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A GENERAL VECTOR-VALUED VARIATIONAL INEQUALITY AND ITS FUZZY EXTENSION

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ABSTRACT. A general vector-valued variational inequality (GVVI) is considered. We establish the existence theorem for (GVVI) in the noncompact setting, which is a noncompact generalization of the existence theorem for (GVVI) obtained by Lee et al., by using the generalized form of KKM theorem due to Park. Moreover, we obtain the fuzzy extension of our existence theorem.

KEY WORDS AND PHRASES: Variational inequalities, fuzzy extension, KKM theorem.
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1. INTRODUCTION

Recently, Giannessi [1] introduced a variational inequality for vector-valued mappings in a Euclidean space. Since then, Chen et al. [2-6] have intensively studied variational inequalities for vector-valued mappings in Banach spaces. Lee et al. [7] have established the existence theorem of a variational inequality for a multifunction with vector values in a Banach space.

On the other hand, Chang and Zhu [8] introduced the concept of variational inequalities for fuzzy mappings in locally convex Hausdorff topological vector spaces and investigated existence theorems for some kinds of variational inequalities for fuzzy mappings, which were the fuzzy extensions of some theorems in [9,10,11,12]. Lee et al. [13] obtained the fuzzy generalizations of new results of Kim and Tan [14], and they [7] established the fuzzy extension of their existence theorem. Our motivation of this paper is to consider the noncompact cases of the existence theorems of variational inequalities for multifunctions with vector values or fuzzy mappings in Banach spaces obtained by Lee et al. [7].

Let X and Y be two normed spaces and D a nonempty convex subset of X. Let $T : X \to 2^{L(X,Y)}$ be a multifunction, where $L(X,Y)$ is the space of all continuous linear maps from X into Y, and C a closed pointed and convex cone of Y such that $Int\, C \neq \emptyset$, where Int denotes the interior.

Consider the following generalized vector-valued variational inequality:

(GVVI) Find $x_0 \in D$ such that for each $x \in D$, there exists an $s_0 \in T(x_0)$ such that

$$\langle s_0, x - x_0 \rangle \notin -Int\, C,$$

where $\langle s_0, y \rangle$ denotes the evaluation of s_0 at y.

When T is a mapping from X into $L(X,Y)$, (GVVI) reduces to the following vector-valued variational inequality (VVI) considered by Chen et al. [3,5,6].

(VVI) Find $x_0 \in D$ such that $\langle T(x_0), x - x_0\rangle \notin -Int\, C$ for all $x \in D$.

The above inequality (VVI) is a generalization of the following classic scalar-valued variational inequality (VI).

(VI) Find $x_0 \in D$ such that $\langle f(x_0), x - x_0\rangle \geq 0$ for all $x \in D$, where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a given mapping.

Our purpose in this paper is to establish the existence theorems for (GVVI) in the noncompact setting, which is the noncompact case of the existence theorem for (GVVI) obtained by Lee et al. [7], by using a particular form of the generalized KKM theorems due to Park [15-17]. Our existence theorem subsumes Theorem 2.1 of Cottle and Yao [18], the part (i) of Theorem 2.1 of Chen and Yang [6], Theorem 2 of Yang [19] and Theorem 2.1 of Lee et al. [7]. Moreover, we obtain the fuzzy extension of our existence theorem. Our fuzzy extension is a generalization of Theorem 3.1 of Lee et al. [7]. Now we give the definition of a KKM map.

DEFINITION 1.1. Let D be a subset of a convex space X. Then a multifunction $G : D \to 2^X$ is called KKM if for each nonempty finite subset N of D, $co\, N \subset G(N)$, where co denotes the convex hull and $G(N) = \bigcup\{Gx : x \in N\}$.

A *convex space* X is a nonempty convex set (in a vector space) with any topology that induces the Euclidean topology on the convex hulls of its finite subsets. Thus, a convex subset X of a topological vector space E with the relative topology is automatically a convex space. For details of the convex space, see Lassonde [9].

We say that a subset A of a topological space X is *compactly closed* in X if for every compact subset $K \subset X$ the set $A \cap K$ is closed in K. We need the following particular form of the generalized KKM theorems due to Park [16-18], which will be used in the proof of our Theorem 2.

THEOREM 1. Let X be a convex space, K a nonempty compact subset of X, and $G : X \to 2^X$ a KKM multifunction. Suppose that

(1) for each $y \in X, G(y)$ is compactly closed; and

(2) for each finite subset N of X, there exists a compact convex subset L_N of X such that $N \subset L_N$ and $L_N \cap \bigcap\{G(y) : y \in L_N\} \subset K$.

Then we have

$$K \cap \bigcap\{G(y) : y \in X\} \neq \emptyset.$$

2. Existence Theorems

First, we give the following definitions for the existence theorems for (GVVI).

DEFINITION 2.1. Let X be a normed space with dual space X^* and $T : X \to X^*$ a mapping.

1. T is said to be ***monotone*** if for any $x, y \in X, \langle T(x) - T(y), x - y\rangle \geq 0$.

2. T is said to be ***pseudomonotone*** if for any $x, y \in X, \langle T(x), y - x\rangle \geq 0$ implies that $\langle T(y), y - x\rangle \geq 0$.

3. T is said to be ***hemicontinuous*** if for any $x, y, z \in X$, the mapping $\alpha \mapsto \langle T(x + \alpha y), z\rangle$ is continuous at 0^+.

DEFINITION 2.2. Let X, Y be two normed spaces, $T : X \to L(X,Y)$ a mapping and C a closed, pointed and convex cone of Y such that $Int\, C \neq \emptyset$.

1. T is said to be *C-monotone* if for any $x, y \in X, \langle T(x) - T(y), x - y\rangle \in C$.

2. T is said to be C-pseudomonotone if for any $x, y \in X, \langle T(x), y - x\rangle \notin -Int\, C$ implies that $\langle T(y), y - x\rangle \notin -Int\, C$.

3. T is said to be *V-hemicontinuous* if for any $x, y, z \in X$, the mapping $\alpha \mapsto \langle T(x+\alpha y), z\rangle$ is continuous at 0^+.

REMARK. When $Y = \mathbb{R}$ and $C = \mathbb{R}_+$, Definition 2.2 becomes Definition 2.1.

DEFINITION 2.3. Let X and Y be two normed spaces, $T : X \to 2^{L(X,Y)}$ a set-valued map and C a closed, pointed and convex cone of Y such that $Int\, C \neq \emptyset$.

1. T is said to be *C-monotone* if for any $x, y \in X, s \in T(x)$ and $t \in T(y), \langle s-t, x-y\rangle \in C$
2. T is said to be *C-pseudomonotone* if for any $x, y \in X, \langle s, y-x\rangle \notin -Int\, C$ for some $s \in T(x)$ implies that $\langle t, y-x\rangle \notin -Int\, C$ for some $t \in T(y)$.
3. T is said to be *V-hemicontinuous* if for any $x, y \in X, \alpha > 0$ and $t_\alpha \in T(x+\alpha y)$, there exists $t_0 \in T(x)$ such that for any $z \in X, \langle t_\alpha, z\rangle \mapsto \langle t_0, z\rangle$ as $\alpha \to 0^+$.

REMARK. 1. Definition 2.3 is a generalization of Definition 2.2.

2. We can easily prove that the C-monotonicity implies the C-pseudomonotonicity.

Now we prove the following existence theorem for the noncompact case of (GVVI).

THEOREM 2. Let X and Y be Banach spaces, C a closed, pointed and convex cone in Y with $Int\, C \neq \emptyset$, D a nonempty convex subset of X, K a nonempty compact subset of X, and $T : X \to 2^{L(X,Y)}$. Suppose that

(1) T is C-pseudomonotone, compact-valued, and V-hemicontinuous; and

(2) for each nonempty finite subset N of D, there exists a nonempty compact convex subset L_N of D such that $N \subset L_N$ and for each $x \in L_N \backslash K$ there exists a $y \in L_N$ such that $\langle t, y-x\rangle \in -Int\, C$ for all $t \in T(y)$. Then (GVVI) is solvable.

PROOF. Define a multifunction $F_1 : D \to 2^D$ by

$$F_1(y) = \{x \in D : \langle s, y-x\rangle \notin -Int\, C \text{ for some } s \in T(x)\}$$

for $y \in D$. Then F_1 is a KKM multifunction on D.

In fact, suppose that $N = \{x_1, \cdots, x_n\} \subset D$, $\Sigma_{i=1}^n \alpha_i = 1$, $\alpha_i \geq 0$, $i = 1, \cdots, n$ and $x = \Sigma_{i=1}^n \alpha_i x_i \notin F_1(N)$. Then for any $s \in T(x)$, we have $\langle s, x_i - x\rangle \in -Int\, C$, $i = 1, \cdots, n$ Thus we have

$$\langle s, x\rangle = \left\langle s, \sum_{i=1}^n \alpha_i x_i \right\rangle = \sum_{i=1}^n \alpha_i \langle s, x_i\rangle \in \sum_{i=1}^n \alpha_i \langle s, x\rangle - Int\, C = \langle s, x\rangle - int\, C.$$

Hence $0 \in Int\, C$, which contradicts the pointedness of C. Therefore, F_1 is a KKM multifunction on D

Define a multifunction $F_2 : D \to 2^D$ by

$$F_2(y) = \{x \in D : \langle t, y-x\rangle \notin -Int\, C \text{ for some } t \in T(y)\}$$

for $y \in D$. For any $x \in F_1(y)$ there exists an $s \in T(x)$ such that $\langle s, y-x\rangle \notin -Int\, C$. By the C-pseudomonotonicity of T, there exists a $t \in T(y)$ such that $\langle t, y-x\rangle \notin -Int\, C$. Thus $x \in F_2(y)$ Hence for any $y \in D, F_1(y) \subset F_2(y)$. Therefore F_2 is also a KKM multifunction on D.

We claim that F_2 is closed-valued. In fact, for any $y \in D$, let $\{x_n\}$ be a sequence in $F_2(y)$ which converges to $x_* \in D$. Since $x_n \in F_2(y)$ for each n, there exists a $t_n \in T(y)$ such that $\langle t_n, y-x_n\rangle \in Y\backslash(-Int\, C)$. Since $T(y)$ is compact, we may assume that $\{t_n\}$ converges to some $t_* \in T(y)$. Note that

$$\begin{aligned} \|\langle t_n, y-x_n\rangle - \langle t_*, y-x_*\rangle\| &= \|\langle t_n, x_*-x_n\rangle - \langle t_n - t_*, y-x_*\rangle\| \\ &\leq \|\langle t_n, x_*-x_n\rangle\| + \|\langle t_n - t_*, y-x_*\rangle\| \\ &\leq \|t_n\|\ \|x_*-x_n\| + \|t_n - t_*\|\ \|y-x_*\|. \end{aligned}$$

Since $\{t_n\}$ is bounded in $L(X,Y)$, $\langle t_n, y-x_n\rangle$ converges to $\langle t_*, y-x_*\rangle$. Hence $\langle t_*, y-x_*\rangle \notin -Int\, C$, whence we have $x_* \in F_2(y)$.

Further, note that assumption (2) implies that, for each $x \in L_N \backslash K$ there exists a $y \in L_N$ such that $x \notin F_2(y)$. Hence $L_N \cap \bigcap \{F_2(y) : y \in L_N\} \subset K$. Therefore, condition (2) of Theorem 1 holds.

Therefore, by Theorem 1, there exists an $x \in K \cap \bigcap \{F_2(y) : y \in D\}$. Then for any $y \in D$, there exists a $t_y \in Ty$ such that $\langle t_y, y - x \rangle \notin -Int\,C$. By the convexity of D, for any $\alpha \in (0, 1)$, there exists a $t_\alpha \in T(\alpha y + (1-\alpha)x)$ such that $\langle t_\alpha, \alpha(y - x) \rangle \notin -Int\,C$. Dividing by α, we have $\langle t_\alpha, y - x \rangle \notin -Int\,C$. By the V-hemicontinuity of T, there exists $t_0 \in T(x)$ such that $\langle t_0, y - x \rangle \notin -Int\,C$. Hence $x \in \bigcap \{F_1(y) : y \in D\} \neq \emptyset$. Consequently, there exists an $x_0 \in K$ such that for each $x \in D$, there exists an $s_0 \in T(x_0)$ such that $\langle s_0, x - x_0 \rangle \notin -Int\,C$.

COROLLARY 2.1. In Theorem 2, if D is closed, then the coercivity (2) can be replaced by the following without affecting its conclusion:

(2') there exists a nonempty compact subset K of D and a $y_0 \in K$ such that

$$\langle t, y_0 - x \rangle \in -Int\,C \quad \text{for} \quad x \in D - K \quad \text{and} \quad t \in T(y_0).$$

PROOF. It suffices to show that (2') implies (2). In fact, for any nonempty finite subset N of D, we let $L_N = \bar{c}o(\{y_0\} \cup N \cup K) \subset D$. By (2'), for any $x \in L_N - K \subset D - K$, there exists a $y_0 \in K \subset L_N$ such that $\langle t, y_0 - x \rangle \in -Int\,C$ for all $t \in T(y_0)$. Hence (2) holds.

REMARK. Even for a single-valued T, Corollary 2.1 is more general than Yang [19, Theorem 2].

For $D = K$, Theorem 2 reduces to the following

COROLLARY 2.2 Let X and Y be Banach spaces, C a closed pointed and convex cone in Y with $Int\,C \neq \emptyset$, D a nonempty compact and convex subset of X and $T : X \to 2^{L(X,Y)}$ C-pseudomonotone, compact-valued, and V-hemicontinuous. Then (GVVI) is solvable.

REMARK. Corollary 2.2 extends Chen and Yang [6, Theorem 2.1, Part (i)].

COROLLARY 2.3 [7]. Let X be a reflexive Banach space, Y a Banach space, C a closed pointed and convex cone in Y with $Int\,C \neq \emptyset$, D a nonempty bounded closed and convex subset of X, and $T : X \to 2^{L(X,Y)}$ C-pseudomonotone, compact-valued and V-hemicontinuous. Then (GVVI) is solvable.

PROOF. Switch to the weak topology on X.

COROLLARY 2.4. Let X be a Banach space with dual space X^*, D a nonempty compact and convex subset of X and $T : X \to X^*$ pseudomonotone and hemicontinuous. Then there exists an $x_0 \in D$ such that $\langle T(x_0), x - x_0 \rangle \geq 0$ for all $x \in D$.

REMARK. Corollary 2.4 generalizes Cottle and Yao [11, Theorem 2.1]. Note that for $Y = \mathbb{R}$ and $C = \mathbb{R}_+$, corollaries extend or reduce to well-known scalar valued variational inequalities due to Hartman and Stampacchia, Browder, Stampacchia, Mosco, Dungundjii and Granas and many others.

3. FUZZY EXTENSION

Let X and Y be two normed spaces and $\mathcal{F}(L(X,Y))$ the collection of all fuzzy sets on $L(X,Y)$. A mapping F from X into $\mathcal{F}(L(X,Y))$ is called a fuzzy mapping.

If $F : X \to \mathcal{F}(L(X,Y))$ is a fuzzy mapping, then $F(x), x \in X$ (denoted by F_x), is a fuzzy set in $\mathcal{F}(L(X,Y))$ and $F_x(s), s \in L(X,Y)$, is the degree of membership of s in F_x. Let $A \in \mathcal{F}(L(X,Y))$ and $\beta \in [0,1]$. Then the set $(A)_\beta = \{s \in L(X,Y) : A(s) \geq \beta\}$ is said to be an *α-cut set* of A.

DEFINITION 3.1 [20]. A fuzzy set A in $L(X,Y)$ is *compact* if for each $\beta \in (0,1], (A)_\beta$ is compact in $L(X,Y)$.

DEFINITION 3.2. Let X and Y be two normed spaces, $F : X \to \mathcal{F}(L(X,Y))$ a fuzzy mapping and C a closed, pointed and convex cone of Y such that $Int\,C \neq \emptyset$.

1. F is said to be *C-monotone* if for any $x, y \in X$ and $s, t \in L(X,Y)$ with $F_x(s) > 0$ and $F_y(t) > 0$, $\langle s - t, x - y \rangle \in C$.

2. F is said to be C-pseudomonotone if for any $x, y \in X$ and $\beta \in (0,1]$, $\langle s, y-x\rangle \notin -Int\,C$ for some $s \in L(X,Y)$ with $F_x(s) \geq \beta$ implies that $\langle t, y-x\rangle \notin -Int\,C$ for some $t \in L(X,Y)$ with $F_y(t) > \beta$.

3. F is said to be *hemicontinuous* if for any $x, y \in X$ and $t_\alpha \in L(X,Y)$ with $F_{x+\alpha y}(t\alpha) \geq \beta$ where $\beta \in (0,1]$, there exists $t_0 \in L(X,Y)$ with $F_x(t_0) \geq \beta$ for any $z \in X, \langle t_\alpha, z\rangle \to \langle t_0, z\rangle$ as $\alpha \to 0^+$.

Now we obtain a fuzzy extension of Theorem 2.

THEOREM 3. Let X and Y be Banach spaces, C a closed, pointed and convex cone in Y with $Int\ C \neq \emptyset$, D a nonempty convex subset of X, K a nonempty compact subset of X, and $F: X \to \mathcal{F}(L(X,Y))$ a fuzzy mapping such that there exists a real number $\beta \in (0,1]$ such that for each $x \in X$, $(F_x)_\beta$ is a nonempty subset of $L(X,Y)$. Suppose that

(1) F is C-pseudomonotone and hemicontinuous, and for each $x \in X$, F_x is a compact fuzzy set in $L(X,Y)$,

(2) for each nonempty finite subset N of D, there exists a compact convex subset L_N of D such that $N \subset L_N$ and for each $x \in L_N \backslash K$ there exists a $y \in L_N$ such that $\langle t, y-x\rangle \in -Int\,C$ for all $t \in L(X,Y)$ with $F_y(t) \geq \beta$.

Then there exists an $x_0 \in D$ such that for each $x \in D$, there exists an $s_0 \in L(X,Y)$ with $F_{x_0}(s_0) \geq \beta$ such that $\langle s_0, x-x_0\rangle \notin -Int\,C$.

PROOF. Define a multifunction $\tilde{F}: X \to 2^{L(X,Y)}$ for any $x \in X, \tilde{F}(x) = F(x)_\beta$. It follows from the C-pseudomonotonicity of F that for any $x, y \in X, \langle s, y-x\rangle \notin -Int\,C$ for some $s \in \tilde{F}(x)$ implies that $\langle t, y-x\rangle \notin -Int\,C$ for some $t \in \tilde{F}(y)$. This implies hat $\tilde{F}$ is C-pseudomonotone. Furthermore, the hemicontinuity of F implies the V-hemicontinuity of $\tilde{F}$. Since for each $x \in X$, F_x is a compact fuzzy set in $L(X,Y)$, then for each $x \in K$, $\tilde{F}(x)$ is compact. Condition (2) implies that assumption (2) in Theorem 2 is satisfied for the multifunction $\tilde{F}$. By Theorem 2.1 there exists $x_0 \in D$ such that for each $x \in D$, there exists $s_0 \in \tilde{F}(x_0)$ such that $\langle s_0, x-x_0\rangle \notin -Int\,C$. Hence there exists an $x_0 \in D$ such that for each $x \in D$, there exists $s_0 \in L(X,Y)$ with $F_{x_0}(s_0) \geq \beta$ such that $\langle s_0, x-x_0\rangle \notin -Int\,C$.

COROLLARY 3.1. In Theorem 3, if D is closed, then the coercivity (2) can be replaced by the following without affecting its conclusion:

(2') there exists a nonempty compact subset K of D and a $y_0 \in K$ such that

$$\langle t, y_0 - x\rangle \in -Int\,C \quad \text{for all} \quad x \in D-K \quad \text{and all} \quad t \in L(X,Y) \quad \text{with} \quad F_{y_0}(t) \geq \beta.$$

For $D = K$, Theorem 3 reduces to the following

COROLLARY 3.2. Let X and Y be Banach space, C a closed pointed and convex cone in Y with $Int\,C \neq \emptyset$, D be a nonempty compact and convex subset of X and $F: X \to \mathcal{F}(L(X,Y))$ a fuzzy mapping such that there exists a real number $\beta \in (0,1]$ such that for each $x \in X, (F_x)_\beta$ is a nonempty subset of $L(X,Y)$. Suppose that F is C-pseudomonotone and hemicontinuous, and that for each $x \in F$, F_x is a compact fuzzy set in $L(X,Y)$. Then there exists an $x_0 \in D$ such that for each $x \in D$, there exists an $s_0 \in L(X,Y)$ with $F_{x_0}(s_0) \geq \beta$ such that $\langle s_0, x-x_0\rangle \notin -Int\,C$.

COROLLARY 3.3 [12]. Let X be a reflexive Banach space and Y a Banach space. Let D be a nonempty, bounded, closed and convex subset of X and C a closed, pointed and convex cone in Y with $Int\,C \neq \emptyset$. Let $F: X \to \mathcal{F}(L(X,Y))$ be a fuzzy mapping such that F is C-pseudomonotone and hemicontinuous and that for each $x \in X$, F_x is a compact fuzzy set in $L(X,Y)$. Suppose further that there exists a real number $\beta \in (0,1]$ such that for each $x \in X$, $(F_x)_\beta$ is a nonempty subset of $L(X,Y)$. Then there exists an $x_0 \in D$ such that for each $x \in D$, there exists an $s_0 \in L(X,Y)$ with $F_{x_0}(s_0) \geq \beta$ such that $\langle s_0, x-x_0\rangle \notin -Int\,C$.

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