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THE GENERAL IKEHATA THEOREM FOR H-SEPARABLE CROSSED PRODUCTS

GEORGE SZETO and LIANYONG XUE

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ABSTRACT. Let B be a ring with 1, C the center of B, G an automorphism group of B of order n for some integer n, C^G the set of elements in C fixed under G, $\Delta = \Delta(B,G,f)$ a crossed product over B where f is a factor set from $G\times G$ to $U(C^G)$. It is shown that Δ is an H-separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$.

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1. Introduction. Let B be a ring with 1, ρ an automorphism of B of order n, $B[x;\rho]$ a skew polynomial ring with a basis $\{1,x,x^2,\dots,x^{n-1}\}$ and $x^n = v \in U(B^\rho)$ for some integer n, where B^ρ is the set of elements in B fixed under ρ and $U(B^\rho)$ is the set of units of B^ρ.

In [4] it was shown that any skew polynomial ring $B[x;\rho]$ of prime degree n is an H-separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle\rho|_C\rangle$ generated by $\rho|_C$ of order n. This theorem was extended to any degree n [5, Theorem 1]. Recently, the theorem was completely generalized by the present authors in [8], that is, let $B[x;\rho]$ be a skew polynomial ring of degree n for some integer n. Then, $B[x;\rho]$ is an H-separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle\rho|_C\rangle \cong \langle\rho\rangle$. The purpose of the present paper is to generalize the above Ikehata theorem to an automorphism group of B (not necessarily cyclic) and f is an factor set from $G\times G$ to $U(C^G)$. We show that Δ is an H-separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$.

2. Preliminaries and basic definitions. Throughout this paper, B represents a ring with 1, C the center of B, G an automorphism group of B of order n for some integer n, B^G the set of elements in B fixed under G, $\Delta = \Delta(B,G,f)$ a crossed product with a free basis $\{U_g \mid g \in G \text{ and } U_1 = 1\}$ over B and the multiplications are given by $U_g b = g(b)U_g$ and $U_gU_h = f(g,h)U_{gh}$ for $b \in B$ and $g,h \in G$ where f is a map from $G\times G$ to $U(C^G)$ such that $f(g,h)f(gh,k) = f(h,k)f(g,hk)$, Z the center of Δ, $\bar{G}$ the inner automorphism group of Δ induced by G, that is, $\bar{g}(x) = U_gxU_g^{-1}$ for each $x \in \Delta$ and $g \in G$. We note that $f(g,1) = f(1,g) = f(1,1) = 1$ for all $g \in G$ and $\bar{G}$ restricted to B is G.

Let A be a subring of a ring S with the same identity 1. We denote $V_s(A)$ the commutator subring of A in S. A ring S is called a G-Galois extension of S^G if there exist elements $\{a_i, b_i \in S,\ i = 1,2,\dots,m\}$ for some integer m such that $\sum_{i=1}^{m} a_i g(b_i) = \delta_{1,g}$. The set $\{a_i, b_i\}$ is called a G-Galois system for S. S is called an H-separable extension of A if there exists an H-separable system $\{x_i \in V_S(A),\ y_i \in V_{S\otimes_A S}(S) \mid i = 1,2,\dots,m\}$ for S over A for some integer m such that $\sum_{i=1}^{m} x_i y_i = 1 \otimes_A 1$.

3. The Ikehata theorem. In this section, we show that Δ is an H-separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_C \cong G$. We begin with a lemma.

Lemma 3.1. (a) $V_\Delta(B) = \sum_{g\in G} J_g U_g$ *where* $J_g = \{b \in B \mid ab = bg(a)$ *for all* $a \in B\}$.

(b) $V_{\Delta\otimes_B\Delta}(\Delta) = \{\sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes_B U_h \mid b(g,h) \in J_{gh}$ *and* $k(b_{(k^{-1}g,h)}) f(k, k^{-1}g) = b_{(g,hk^{-1})} f(hk^{-1}, k)$ *for all* $g,k \in G\}$.

(c) *If* $\sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes U_h \in V_{\Delta\otimes_B\Delta}(\Delta)$, *then* $b_{(g,h)} U_{gh} \in V_\Delta(B)$.

(d) *If* $\sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes U_h \in V_{\Delta\otimes_B\Delta}(\Delta)$, *then* $b_{(g,g^{-1})} = g(b_{1,1})(f(g^{-1}, g))^{-1}$ *for all* $g \in G$.

Proof. (a) Let $b \in J_g$. Then $a(bU_g) = (ab)U_g = bg(a)U_g = (bU_g)a$ for all $a \in B$. Hence $J_g U_g \subset V_\Delta(B)$. Therefore, $\sum_{g\in G} J_g U_g \subset V_\Delta(B)$. Conversely, let $\sum_{g\in G} b_g U_g \in V_\Delta(B)$. Then $a\sum_{g\in G} b_g U_g = \sum_{g\in G} b_g U_g a = \sum_{g\in G} b_g g(a) U_g$ for all $a \in B$, and so $ab_g = b_g g(a)$ for all $a \in B$ and $g \in G$, that is, $b_g \in J_g$ for all $g \in G$. Thus $V_\Delta(B) \subset \sum_{g\in G} J_g U_g$.

(b) $x = \sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes_B U_h \in V_{\Delta\otimes_B\Delta}(\Delta)$ if and only if $bx = xb$ and $U_k x = xU_k$ for all $a \in B$ and $k \in G$. But

$$
\begin{aligned}
bx &= \sum_{g\in G}\sum_{h\in G} bb_{(g,h)} U_g \otimes_B U_h, \\
xb &= \sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes_B U_h b = \sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes_B h(b) U_h \\
&= \sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g h(b) \otimes_B U_h = \sum_{g\in G}\sum_{h\in G} b_{(g,h)} (gh)(b) U_g \otimes_B U_h,
\end{aligned}
\tag{3.1}
$$

so $bx = xb$ if and only if $bb_{(g,h)} = b_{(g,h)}((gh)(b))$ for all $b \in B$ and $g,h \in G$, that is, $b_{(g,h)} \in J_{gh}$ by noting that $\{U_g \otimes_B U_h \mid g,\ h \in G\}$ is a basis for Δ over B. Moreover,

$$
\begin{aligned}
U_k x &= U_k \sum_{g\in G}\sum_{h\in G} b_{(g,h)} U_g \otimes_B U_h = \sum_{g\in G}\sum_{h\in G} k(b_{(g,h)}) U_k U_g \otimes_B U_h \\
&= \sum_{g\in G}\sum_{h\in G} k(b_{(g,h)}) f(k,g) U_{kg} \otimes_B U_h \\
&= \sum_{g\in G}\sum_{h\in G} k(b_{(k^{-1}(kg),h)}) f(k, k^{-1}(kg)) U_{(kg)} \otimes_B U_h \\
&= \sum_{l\in G}\sum_{h\in G} k(b_{(k^{-1}l,h)}) f(k, k^{-1}l) U_l \otimes_B U_h \\
&= \sum_{g\in G}\sum_{h\in G} k(b_{(k^{-1}g,h)} f(k, k^{-1}g)) U_g \otimes_B U_h,
\end{aligned}
\tag{3.2}
$$

and

$$\begin{aligned} xU_k &= \sum_{g\in G}\sum_{h\in G} b_{(g,h)}U_g \otimes_B U_hU_k = \sum_{g\in G}\sum_{h\in G} b_{(g,h)}U_g \otimes_B f(h,k)U_{hk} \\ &= \sum_{g\in G}\sum_{h\in G} b_{(g,h)}U_g f(h,k) \otimes_B U_{hk} = \sum_{g\in G}\sum_{h\in G} b_{(g,h)} f(h,k)U_g \otimes_B U_{hk} \\ &= \sum_{g\in G}\sum_{h\in G} b_{(g,(hk)k^{-1})} f((hk)k^{-1},k)U_g \otimes_B U_{hk} \\ &= \sum_{g\in G}\sum_{h\in G} b_{(g,lk^{-1})} f(lk^{-1},k)U_g \otimes_B U_l = \sum_{g\in G}\sum_{h\in G} b_{(g,hk^{-1})} f(hk^{-1},k)U_g \otimes_B U_h. \end{aligned} \tag{3.3}$$

Hence, $U_kx = xU_k$ if and only if $k(b_{(k^{-1}g,h)})f(k,k^{-1}g) = b_{(g,hk^{-1})}f(hk^{-1},k)$ for all $g,h,k \in G$.

(c) If $\sum_{g\in G}\sum_{h\in G} b_{g,h}U_g \otimes U_h \in V_{\Delta\otimes_B\Delta}(\Delta)$, then $b_{(g,h)} \in J_{gh}$ by (b); and so $b_{(g,h)}U_{gh} \in V_\Delta(B)$ by (a).

(d) If $\sum_{g\in G}\sum_{h\in G} b_{(g,h)}U_g \otimes U_h \in V_{\Delta\otimes B\Delta}(\Delta)$, then $k(b_{(k^{-1}g,h)})f(k,k^{-1}g) = b_{(g,hk^{-1})}f(hk^{-1},k)$ for all $g,h,k \in G$ by (b). Let $k = g$ and $h = 1$. Then $b_{(g,g^{-1})}f(g^{-1},g) = g(b_{1,1})f(g,1) = g(b_{1,1})$ for all $g \in G$. This implies that $b_{(g,g^{-1})} = g(b_{1,1})(f(g^{-1},g))^{-1}$ for all $g \in G$. □

Theorem 3.2. *Δ is an H-separable extension of B and $V_\Delta(B)$ is a commutative subring of Δ if and only if C is a Galois algebra over C^G with Galois group $G|_c \cong G$.*

Proof. ($\Longrightarrow$) Since Δ is an H-separable extension of B and B is a direct summand of Δ as a left B-module, $V_\Delta(V_\Delta(B)) = B$ [7, Proposition 1.2]. But $V_\Delta(B)$ is commutative, so $V_\Delta(B) \subset V_\Delta(V_\Delta(B)) = B$. Thus $V_\Delta(B) = C$.

Since Δ is an H-separable extension of B again, there exists an H-separable system $\{x_i \in V_\Delta(B),\ y_i \in V_{\Delta\otimes B\Delta}(\Delta) \mid i = 1,2,\dots,m\}$ for some integer m such that $\sum_{i=1}^m x_iy_i = 1 \otimes_B 1$. Let $y_i = \sum_{g\in G}\sum_{h\in G} b^{(i)}_{(g,h)}U_g \otimes_B U_h$. We claim that $\{a_i = x_i,\ b_i = b^{(i)}_{(1,1)} \mid i = 1,2,\dots,m\}$ is a G-Galois system for C. In fact, $a_i = x_i \in V_\Delta(B) = C$ and by Lemma 3.1(b), $b_i = b^{(i)}_{(1,1)} \in J_1 = C$. Moreover, since $y_i = \sum_{g\in G}\sum_{h\in G} b^{(i)}_{(g,h)}U_g \otimes_B U_h \in V_{\Delta\otimes_B\Delta}(\Delta)$, $b^{(i)}_{(g,h)}U_{gh} \in V_\Delta(B)$ by Lemma 3.1(c). But $V_\Delta(B) = C$, so $b^{(i)}_{(g,h)} = 0$ when $gh \neq 1$. Thus, $y_i = \sum_{g\in G} b^{(i)}_{(g,g^{-1})}U_g \otimes_B U_{g^{-1}}$. By Lemma 3.1(d), $b^{(i)}_{(g,g^{-1})} = g(b^{(i)}_{(1,1)})(f(g^{-1},g))^{-1} = g(b_i)(f(g^{-1},g))^{-1}$, so $y_i = \sum_{g\in G} g(b_i)(f(g^{-1},g))^{-1}U_g \otimes_B U_{g^{-1}}$. Therefore,

$$\begin{aligned} 1 \otimes_B 1 &= \sum_{i=1}^m x_iy_i = \sum_{i=1}^m a_i \sum_{g\in G} g(b_i)\left(f(g^{-1},g)\right)^{-1}U_g \otimes_B U_{g^{-1}} \\ &= \sum_{g\in G}\sum_{i=1}^m a_ig(b_i)\left(f(g^{-1},g)\right)^{-1}U_g \otimes_B U_{g^{-1}}. \end{aligned} \tag{3.4}$$

This implies that $\sum_{i=1}^m a_ig(b_i)(f(g^{-1},g))^{-1} = \delta_{1,g}$, so $\sum_{i=1}^m a_ig(b_i) = \delta_{1,g}$, that is $\{a_i,b_i \mid i = 1,2,\dots,m\}$ is a G-Galois system for C. Therefore, C is a Galois algebra over C_G with Galois group $G\,|_C \cong G$.

($\Longleftarrow$) Since C is a Galois algebra over C^G with Galois group with $G|_C \cong G$, there exists a G-Galois system $\{a_i,b_i \in C \mid i = 1,2,\dots,m\}$ for some integer m such that $\sum_{i=1}^m a_ig(b_i) = \delta_{1,g}$. Let $x_i = a_i$ and $y_i = \sum_{g\in G} g(b_i)U_g \otimes_B U_g^{-1}$. We claim that $\{x_i \in$

$V_\Delta(B)$, $y_i \in V_{\Delta\otimes_B\Delta}(\Delta) \mid i = 1,2,\dots,m\}$ is an H-separable system for Δ over B. In fact, $x_i = a_i \in C \subset V_\Delta(B)$. Noting that $U_g^{-1} = f(g,g^{-1})^{-1}U_{g^{-1}}$, we have $U_g^{-1}b = f(g,g^{-1})^{-1}$ $U_{g^{-1}}b = f(g,g^{-1})^{-1}g^{-1}(b)U_{g^{-1}} = g^{-1}(b)f(g,g^{-1})^{-1}U_{g^{-1}} = g^{-1}(b)U_g^{-1}$ for any $b \in B$. Hence

$$\begin{aligned} by_i &= b\sum_{g\in G} g(b_i)U_g \otimes_B U_g^{-1} = \sum_{g\in G} g(b_i)bU_g \otimes_B U_g^{-1} \\ &= \sum_{g\in G} g(b_i)U_g g^{-1}(b) \otimes_B U_g^{-1} = \sum_{g\in G} g(b_i)U_g \otimes_B g^{-1}(b)U_g^{-1} \\ &= \sum_{g\in G} g(b_i)U_g \otimes_B U_g^{-1}b = y_i b. \end{aligned} \tag{3.5}$$

for any $h \in G$,

$$\begin{aligned} U_h y_i &= U_h \sum_{g\in G} g(b_i)U_g \otimes_B U_g^{-1} = \sum_{g\in G} (hg)(b_i)U_hU_g \otimes_B U_g^{-1} \\ &= \sum_{g\in G} (hg)(b_i)f(h,g)U_{hg} \otimes_B U_g^{-1} = \sum_{g\in G} (hg)(b_i)U_{hg} \otimes_B f(h,g)U_g^{-1} \\ &= \sum_{g\in G} (hg)(b_i)U_{hg} \otimes_B U_{hg}^{-1}U_{hg}f(h,g)U_g^{-1} \\ &= \sum_{g\in G} (hg)(b_i)U_{hg} \otimes_B U_{hg}^{-1}U_hU_gU_g^{-1} = \sum_{g\in G} (hg)(b_i)U_{hg} \otimes_B U_{hg}^{-1}U_h \\ &= \sum_{k\in G} k(b_i)U_k \otimes_B U_k^{-1}U_h = y_iU_h. \end{aligned} \tag{3.6}$$

Thus $y_i \in V_{\Delta\otimes_B\Delta}(\Delta)$. Moreover, $\sum_{i=1}^m x_iy_i = \sum_{i=1}^m a_i \sum_{g\in G} g(b_i)U_g \otimes_B U_g^{-1} = \sum_{g\in G}\sum_{i=1}^m a_i$ $g(b_i)U_g \otimes_B U_g^{-1} = \sum_{g\in G} \delta_{1,g}U_g \otimes_B U_g^{-1} = 1 \otimes 1$. This implies that $\{x_i \in V_\Delta(B),\ y_i \in V_{\Delta\otimes_B\Delta}(\Delta) \mid i = 1,2,\dots,m\}$ is an H-separable system for Δ over B. Thus, Δ is an H-separable extension of B. Moreover, B is a direct summand of Δ as a left B-module, so $V_\Delta(V_\Delta(B)) = B$ [7, Proposition 1.2]. But then, the center of Δ, $Z \subset B$; and so $Z = C^G$. Clearly, $V_\Delta(B)^{\bar{G}} = Z = C^G$ and $C \subset V_\Delta(B)$, so $V_\Delta(B)$ is a G-Galois algebra over C^G with the same Galois system as C. Therefore, $V_\Delta(B) = C$ which is commutative. The proof is completed. □

The Ikehata theorem is an immediate consequence of Theorem 3.2 by the fact that any Galois algebra with a cyclic Galois group is a commutative ring [1, Theorem 11].

Corollary 3.3 (the Ikehata theorem)**.** *Let ρ be an automorphism of B of order n and $B[x;\rho]$ a skew polynomial ring of degree n with $x^n = v \in U(B^\rho)$ for some integer n. Then, $B[x;\rho]$ is an H-separable extension of B if and only if C is a Galois algebra over C^ρ with Galois group $\langle\rho \mid c\rangle \cong \langle\rho\rangle$.*

Proof. It is easy to check that if ρ has order n, then $x^n = v \in U(C^\rho)$. Let $B[x;\rho]$ be an H-separable extension of B. Then $V_{B[x;\rho]}(B)$ is a Galois algebra over C^ρ with cyclic Galois algebra group $\langle\bar{\rho}\rangle$ generated by $\bar{\rho}$ [6, Theorem 3.2]; and so $V_{B[x;\rho]}(B)$ is a commutative ring by [1, Theorem 11]. On the other hand, $B[x;\rho]$ is a crossed product $\Delta(B,\langle\rho\rangle,f)$ where $f : \langle\rho\rangle\times\langle\rho\rangle \to U(C^\rho)$ by $f(\rho^i,\rho^j) = 1$ if $i+j < n$, $f(\rho^i,\rho^j) = v$ if $i+j \ge n$, and $U_{\rho^i} = x^i$ for $i = 0,1,2,\dots,n-1$. Thus the corollary is immediate from Theorem 3.2. □

Next we prove more characterizations of the ring B as given in Theorem 3.2.

Theorem 3.4. *Assume Δ is an H-separable extension of B. Then the following statements are equivalent:*

(1) $V_\Delta(B)$ *is a commutative subring of* Δ.

(2) $V_\Delta(B) = C$.

(3) $V_\Delta(C) = B$.

(4) $J_g = \{0\}$ *for each* $g \neq 1$ *where* $J_g = \{b \in B \mid ab = bg(a)$ *for all* $a \in B\}$.

(5) $I_g = \{0\}$ *for each* $g \neq 1$ *where* $I_g = \{b \in B \mid cb = bg(c)$ *for all* $c \in C\}$.

Proof. We prove (1)⟹(2)⟹(3)⟹(4)⟹(5)⟹(1).

(1)⟹(2). This was given in the proof of the necessity of Theorem 3.2.

(2)⟹(3). Clearly, $B \subseteq V_\Delta(C)$. Conversely, for each $\sum_{g\in G} b_g U_g$ in $V_\Delta(C)$, we have $c(\sum_{g\in G} b_g U_g) = (\sum_{g\in G} b_g U_g)c$ for each c in C, so $cb_g = b_g g(c)$, that is $b_g(c - g(c)) = 0$ for each $g \in G$ and $c \in C$. But C is a commutative G-Galois extension of C^G, so the ideal of C generated by $\{c - g(c) \mid c \in C\}$ is C when $g \neq 1$ [2, Proposition 1.2(5)]. Hence $b_g = 0$ for each $g \neq 1$. But then $\sum_{g\in G} b_g U_g = b_1 \in B$. Thus $V_\Delta(C) \subseteq B$, and so $V_\Delta(C) = B$.

(3)⟹(4). By hypothesis, $V_\Delta(C) = B$ so $V_\Delta(B) \subset V_\Delta(C) = B$. But $V_\Delta(B) = \sum_{g\in G} J_g U_g$ by Lemma 3.1(a), so $\sum_{g\in G} J_g U_g = V_\Delta(B) \subset B$. Thus $J_g = \{0\}$ for each $g \neq 1$.

(4)⟹(5). By Lemma 3.1(a) again, $V_\Delta(B) = \sum_{g\in G} J_g U_g$, and by hypothesis, $J_g = \{0\}$ for each $g \neq 1$, so $V_\Delta(B) = J_1 = C$. Hence part (2) holds; and so $V_\Delta(C) = B$ by (2)⟹(3). Clearly, $V_\Delta(C) = \sum_{g\in G} I_g U_g$, so $\sum_{g\in G} I_g U_g = B$. Thus $I_g = \{0\}$ for each $g \neq 1$.

(5)⟹(1). Since $C \subset B$, $J_g \subset I_g$ for all $g \in G$. Hence $I_g = \{0\}$ implies $J_g = \{0\}$. But then $V_\Delta(B) = \sum_{g\in G} J_g U_g = J_1 = C$ which is commutative. □

Corollary 3.5. *C is a Galois algebra over C^G with Galois group $G|_c \cong G$ if and only if Δ is an H-separable extension of B and anyone of the equivalent conditions in Theorem 3.4 holds.*

We conclude the present paper with two examples of crossed products Δ to demonstrate our results:

(1) Δ is an H-separable extension of B, but $V_\Delta(B)$ is not commutative,

(2) $V_\Delta(B)$ is commutative, but Δ is not an H-separable extension of B.

Hence C is not a Galois algebra over C^G with $G|_C \cong G$ in either example by Theorem 3.2.

Example 3.6. Let $B = Q[i,j,k] = Q + Qi + Qj + Qk$ be the quaternion algebra over the rational field Q, $G = \{g_1 = 1,\ g_i,\ g_j,\ g_k \mid g_i(x) = ixi^{-1},\ g_j(x) = jxj^{-1},\ g_k(x) = kxk^{-1}$ for all $x \in B\}$, and $\Delta = \Delta(B,G,1)$. Then

(1) The center of Δ, $Z = Q = C$, the center of B.

(2) Δ is a separable extension of B and B is an Azumaya Q-algebra, so Δ is an Azumaya Q-algebra. Since Δ is a free left B-module, Δ is an H-separable extension of B [3, Theorem 1].

(3) $V_\Delta(B) = Q + QiU_{g_i} + QjU_{g_j} + QkU_{g_k}$ which is not commutative, so C is not a Galois algebra over C^G with Galois group $G|_C \cong G$ by Theorem 3.2.

Example 3.7. Let $B = Q[i,j,k] = Q + Qi + Qj + Qk$ be the quaternion algebra over the rational field Q, $G = \{g_1 = 1,\ g_i \mid g_i(x) = ixi^{-1}$ for all $x \in B\}$, and $\Delta = \Delta(B,G,1)$.

Then

(1) The center of B, $C = Q = C^G$.

(2) $V_\Delta(B) = Q + QiU_{g_i}$ which is commutative.

(3) The center of Δ, $Z = Q + QiU_{g_i} \neq C^G$. On the other hand, assume that Δ is an H-separable extension of B. Since B is a direct summand of Δ as a left B-module, $V_\Delta(V_\Delta(B)) = B$ [7, Proposition 1.2]. This implies that the center of Δ, $Z = C^G$, a contradiction. Thus Δ is not an H-separable extension of B. Therefore, C is not a G-Galois algebra over C^G with $G|_c \cong G$ by Theorem 3.2.

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Szeto: Department of Mathematics, Bradley University, Peoria, Illinois 61625, USA
E-mail address: szeto@bradley.bradley.edu

Xue: Department of Mathematics, Bradley University, Peoria, Illinois 61625, USA
E-mail address: lxue@bradley.bradley.edu