to (Y,\sigma)$ is ap-irresolute. □

Observe that restrictions of ap-semi-closed maps can fail to be ap-semi-closed.

Really, as in [1], let X be an indiscrete space. Then X and $\emptyset$ are the only semi-open subsets of X. Hence the semi-closed subsets of X are also X and $\emptyset$. Let A a nonempty proper subset of X. The identity map $f : X \to X$ is ap-semi-closed, but $f_A : A \to X$ fails to be ap-semi-closed. In fact, $f(A)$ is sg-open (every subset of X is sg-open) and A is closed in A. Therefore semi-closed in (A,τ_A), but $f(A) \subseteq \mathrm{sInt}(f(A))$.

4. A characterization of semi-$T_{1/2}$ spaces. In the following theorem, we give a characterization of a class of topological space called semi-$T_{1/2}$ space by using the concepts of ap-irresolute maps and ap-semi-closed maps.

We recall that a topological space (X,τ) is said to be semi-$T_{1/2}$ space [2], if every sg-closed set is semi-closed.

Theorem 4.1. *Let (X,τ) be a topological space. Then the following statements are equivalent:*

(i) *(X,τ) is a semi-$T_{1/2}$ space.*

(ii) *For every space (Y,σ) and every map $f : (X,\tau) \to (Y,\sigma)$, f is ap-irresolute.*

Proof. (i)⟹(ii): Let F be a sg-closed subset of (X,τ) and suppose that $F \subseteq f^{-1}(O)$, where $O \in \mathrm{SO}(Y,\sigma)$. Since (X,τ) is a semi-$T_{1/2}$ space, F is semi-closed (i.e., $F = \mathrm{sCl}(F)$). Therefore $\mathrm{sCl}(F) \subseteq f^{-1}(O)$. Then f is ap-irresolute.

(ii)⟹(i): Let B be a sg-closed subset of (X,τ) and let Y be the set X with the topology $\sigma = \{\emptyset, B, Y\}$. Finally let $f : (X,\tau) \to (Y,\sigma)$ be the identity map. By assumption f is ap-irresolute. Since B is sg-closed in (X,τ) and semi-open in (Y,σ) and $B \subseteq f^{-1}(B)$, it follows that $\mathrm{sCl}(B) \subseteq f^{-1}(B) = B$. Hence B is semi-closed in (X,τ) and therefore is semi-$T_{1/2}$. □

Theorem 4.2. *Let (Y,σ) be a topological space. Then the following statements are equivalent:*

(i) *(Y,σ) is a semi-$T_{1/2}$ space.*

(ii) *For every space (X,τ) and every map $f:(X,\tau)\to(Y,\sigma)$, f is ap-semi-closed.*

Proof. Analogous to Theorem 4.1 making the obvious changes. □

We refer the reader to [2, 4, 5, 15] for other results on semi-$T_{1/2}$ spaces.

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