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Research Article

Univalence of a New General Integral Operator Associated with the q-Hypergeometric Function

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Motivated by the familiar q-hypergeometric functions, we introduce a new family of integral operators and obtain new sufficient conditions of univalence criteria. Several corollaries and consequences of the main results are also pointed out.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} c_n z^n, \quad c_n \geq 0, \tag{1}$$

which are analytic in the open unit disk $\mathcal{U} = \{z \in \mathbb{C} : |z| < 1\}$, and $\mathcal{S}$ the class of functions $f \in \mathcal{A}$ which are univalent in $\mathcal{U}$.

Let $f, g \in \mathcal{A}$, where f is defined by (1) and g is given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \quad b_n \geq 0. \tag{2}$$

Then the Hadamard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} c_n b_n z^n. \tag{3}$$

For complex parameters a_i, b_j, and q ($i = 1, \ldots, r$, $j = 1, \ldots, s$, $b_j \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, $|q| < 1$), we define the q-hypergeometric function ${}_r\Phi_s(a_1, \ldots, a_r; b_1, \ldots, b_s; q, z)$ by

$${}_r\Phi_s(a_i; b_j; q, z) = \sum_{n=0}^{\infty} \frac{(a_1, q)_n \cdots (a_r, q)_n}{(q, q)_n (b_1, q)_n \cdots (b_s, q)_n} z^n \tag{4}$$

($r = s + 1$; $r, s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$; $z \in \mathcal{U}$), where $\mathbb{N}$ denotes the set of positive integers and $(a, q)_n$ is the q-shifted factorial defined by

$$(a, q)_n = \begin{cases} 1, & n = 0; \\ (1 - a)(1 - aq)(1 - aq^2) \cdots (1 - aq^{n-1}), & n \in \mathbb{N}. \end{cases} \tag{5}$$

By using the ratio test, we should note that, if $|q| < 1$, the series (4) converges absolutely for $|z| < 1$ if $r = s + 1$. For more mathematical background of these functions, one may refer to [1].

Corresponding to the function defined by (4), consider

$${}_r\mathcal{G}_s(a_i; b_j; q, z) = z \quad {}_r\Phi_s(a_i; b_j; q, z). \tag{6}$$

Recently, the authors [2] defined the linear operator $\mathcal{M}(a_i, b_j; q) f : \mathcal{A} \to \mathcal{A}$ by

$$\begin{aligned} \mathcal{M}(a_i, b_j; q) f(z) &= {}_r\mathcal{G}_s(a_i; b_j; q, z) * f(z) \\ &= z + \sum_{n=2}^{\infty} \Upsilon_n c_n z^n, \end{aligned} \tag{7}$$

where

$$\Upsilon_n = \frac{(a_1, q)_{n-1} \cdots (a_r, q)_{n-1}}{(q, q)_{n-1} (b_1, q)_{n-1} \cdots (b_s, q)_{n-1}}, \quad (|q| < 1). \tag{8}$$

It should be remarked that the linear operator (7) is a generalization of many operators considered earlier. For $a_i = q^{\alpha_i}$, $b_j = q^{\beta_j}$, α_i, $\beta_j \in \mathbb{C}$, $\beta_j \neq 0, -1, -2, \ldots, (i = 1, \ldots, r,\ j = 1, \ldots, s)$, and $q \to 1$, we obtain the Dziok-Srivastava linear operator [3] (for $r = s + 1$), so that it includes (as its special cases) various other linear operators introduced and studied by Ruscheweyh [4], Carlson and Shaffer [5] and the Bernardi-Libera-Livingston operators [6–8].

The q-difference operator is defined by

$$d_q h(z) = \frac{h(qz) - h(z)}{(q-1)z}, \quad q \neq 1,\ z \neq 0,$$
$$\lim_{q \to 1} d_q h(z) = h'(z), \tag{9}$$

where $h'(z)$ is the ordinary derivative. For more properties of d_q see [9, 10].

Lemma 1 (see [2]). *Let $f \in \mathcal{A}$; then*

(i) *for $r = 1$, $s = 0$, and $a_1 = q$, one has $\mathcal{M}(q, -; q) f(z) = f(z)$.*

(ii) *For $r = 1, s = 0$, and $a_1 = q^2$, one has $\mathcal{M}(q^2, -; q) f(z) = z d_q f(z)$ and $\lim_{q \to 1} \mathcal{M}(q^2, -; q) f(z) = z f'(z)$, where d_q is the q-derivative defined by* (9).

Definition 2. A function $f \in \mathcal{A}$ is said to be in the class $\mathfrak{B}_s^r(a_i, b_j; q; \mu)$ if it is satisfying the condition

$$\left| \frac{z^2 \left(\mathcal{M}(a_i, b_j; q) f(z) \right)'}{\left[\mathcal{M}(a_i, b_j; q) f(z) \right]^2} - 1 \right| < 1 - \mu \quad (z \in \mathcal{U};\ 0 \leq \mu < 1), \tag{10}$$

where $\mathcal{M}(a_i, b_j; q) f$ is the operator defined by (7).

Note that $\mathfrak{B}_0^1(q, -; q; \mu) = \mathfrak{B}(\mu)$, where the class $\mathfrak{B}(\mu)$ of analytic and univalent functions was introduced and studied by Frasin and Darus [11].

Using the operator $\mathcal{M}(a_i, b_j; q) f(z) f$, we now introduce the following new general integral operator.

For $m \in \mathbb{N} \cup \{0\}$, $\gamma_1, \gamma_2, \ldots, \gamma_m$, $\delta \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, and $|q| < 1$, we define the integral operator $I_{\gamma_k, \delta}(a_i, b_j; q; z) : \mathcal{A}^n \to \mathcal{A}^n$ by

$$I_{\gamma_k, \delta}(a_i, b_j; q; z) = \left(\delta \int_0^z t^{\delta - 1} \prod_{k=1}^m \left(\frac{\mathcal{M}(a_i, b_j; q) f(z) f_k(t)}{t} \right)^{1/\gamma_k} dt \right)^{1/\delta}, \tag{11}$$

where $f_k \in \mathcal{A}$.

Remark 3. It is interesting to note that the integral operator $I_{\gamma_k, \delta}(a_i, b_j; q; z)$ generalizes many operators introduced and studied by several authors, for example,

(1) for $r = s + 1$, $a_i = q^{\alpha_i}$, $b_j = q^{\beta_j}$, $i = 1, \ldots, r$, $j = 1, \ldots, s$, $q \to 1$, $\gamma_k = 1/(\alpha - 1)$, and $\delta = 1 + m(\alpha - 1)$, where $\alpha \in \mathbb{C}$ and $\mathfrak{R}(\alpha) > 0$, we obtain the following integral operator introduced and studied by Selvaraj and Karthikeyan [12]:

$$F_\alpha(\alpha_1, \beta_1; z) = \left(1 + m(\alpha - 1) \int_0^z \left(H_s^r(\alpha_1, \beta_1) f_1(t) \right)^{\alpha - 1} \cdots \left(H_s^r(\alpha_1, \beta_1) f_m(t) \right)^{\alpha - 1} dt \right)^{1/(1 + m(\alpha - 1))}, \tag{12}$$

where for convenience $H_s^r(\alpha_1, \beta_1) f := H(\alpha_1, \ldots, \alpha_r; \beta_1, \ldots, \beta_s; z) f(z)$, and $H_s^r(\alpha_1, \beta_1) f(z) = z + \sum_{n=2}^{\infty} ((\alpha_1)_{n-1} \cdots (\alpha_r)_{n-1} / (\beta_1)_{n-1} \cdots (\beta_s)_{n-1} (n-1)!) a_n z^n$ is the Dziok-Srivastava operator [3].

(2) For $r = 1$, $s = 0$, $a_1 = q$, $\gamma_k = 1/(\alpha - 1)$, and $\delta = 1 + m(\alpha - 1)$, we obtain the integral operator

$$F_{m,\alpha}(z) = \left(1 + m(\alpha - 1) \times \int_0^z (f_1(t))^{\alpha - 1} \cdots (f_m(t))^{\alpha - 1} dt \right)^{1/(1 + m(\alpha - 1))} \tag{13}$$

studied recently by Breaz et al. [13].

(3) For $r = 1$, $s = 0$, $a_1 = q$, $\gamma_k = 1/\alpha_k$, and $\delta = 1$, we obtain the integral operator

$$F_\alpha(z) = \int_0^z \left(\frac{f_1(t)}{t} \right)^{\alpha_1} \cdots \left(\frac{f_m(t)}{t} \right)^{\alpha_m} dt \tag{14}$$

introduced and studied by D. Breaz and N. Breaz [14].

(4) For $r = 1$, $s = 0$, $a_1 = q^2$, $\gamma_k = 1/(\alpha - 1)$, and $\delta = 1 + m(\alpha - 1)$, we obtain the integral operator

$$G_\alpha(z) = \left(1 + m(\alpha - 1) \times \int_0^z t^{m(\alpha - 1)} \left(f_1'(t) \right)^{\alpha - 1} \cdots \left(f_m'(t) \right)^{\alpha - 1} dt \right)^{1/(1 + m(\alpha - 1))} \tag{15}$$

introduced by Selvaraj and Karthikeyan [12].

(5) For $r = 1$, $s = 0$, $a_1 = q^2$, $\gamma_k = 1/\alpha$, and $\delta = 1$, we obtain the integral operator

$$G_\alpha(z) = \int_0^z \left(f_1'(t) \right)^{\alpha} \cdots \left(f_m'(t) \right)^{\alpha} dt, \tag{16}$$

recently introduced and studied by Breaz and Güney [15].

(6) For $r = 1$, $s = 0$, $a_1 = q$, $f_1 = \cdots = f_m = f \in \mathcal{A}$, $\gamma_k = 1/(\alpha - 1)$, and $\delta = \alpha$, where $\alpha \in \mathbb{C}$ and $\mathfrak{R}(\alpha) > 0$, we obtain the integral operator

$$G_\alpha(z) = \left(\alpha \int_0^z (f(t))^{\alpha - 1} \right)^{1/\alpha} dt, \tag{17}$$

introduced and studied by Pescar [16].

In order to derive our main results, we have to recall the following univalence criteria.

Lemma 4 (see [17, 18]). *Let $\delta \in \mathbb{C}$ with $\mathrm{Re}(\delta) > 0$. If $f \in \mathscr{A}$ satisfies*

$$\frac{1-|z|^{2\,\mathrm{Re}(\delta)}}{\mathrm{Re}(\delta)}\left|\frac{zf''(z)}{f'(z)}\right| \le 1, \tag{18}$$

for all $z \in \mathscr{U}$, then the integral operator

$$F_\delta(z) = \left\{\delta \int_0^z t^{\delta-1} f'(t)\,dt\right\}^{1/\delta} \tag{19}$$

is in the class $\mathscr{S}$.

Lemma 5 (see [16]). *Let $\delta \in \mathbb{C}$ with $\mathrm{Re}(\delta) > 0$, $c \in \mathbb{C}$, with $|c| \le 1$, $c \ne -1$. If $f \in \mathscr{A}$ satisfies*

$$\left|c|z|^{2\delta} + \left(1-|z|^{2\delta}\right)\frac{zf''(z)}{\delta f'(z)}\right| \le 1, \tag{20}$$

for all $z \in \mathscr{U}$ then the integral operator

$$F_\delta(z) = \left\{\delta \int_0^z t^{\delta-1} f'(t)\,dt\right\}^{1/\delta} \tag{21}$$

is in the class $\mathscr{S}$.

Lemma 6 (Generalized Schwarz Lemma, see [19]). *(Generalized Schwarz Lemma) Let the function f be analytic in the disk $\mathscr{U}_R = \{z : |z| < R\}$, with $|f(z)| < M$ for fixed M. If $f(z)$ has one zero with multiplicity order bigger that m for $z = 0$, then*

$$|f(z)| \le \frac{M}{R^m}|z|^m, \quad (z \in \mathscr{U}_R). \tag{22}$$

Equality can hold only if

$$f(z) = e^{i\theta}\left(\frac{M}{R^m}\right)z^m, \tag{23}$$

where θ is constant.

2. Univalence Conditions for $I_{\gamma_k,\delta}(a_i,b_j;q;z)$

Theorem 7. *Let $f_k \in \mathscr{A}$ for all $k = 1,\dots,m$, $\gamma_k \in \mathscr{C}$, and $M \ge 1$ with*

$$\frac{1}{\mathrm{Re}(\delta)}\sum_{k=1}^{m}\frac{[(2-\mu_k)M+1]}{|\gamma_k|} \le 1. \tag{24}$$

If for all $k = 1,\dots,m$, $f_k \in \mathfrak{B}_s^r(a_i,b_j,q,\mu_k)$, $0 \le \mu_k < 1$, and

$$\left|\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right| \le M, \quad (z \in \mathscr{U}) \tag{25}$$

then the integral operator $I_{\gamma_k,\delta}(a_i,b_j;q;z)$ defined by (11) is analytic and univalent in $\mathscr{U}$.

Proof. From the definition of the operator $\mathscr{M}(a_i,b_j;q)f(z)f$ it can be observed that

$$\frac{\mathscr{M}(a_i,b_j;q)\,f(z)}{z} \ne 0, \quad (z \in \mathscr{U}), \tag{26}$$

and for $z = 0$, we have

$$\left(\frac{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_1(z)}{z}\right)^{1/\gamma_1} \cdots \left(\frac{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_m(z)}{z}\right)^{1/\gamma_m} = 1. \tag{27}$$

We define the function $h(z)$ by the form

$$h(z) = \int_0^z \prod_{k=1}^{m}\left(\frac{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(t)}{t}\right)^{1/\gamma_k} dt. \tag{28}$$

Therefore

$$h'(z) = \prod_{k=1}^{m}\left(\frac{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)}{z}\right)^{1/\gamma_k}. \tag{29}$$

Differentiating logarithmically and multiplying by z on both sides of (29)

$$\frac{zh''(z)}{h'(z)} = \sum_{k=1}^{m}\frac{1}{\gamma_k}\left(\frac{z\left(\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right)'}{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)} - 1\right). \tag{30}$$

Thus we have

$$\left|\frac{zh''(z)}{h'(z)}\right| \le \sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left|\frac{z\left(\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right)'}{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)} - 1\right|. \tag{31}$$

So

$$\begin{aligned}
&\frac{1-|z|^{2\,\mathrm{Re}(\delta)}}{\mathrm{Re}(\delta)}\left|\frac{zh''(z)}{h'(z)}\right| \\
&\le \frac{1-|z|^{2\,\mathrm{Re}(\delta)}}{\mathrm{Re}(\delta)} \times \left[\sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left(\left|\frac{z\left(\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right)'}{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)}\right| + 1\right)\right] \\
&\le \frac{1-|z|^{2\,\mathrm{Re}(\delta)}}{\mathrm{Re}(\delta)} \times \left[\sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left(\left|\frac{z^2\left(\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right)'}{\left[\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)\right]^2}\right| \times \left|\frac{\mathscr{M}(a_i,b_j;q)\,f(z)\,f_k(z)}{z}\right| + 1\right)\right].
\end{aligned} \tag{32}$$

Since $|\mathscr{M}(a_i, b_j; q) f(z) f_k(z)| \leq M$, $(z \in \mathscr{U}, k = 1, \ldots, m)$, and $f_k \in \mathfrak{B}_s^r(a_i, b_j, q, \mu_k)$ for all $k = 1, \ldots, m$, then from the Schwarz Lemma and (10), we obtain

$$\begin{aligned}
&\frac{1-|z|^{2\operatorname{Re}(\delta)}}{\operatorname{Re}(\delta)}\left|\frac{zh''(z)}{h'(z)}\right| \\
&\leq \frac{1-|z|^{2\operatorname{Re}(\delta)}}{\operatorname{Re}(\delta)} \\
&\quad\times\left[\left(\sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left|\frac{z^2\left(\mathscr{M}\left(a_i, b_j; q\right) f(z) f_k(z)\right)'}{\left[\mathscr{M}\left(a_i, b_j; q\right) f(z) f_k(z)\right]^2}\right| M\right.\right. \\
&\quad\left.\left. + M + 1\right)\right] \\
&\leq \frac{1}{\operatorname{Re}(\delta)}\sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left[(2-\mu_k) M + 1\right], \quad (z \in \mathscr{U})
\end{aligned} \tag{33}$$

which, in the light of the hypothesis (24), yields

$$\frac{1-|z|^{2\operatorname{Re}(\delta)}}{\operatorname{Re}(\delta)}\left|\frac{zh''(z)}{h'(z)}\right| \leq 1, \quad (z \in \mathscr{U}). \tag{34}$$

Applying Lemma (1) for the function $h(z)$ we obtain that $I_{\gamma_k,\delta}(a_i, b_j; q; z)$ is univalent. □

Taking $\mu_k = 0$ (for all $k = 1, \ldots, m$), $M = 1$, $a_i = q^{\alpha_i}$, $b_j = q^{\beta_j}$, $q \to 1$, and $\gamma_k = 1/(\alpha - 1)$, $\delta = 1 + m(\alpha - 1)$ in Theorem 7, we have the following.

Corollary 8 (see [12]). *Let $f_k \in \mathscr{A}$ for all $k = 1, \ldots, m$ and $\alpha \in \mathbb{C}$ with*

$$|\alpha - 1| \leq \frac{\operatorname{Re}(\alpha)}{3m}. \tag{35}$$

If

$$\left|\frac{z^2\left(H_s^r(\alpha_1, \beta_1) f_k(z)\right)'}{\left(H_s^r(\alpha_1, \beta_1) f_k(z)\right)^2} - 1\right| < 1, \quad (z \in \mathscr{U}) \tag{36}$$

and for all $k = 1, \ldots, m$, then the integral operator $F_\alpha(\alpha_1, \beta_1; z)$ defined by (12) *is analytic and univalent in $\mathscr{U}$.*

Taking $\mu_k = 0$ (for all $k = 1, \ldots, m$), $M = 1$, $r = 1$, $s = 0$, $a_1 = q$, and $\gamma_k = 1/(\alpha - 1)$, $\delta = 1 + m(\alpha - 1)$ in Theorem 7, we have the following.

Corollary 9. *Let $f_k \in \mathscr{A}$ for all $k = 1, \ldots, m$ and $\alpha \in \mathbb{C}$ with*

$$|\alpha - 1| \leq \frac{\operatorname{Re}(\alpha)}{3m}. \tag{37}$$

If

$$\left|\frac{z^2 f_k'(z)}{\left(f_k(z)\right)^2} - 1\right| < 1, \quad (z \in \mathscr{U}) \tag{38}$$

and for all $k = 1, \ldots, m$, then the integral operator $F_{m,\alpha}(z)$ defined by (13) *is analytic and univalent in $\mathscr{U}$.*

Theorem 10. *Let $f_k \in \mathscr{A}$ for all $k = 1, \ldots, m, \delta$, $\gamma_k \in \mathbb{C}$, and $M \geq 1$ with*

$$|c| \leq 1 - \frac{1}{\delta}\sum_{k=1}^{m}\frac{\left[(2-\mu_k) M + 1\right]}{|\gamma_k|}, \quad c \in \mathbb{C}. \tag{39}$$

If for all $k = 1, \ldots, m$, $f_k \in \mathfrak{B}_s^r(a_i, b_j, q, \mu_k)$, $0 \leq \mu_k < 1$, and

$$\left|\mathscr{M}\left(a_i, b_j; q\right) f(z) f_k(z)\right| \leq M, \quad (z \in \mathscr{U}), \tag{40}$$

then the integral operator $I_{\gamma_k,\delta}(a_i, b_j; q)$ defined by (11) *is analytic and univalent in $\mathscr{U}$.*

Proof. From the proof of Theorem 7, we have

$$\frac{zh''(z)}{h'(z)} = \sum_{k=1}^{m}\frac{1}{\gamma_k}\left(\frac{z\left(\mathscr{M}\left(a_i, b_j; q\right) f(z) f_k(z)\right)'}{\mathscr{M}\left(a_i, b_j; q\right) f(z) f_k(z)} - 1\right). \tag{41}$$

Thus we have

$$\left|c|z|^{2\delta} + \left(1-|z|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right| \leq |c| + \left|\left(1-|z|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right|. \tag{42}$$

From this result and using the proof of Theorem 7 we obtain

$$\begin{aligned}
&\left|c|z|^{2\delta} + \left(1-|z|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right| \\
&\quad\leq |c| + \frac{1}{\delta}\sum_{k=1}^{m}\frac{1}{|\gamma_k|}\left[(2-\mu_k) M + 1\right].
\end{aligned} \tag{43}$$

Since $|c| \leq 1 - (1/\delta)\sum_{k=1}^{m}(1/\gamma_k)[(2-\mu_k)M + 1]$, then we have

$$\left|c|z|^{2\delta} + \left(1-|z|^{2\delta}\right)\frac{zh''(z)}{\delta h'(z)}\right| \leq 1, \quad (z \in \mathscr{U}). \tag{44}$$

Applying Lemma (4) for the function $h(z)$ we obtain that $I_{\gamma_k,\delta}(a_i, b_j; q; z)$ is univalent. □

Taking $\mu_k = 0$ (for all $k = 1, \ldots, m$), $r = 1$, $s = 0$, $a_1 = q$, and $\gamma_k = 1/(\alpha - 1)$, $\delta = 1 + m(\alpha - 1)(\alpha \in \mathbf{R})$ in Theorem 10, we have the following.

Corollary 11. *Let $f_k \in \mathscr{A}$ for all $k = 1, \ldots, m$; $c \in \mathbb{C}$, $\alpha \in \mathbf{R}$, and $M \geq 1$ with*

$$\begin{gathered}
|c| \leq 1 + \left(\frac{1-\alpha}{1+m(\alpha-1)}\right)(2M+1) m, \\
\alpha \in \left[1, \frac{2Mm+1}{2Mm}\right].
\end{gathered} \tag{45}$$

If for all $k = 1, \ldots, m$

$$\begin{gathered}
\left|\frac{z^2 f_k'(z)}{f_k^2(z)} - 1\right| < 1, \quad (z \in \mathscr{U}), \\
\left|f_k(z)\right| \leq M, \quad (z \in \mathscr{U}; k = 1, \ldots, m),
\end{gathered} \tag{46}$$

then the integral operator $F_{m,\alpha}(z)$ defined by (13) *is analytic and univalent in $\mathscr{U}$.*

Letting $m = 1$, $M = 1$, and $f_1 = f$ in Corollary 11, we have the following.

Corollary 12. *Let $f \in \mathcal{A}$, $c \in \mathbb{C}$ and $\alpha \in \mathbf{R}$ with*

$$|c| \leq \frac{3-2\alpha}{\alpha}, \quad (c \neq -1), \qquad \alpha \in \left[1, \frac{3}{2}\right]. \tag{47}$$

If

$$\left|\frac{z^2 f'(z)}{f^2(z)} - 1\right| < 1, \quad (z \in \mathcal{U}), \qquad |f(z)| \leq 1, \quad (z \in \mathcal{U}), \tag{48}$$

then the integral operator $G_\alpha(z)$ defined by (17) *is analytic and univalent in $\mathcal{U}$.*

Remark 13. Many other interesting corollaries and results can be obtained by specializing the parameters in Theorem 10; for example, see [13, 20, 21].

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