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ACOUSTIC-GRAVITY WAVES IN A VISCOUS AND THERMALLY CONDUCTING ISOTHERMAL ATMOSPHERE

(Part III: For Arbitrary Prandtl Number)

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ABSTRACT. In this paper we will investigate the combined effect of Newtonian cooling, viscosity and thermal condition on upward propagating acoustic waves in an isothermal atmosphere. In part one of this series we considered the case of large Prandtl number, while in part two we investigated the case of small Prandtl number In those parts we examined only the limiting cases, i e. the cases of small and large Prandtl number, and it is more interesting to consider the case of arbitrary Prandtl number, which is the subject of this paper, because it is a better representative model. It is shown that if the Newtonian cooling coefficient is small compared to the frequency of the wave, the effect of the thermal conduction is dominated by that of the viscosity. Moreover, the solution can be written as a linear combination of an upward and a downward propagating wave with equal wavelengths and equal damping factors On the other hand if Newtonian cooling is large compared to the frequency of the wave the effect of thermal conduction will be eliminated completely and the atmosphere will be transformed from the adiabatic form to an isothermal. In addition, all the linear relations among the perturbations quantities will be modified. It follows from the above conclusions and those of the first two parts, that when the effect of Newtonian cooling is negligible thermal conduction influences the propagation of the wave only in the case of small Prandtl number.

KEY WORDS AND PHRASES: Acoustic-Gravity Waves, Atmospheric Waves, Wave Propagation
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1. INTRODUCTION

It is well known that upward propagating acoustic waves of small amplitude may be reflected downward if the Brunt-*Väisälä* frequency varies with altitude. However, even when the Brunt-*Väisälä* frequency is constant, additional reflection is possible because of the exponential decrease of the density with height. This type of reflection is most important when the wavelength is large compared to the density scale height.

The reflection properties of a viscous isothermal atmosphere were examined by Yanowitch [27], Alkahby and Yanowitch [3], Campos [14]. It was shown that the viscosity creates a transition region, which connect two distinct regions and acts like an absorbing and reflecting barrier In the lower region the effect of the kinematic viscosity is negligible and the solution can be written, for frequencies greater than the adiabatic cutoff frequency, as a linear combination of an upward and a downward propagating wave In the upper region the effect of the kinematic viscosity is large and the solution decays exponentially with altitude to a constant value.

The presence of thermal conduction also produces a reflecting layer, with different mechanism from that of the viscosity. The exponential increase of the thermal diffusivity with height creates a semitransparent layer allowing part of the energy to propagate upward As a result, the reflecting layer separates two distinct regions with different sound speeds, because the signals propagate with Newtonian sound speed in the isothermal region. Consequently, the wavelengths in the two regions are different and this will account for the reflection (Alkahby [7], Alkahby and Yanowitch [3,4], Lyons and Yanowitch [18]).

The combined effect of Newtonian cooling, viscosity and thermal conduction, for large Prandtl number, is investigated in Alkahby [8]. It was shown that the effect of thermal conduction can be excluded and the solution, above the reflecting layer that is created by the viscosity, decays exponentially with altitude before it is influenced by the effect of thermal conduction. Moreover, when the Newtonian cooling coefficient is large compared to the frequency of the wave the lower region will be transformed from the adiabatic form to the isothermal one. The effect of Newtonian cooling, viscosity and thermal conduction, for small Prandtl number, on upward propagating acoustic waves in an isothermal atmosphere is investigated by Alkahby [9]. It was shown that when the Newtonian cooling coefficient is small compared to the frequency of the wave the atmosphere may be divided into three distinct regions connected by two different reflecting layers In the lower region the oscillatory process is approximately adiabatic, it is isothermal in the middle region and in the upper region the solution will decay exponentially with altitude. On the other hand if the Newtonian cooling coefficient is large compared to the frequency of the wave the oscillatory process in the lower region will be transformed to an isothermal one As a result, the two lower regions become one because the reflecting layer, which is created by thermal conduction, will be eliminated.

In the above two limiting cases, which were discussed in part I and part II of this series, our conclusions are presumed and it is more important to consider the case of the effect of Newtonian cooling, viscosity and thermal conduction for arbitrary Prandtl number because this case is a more representative model for the reflection and dissipation of acoustic waves in an isothermal atmosphere. It is shown that the atmosphere can be divided into two distinct regions connected by a reflecting and absorbing layer. when the Newtonian cooling coefficient is small compared to the frequency of the wave, the oscillatory process in the lower region is adiabatic and above the reflecting barrier the solution will decay exponentially with altitude. When the Newtonian cooling coefficient is large compared to the frequency of the wave, the atmosphere will be transformed to an isothermal one and the effect of thermal conduction will be eliminated completely while the influence of the viscosity will remain the same. Consequently, thermal conduction influences the reflection and dissipation of the acoustic wave only in the case of small Prandtl number and the effect of Newtonian cooling is negligible. Moreover, the atmosphere can only be divided into three distinct regions in the case of small Prandtl number and negligible effect of Newtonian cooling. It is shown that if the Newtonian cooling coefficient is large compared to the adiabatic cutoff frequency, it will act directly to eliminate the temperature perturbation quantity associated with the wave in a time which is small compared to the period of oscillations Since Newtonian cooling adds an additional term to the linearized equation of the energy, damping modifies all linear relations among perturbation quantities. In particular, it causes attenuation in the amplitude of the wave and thereby the energy flux as well Also the attenuation in the amplitude of the wave will vanish not only when Newtonian cooling is eliminated but also when Newtonian cooling becomes large compared to the adiabatic cutoff frequency. The reflection coefficient and the damping factors are obtained and the conclusions are discussed in connection with the heating of the solar atmosphere

2. MATHEMATICAL FORMULATION OF THE PROBLEM

In this section we will indicate the main steps of the formulation of the problem and the details can be found in Part I or Part II. Suppose that an isothermal atmosphere, which is viscous and thermally conducting, and occupies the upper half-space $z > 0$ We will investigate the problem of small vertical oscillations about equilibrium, i.e. oscillations which depend only on the time t and on the vertical coordinate z.

Let the equilibrium pressure, density and temperature be denoted by P_0, ρ_0, and T_0, where P_0 and T_0 satisfy the gas law $P_0 = RT_0\rho_0$ and the hydrostatic equation $P_0' + g\rho_0 = 0$. Here R is the gas constant, g is the gravitational acceleration and the prime denotes differentiation with respect to z The equilibrium pressure and density,

$$P_0(z) = P_0(0)\exp(-1/H), \quad \rho_0(z) = \rho_0(0)\exp(-z/H),$$

where $H = RT_0/g$ is the density scale height

Let p, ρ, w, and T be the perturbations in the pressure, density, vertical velocity, and temperature. The linearized equations of motion (conservation of momentum and mass, the heat flow equation and the gas law) are

$$\rho_0 w_t + p_z + g\rho = (4/3)\mu w_{zz}, \tag{2 1}$$
$$\rho_t + (\rho_0 w)_z = 0, \tag{2.2}$$
$$\rho_0(c_V(T_t + qT) + gHw_z) = \kappa T_{zz}, \tag{2.3}$$
$$p = R(\rho_0 T + T_{0\rho}). \tag{2.4}$$

Here c_V is the specific heat at constant volume, μ is the dynamic viscosity coefficient, q is the Newtonian cooling coefficient which refers to the heat exchange and κ is the thermal conductivity, all assumed to be constants. The subscript z and t denote differentiation with respect to z and t respectively Equation (2.4) includes the heat flux term $c_V\rho_0 qT$, which comes from the linearized form of the Stefan-Boltzmann law We will consider solutions which are harmonic in time, i.e

$$w(z,t) = W(z0\exp(-i\omega t), \quad T(z,t) = T(z)\exp(-i\omega t), \tag{2 5}$$

where w denotes the frequency of the wave.

It is more convenient to rewrite the equations in dimensionless form; $z^* = z/H$, $w_a = c/2H, W^* = w/c$, $\omega^* = \omega/\omega_a$, $t^* = t\omega_a$, $\kappa^* = 2\kappa/c_V cH\rho_0(0)$, $T^* = T/2\gamma T_0$, $q^* = q/\omega_a$, where $c = \sqrt{\gamma RT_0} = \sqrt{\gamma gH}$ is the adiabatic sound speed, and ω_a is the adiabatic cutoff frequency. The primes can be omitted, since all variables will be written in dimensionless form from now on

One can eliminate p, w and ρ from equation (2.1) by differentiating it with respect to t, then with respect to z and substituting equations (2.2-2.5) to obtain a single fourth-order differential equation for $T(z)$ only:

$$\begin{aligned}&\big[(D^2 - D + \tau\omega^2/4) - i(\kappa/m)e^z D^2(D^2 + D + \gamma\omega^2/4)\\ &- iP_r m\tau(\kappa/m)e^z D(D+1) - (\gamma P_r m)(\kappa e^z/m)^2 D^2(D+1)(D+2)\big]T(z) = 0.\end{aligned} \tag{2.6}$$

where $\tau = \gamma(\omega + iq)/(\gamma\omega + iq) = \gamma(\omega + iq)/m$, $m = \gamma\omega + iq$, $D = d/dz$ and $P_r = \mu/\kappa$

BOUNDARY CONDITIONS: To complete the formulation of the problem certain boundary conditions must be imposed to ensure a unique solution. Physically relevant solutions must satisfy the following two conditions (Alkahby [7], Alkahby and Yanowitch [4], Lyons and Yanowitch [18])

$$\mu\int_0^\infty |W_z|^2 dz < \infty, \tag{2 7}$$

$$\kappa \int_0^\infty |T_z|^2 dz < \infty. \tag{2 8}$$

The first of these is the dissipation (DC), which follows from the finiteness of the energy dissipation rate in a column of fluid of a unit cross-section. The second one, the entropy condition (EC), is a consequence of the finiteness of the entropy growth rate in a column of fluid of a unit cross section

Boundary conditions are required at $z = 0$, and we shall adopt the lower boundary condition (LBC) In a fixed interval $0 < z < z_0$, the solution of the differential equation (2.10) should approach some solution of the limiting differential equation ($\kappa \to 0$ and $\mu \to 0$), i.e the solution can be written in the form

$$T(z) = \text{Const.}\left[\exp\left[\left(1+\sqrt{1-\tau\omega^2}\right)z/2\right] + K_q \exp\left[\left(\sqrt{1-\tau\omega^2}z/2\right)\right]\right], \tag{2.9}$$

where K_q is a constant. Considering the lower boundary condition is simpler than prescribing $T(z)$ and $W(z)$ at $z = 0$ because we avoid the computation of the boundary layer which has no effect on the reflection and dissipation processes that take place at high altitudes.

3. THE EFFECT OF NEWTONIAN COOLING ALONE

In this section we will review the effect of Newtonian cooling alone on the wave propagation to make the paper more self contained. Also the results of this section are needed for the results and the analysis of section (4). For this case, the differential equation can be obtained by setting $\kappa = \mu = 0$ in the differential equation (2.10). The resulting differential equation is

$$\left[D^2 - D + \tau\omega_2/4\right]T(z) = 0. \tag{3.1}$$

where $\tau = \gamma(\omega + iq)/m$, and $m = \gamma\omega + iq$. The solution of the differential equation (3.1) can be written in the following form

$$T(z) = c_1 \exp\left[\left(1+\sqrt{1-\tau\omega_2}\right)z/2\right] + c_2 \exp\left[\left(1-\sqrt{1-\tau\omega_2}\right)z/2\right], \tag{3.2}$$

where c_1 and c_2 are constants and they will be determined from the boundary condition To examine the effect of Newtonian cooling on the wave propagation and dissipation, let

$$\left(\sqrt{1-\tau\omega_2}\right)/2 = \pm\left(-d(q,\omega) + i\beta\right)). \tag{3.3}$$

To investigate the behavior of $d(q,\omega)$ and β, we have to study the following cases

(N1) When $q = 0$ and $\omega > 1$, the solution of the differential equation (3.1), defined in equation (3.2), becomes a linear combination of an upward and a downward propagating wave with equal adiabatic wave number $\beta_a = \left(\sqrt{\omega_2 - 1}\right)/2$ and $d(q,\omega) = 0$.

(N2) When $\omega/q \to 0$ and $\omega > 1/\sqrt{\gamma}$, equation (3.2) which defines the solution of the differential q (3.1) can be written as a linear combination of an upward and downward traveling wave with equal isothermal wave number $\beta_i = \left(\sqrt{\gamma\omega_2 - 1}\right)/2$ and $d(q,\omega) = 0$. This can easily be seen from limit of τ as $q \to \infty$.

(N3) When $q/\omega \ll 1$ the solution of the problem can be described as follows: the first term on the right of equation (3.2) will be an upward propagating wave decaying exponentially like $\exp(-d(q,\omega)z)$ and the second term is a downward traveling wave decaying in the same rate

(N4) As a result of (N1) and (N2), the damping factor, $d(q,\omega)$ becomes zero not only when the effect of Newtonian cooling is eliminated but also when the Newtonian cooling coefficient becomes large compared to the adiabatic cutoff frequency of the wave. Also the wave number β increases from the adiabatic value β_a to the isothermal β_i. At the same time the oscillatory process changes from the adiabatic form to the isothermal one.

(N5) As a result of the above discussion we have three ranges for the frequency of the wave. above the adiabatic cutoff frequency ω_a, below the isothermal cutoff frequency ω_i, and between ω_a and ω_i

(N6) When the frequency of the wave is greater than the adiabatic cutoff frequency the damping factor $d(q,\omega)$ is positive and equals zero at the extreme limits, i e. when the Newtonian cooling coefficient equals zero and when it is large compared to the adiabatic cutoff frequency The damping factor increases to its maximum value, $d(q,\omega) = 0.1$, when $(q/\omega) = O(1)$ and decays to zero as $q \to 0$

4. SOLUTION OF THE PROBLEM

In this section we will investigate the singular perturbation boundary value problem for the following differential equation

$$\begin{aligned}&\big[(D^2 - D + \tau\omega^2/4) - i(\kappa/m)e^z D^2(D^2 + D + \gamma\omega^2/4)\\ &- iP_r m\tau(\kappa/m)e^z D(D+1) - (\gamma P_r m)(\kappa e^z/m)^2 D^2(D+1)(D+2)\big]T(z) = 0, \qquad (4.1)\end{aligned}$$

where $\tau = \gamma(\omega + iq)/(\gamma\omega + iq) = \gamma(\omega + iq)/m$ and $m = \gamma\omega + iq$, subjected to the boundary condition (2.7), (2.8), and the lower boundary condition. At the outset we have to indicate that the parameters μ and κ are sufficiently small and proportional to the values at $z = 0$ of the kinematic viscosity and thermal diffusivity Prandtl number P_r can be written as

$$P_r = \mu/\kappa = (\mu/\rho_0)/(\kappa/\rho_0) = |(\mu/m\rho_0)/(\kappa/m\rho_0)|\,. \qquad (4.2)$$

It is clear that Prandtl number P_r measures the relative strength of the viscosity with respect to thermal conduction. As a result, small Prandtl number means that thermal conduction dominates the oscillatory process and large Prandtl number indicates that the viscosity dominates the motion. For small Prandtl number the atmosphere may be divided into three distinct regions because thermal conduction creates a semitransparent reflecting layer. In the case of large Prandtl number the atmosphere may be divided into two different regions connected by an absorbing and reflecting layer. For arbitrary Prandtl number the reflection and dissipation process depends mainly on the viscosity and Newtonian cooling. To obtain the solution of the differential equation (4.1) it is convenient to introduce a new independent dimensionless variable ξ defined by

$$\xi = \exp(-z)/(i\kappa/m) = \exp[-z - \log|\kappa/m| + i\theta_m + 3\pi i/2], \qquad (4\ 3)$$

where $\theta_m = \arg(m)$. As a result, the differential equation (4.1) becomes

$$\begin{aligned}&\big[\xi^2(\theta_2 + \theta + \tau\omega^2/4) - \xi\theta_2(\theta_2 - \theta + \gamma\omega^2/4)\\ &- mP_r\tau\xi(\theta^2 - \theta) + \gamma P_r m\theta^2(\theta - 1)(\theta - 2)\big]T(\xi) = 0. \qquad (4\ 4)\end{aligned}$$

where $\theta = \xi d/d\xi$. The point $\xi = 0$ is a regular singular point of this differential q
(4.4). Consequently, there are four linearly independent solutions, which in the neighborhood of $\xi = 0$ can be written in the following form

$$T_1(\xi) = \sum a_n(e_1)\xi^{n+e_1}, \quad T_2(\xi) = \sum a_n'(e_2)\xi^{n+e_2} + T_1(\xi)\log(\xi),$$

$$T_3(\xi) = \sum a_n''(e_3)\xi^{n+e_3}, \quad T_4(\xi) = \sum a_n'''(e_4)\xi^{n+e_4} + T_3(\xi)\log(\xi), \qquad (4\ 5)$$

where $e_1 = 2$, $e_2 = 1$, $e_3 = e_4 = 0$ The prime denotes differentiation of a_n and the sums are taken from $n = 0$ to $n = \infty$. The coefficients $a_n(e_i)$ are determined from the following three term recursion formula

$$p_0(n+2+e)a_{n+2} + p_1(n+1+e)a_{n+1} + p_2(n+e)a_n = 0, \qquad (4.6)$$

where

$$p_0(y) = \gamma P_r m y^2 (y-1)(y-2),$$
$$p_1(y) = -m\tau P_r y(y-1) - y^2(y^2 - y + \gamma\omega^2/4),$$
$$p_2(y) = (y^2 + y + \tau\omega^2/4). \tag{4 7}$$

Following the same procedure as in Part II (Alkahby [9]), the solution of the differential equation, which satisfies the prescribed boundary conditions can be written in the following form

$$T(z) = c_1 T_1(z) + c_2 T_3(z). \tag{4 8}$$

To determine the linear combination of $T(z)$ in equation (4.8), the behavior of $T_1(z)$ and $T_3(z)$ for small z must be found Since small z corresponds to large $|\xi|$ with $\arg(\xi) = 3\pi/2 + \theta_m$, the asymptotic expansions of $T_1(\xi)$ and $T_3(\xi)$ about infinity should be found. The differential equation (4 8) is similar to equation (26) in Alkahby [10]. Since the calculations are similar to those in Alkahby [10], we will omit them and merely indicate the results Although these problems are mathematically similar, the physical conclusions are completely different. In Alkahby [8,9], the differential equation (4 4) is solved, only for small and for large Prandtl number, by matching inner and outer approximations in an overlapping domain. The matching procedure reduces the three terms recursion formula, equation (4.6), to a two term one. This will simplify the computations for obtaining the asymptotic behavior of the solution defined in equation (4.8). For arbitrary Prandtl number the solution of the problem by Laplace integration (Alkahby [10]). It follows that the solution of the differential equation (4.8) for arbitrary Prandtl number can be written in the following form

$$T(z) \sim \text{Const.}\left[\exp[(1/2 - d(q,\omega) + i\beta)z] + K\exp[(1/2 + d(q,\omega) - i\beta)z]\right], \tag{4 9}$$

where K denotes the reflection coefficient and defined by

$$K = U(d(q,\omega,\beta,\beta_\imath)\exp[L_1 - iL_2], \tag{4 10}$$

$$U = \left[\frac{\Gamma(\alpha_1-\alpha_2)\Gamma^2(\alpha_2+1)\Gamma(\alpha_2-\beta_1)\Gamma(\alpha_2-\beta_2)}{\Gamma(\alpha_2-\alpha_1)\Gamma^2(\alpha_1+1)\Gamma(\alpha_1-\beta_1)\Gamma(\alpha_2-\beta_2)}\right],$$
$$L_1 = -\pi\beta + 2d(q,\omega(\ln(|\kappa/m|) - 2\beta\theta_m,$$
$$L_2 = 2\beta\ln(|\kappa/m|) + \pi d(q,\omega) + 2d(q,\omega)\theta_m,$$
$$\alpha_1 - 1/2 - d(q,\omega) + i\beta, \quad \alpha_2 - 1/2 + d(q,\omega) - i\beta,$$
$$\beta_1 = -1/2 + i\beta_\imath, \quad \beta_2 = -1/2 - i\beta_\imath.$$

When Newtonian cooling is eliminated we retain the results of Alkahby [10].

From the above results and discussion, we have the following conclusions

[I] All the conclusions of section (3) can be restated in this section. In addition, we indicate the following observations.

[II] Equation (4.9) represents the behavior of the solution below the reflecting layer. It indicates that the oscillatory process, below the reflecting layer, depends on the effect of Newtonian cooling and it will be changed to an isothermal if the Newtonian cooling coefficient is large compared to the adiabatic cutoff frequency of the wave.

[III] If the Newtonian cooling coefficient is large compared to the adiabatic cutoff frequency of the wave, it will act directly to eliminate the temperature perturbation quantity in a time which is small compared to a period of oscillations. This can easily be seen from equation (2.3) where $T/w = O(1/q)$. Thus, as $q \to \infty$ the temperature perturbation vanishes, $\tau \to \gamma$ and the equation for $W(z)$ reduced to

$$(1 - i\delta\exp(z))D^2W(z) - DW(z) + (\gamma\omega^2/4)W(z) = 0, \tag{4.11}$$

where $\delta = 2\gamma\omega\mu/3cH\rho_0(0)$. the equation for T is superfluous since $\kappa = 0$. Moreover, the equation for $W(z)$ can be transformed to the hypergeometric differential equation which has two linearly independent solutions. One solution will be eliminated by the dissipation condition because it increases linearly with z The second solution has the following asymptotic form

$$W(z) \sim \frac{1}{1+K_\mu}[\exp[(1/2+i\beta_1)z + K_\mu \exp[(1/2-i\beta_1)z]], \tag{4 12}$$

where K_μ denotes the reflection coefficient and defined by

$$K_\mu = \exp(-i\beta_1)[\cos\theta + isin\theta], \tag{4 12}$$

$$\theta = 2\arg\Gamma(2i\beta_1) - 4\arg\Gamma(1/2 + i\beta_1 0.$$

It is clear that the magnitude of the reflection coefficient $|K_\mu| = \exp(-i\beta_1)$ Moreover, if $q = 0$the magnitude of the reflection coefficient will be $\exp(-i\beta)$.

[IV] It follows from [III] and the results, which were obtained in Alkahby [8,9], that the effect of thermal conduction on the reflection and dissipation of the wave will be eliminated if the heat exchange between the hotter and cooler region in the atmosphere is intense and the oscillatory process is transformed from the adiabatic form to the isothermal one.

[V] The reflected wave, from the reflecting layer, will be reflected upward at $z = 0$. the reflection of the waves and the dissipation of the energy will continue until the energy of the waves dissipates completely. The dissipated energy of the acoustic waves may contribute to the process of the heating of the solar atmosphere.

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