Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2013, Article ID 238490, 6 pages
http://dx.doi.org/10.1155/2013/238490

Research Article

On Symmetric Left Bi-Derivations in *BCI*-Algebras

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Received 15 February 2013; Accepted 30 May 2013

Academic Editor: Aloys Krieg

The notion of *symmetric left bi-derivation* of a *BCI*-algebra X is introduced, and related properties are investigated. Some results on componentwise regular and d-regular *symmetric left bi-derivations* are obtained. Finally, characterizations of a p-semisimple *BCI*-algebra are explored, and it is proved that, in a p-semisimple *BCI*-algebra, F is a *symmetric left bi-derivation* if and only if it is a *symmetric bi-derivation*.

1. Introduction

BCK-algebras and *BCI*-algebras are two classes of nonclassical logic algebras which were introduced by Imai and Iséki in 1966 [1, 2]. They are algebraic formulation of *BCK*-system and *BCI*-system in combinatory logic. Later on, the notion of *BCI*-algebras has been extensively investigated by many researchers (see [3–6], and references therein). The notion of a *BCI*-algebra generalizes the notion of a *BCK*-algebra in the sense that every *BCK*-algebra is a *BCI*-algebra but not vice versa (see [7]). Hence, most of the algebras related to the t-norm-based logic such as MTL [8], BL, hoop, MV [9] (i.e lattice implication algebra), and Boolean algebras are extensions of *BCK*-algebras (i.e. they are subclasses of *BCK*-algebras) which have a lot of applications in computer science (see [10]). This shows that *BCK*-/*BCI*-algebras are considerably general structures.

Throughout our discussion, X will denote a *BCI*-algebra unless otherwise mentioned. In the year 2004, Jun and Xin [11] applied the notion of derivation in ring and near-ring theory to *BCI*-algebras, and as a result they introduced a new concept, called a (regular) derivation, in *BCI*-algebras. Using this concept as defined they investigated some of its properties. Using the notion of a regular derivation, they also established characterizations of a p-semisimple *BCI*-algebra. For a self-map d of a *BCI*-algebra, they defined a d-invariant ideal and gave conditions for an ideal to be d-invariant. According to Jun and Xin, a self map $d : X \to X$ is called a left-right derivation (briefly (l, r)-derivation) of X if $d(x * y) = d(x) * y \wedge x * d(y)$ holds for all $x, y \in X$. Similarly, a self map $d : X \to X$ is called a right-left derivation (briefly (r, l)-derivation) of X if $d(x * y) = x * d(y) \wedge d(x) * y$ holds for all $x, y \in X$. Moreover, if d is both (l, r)- and (r, l)-derivation, it is a derivation on X. After the work of Jun and Xin [11], many research articles have appeared on the derivations of *BCI*-algebras and a greater interest has been devoted to the study of derivations in *BCI*-algebras on various aspects (see [12–17]).

Inspired by the notions of σ-derivation [18], left derivation [19], and symmetric bi-derivations [20, 21] in rings and near-rings theory, many authors have applied these notions in a similar way to the theory of *BCI*-algebras (see [12, 13, 17]). For instantce in 2005 [17], Zhan and Liu have given the notion of f-derivation of *BCI*-algebras as follows: a self map $d_f : X \to X$ is said to be a left-right f-derivation or (l, r)-f-derivation of X if it satisfies the identity $d_f(x * y) = d_f(x) * f(y) \wedge f(x) * d_f(y)$ for all $x, y \in X$. Similarly, a self map $d_f : X \to X$ is said to be a right-left f-derivation or (r, l)-f-derivation of X if it satisfies the identity $d_f(x * y) = f(x) * d_f(y) \wedge d_f(x) * f(y)$ for all $x, y \in X$. Moreover, if d_f is both (l, r)- and (r, l)-f-derivation, it is said that d_f is an f-derivation, where f is an endomorphism. In the year 2007, Abujabal and Al-Shehri [12] defined and studied the notion of left derivation of *BCI*-algebras as follows: a self map $D : X \to X$ is said to be a left derivation of X if satisfying $D(x*y) = x*D(y) \wedge y*D(x)$ for all $x, y \in X$. Furthermore, in 2011 [13], Ilbira et al. have introduced the notion of symmetric bi-derivations in *BCI*-algebras. Following [13], a mapping $D(\cdot,\cdot) : X \times X \to X$ is said to be symmetric if $F(x, y) = F(y, x)$ holds for all pairs $x, y \in X$. A symmetric mapping $D(\cdot,\cdot) : X \times X \to X$ is called left-right symmetric bi-derivation (briefly (l, r)-symmetric bi-derivation) if it satisfies the identity $D(x * y, z) = D(x, z) * y \wedge x * D(y, z)$ for all $x, y, z \in X$. D is called right-left symmetric bi-derivation (briefly (r, l)-symmetric bi-derivation) if it satisfies the identity $D(x * y, z) = x * D(y, z) \wedge D(x, z) * y$ for all $x, y, z \in X$. Moreover, if D is both a (l, r)- and a (r, l)-symmetric bi-derivation, it is said that D is a symmetric bi-derivation on X.

Motivated by the notion of *symmetric bi-derivations* [13] in the theory of *BCI*-algebras, in the present analysis, we introduced the notion of *symmetric left bi-derivations* on *BCI*-algebras and investigated related properties. Further, we obtain some results on componentwise regular and d-regular *symmetric left bi-derivations*. Finally, we characterize the notion of p-semisimple *BCI*-algebra X by using the concept of *symmetric left bi-derivation* and show that, in a p-semisimple *BCI*-algebra X, F is a *symmetric left bi-derivation* if and only if it is a *symmetric bi-derivation*.

2. Preliminaries

We begin with the following definitions and properties that will be needed in the sequel.

A nonempty set X with a constant 0 and a binary operation $*$ is called a *BCI-algebra* if for all $x, y, z \in X$ the following conditions hold:

(I) $((x * y) * (x * z)) * (z * y) = 0$,

(II) $(x * (x * y)) * y = 0$,

(III) $x * x = 0$,

(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$.

Define a binary relation $\leq$ on X by letting $x * y = 0$ if and only if $x \leq y$. Then $(X, \leq)$ is a partially ordered set. A *BCI*-algebra X satisfying $0 \leq x$ for all $x \in X$ is called *BCK*-algebra.

A *BCI*-algebra X has the following properties for all $x, y, z \in X$.

(a1) $x * 0 = x$.

(a2) $(x * y) * z = (x * z) * y$.

(a3) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$.

(a4) $(x * z) * (y * z) \leq x * y$.

(a5) $x * (x * (x * y)) = x * y$.

(a6) $0 * (x * y) = (0 * x) * (0 * y)$.

(a7) $x * 0 = 0$ implies $x = 0$.

For a *BCI*-algebra X, denote by X_+ (resp., $G(X)$) the *BCK*-part (resp., the *BCI*-G part) of X; that is, X_+ is the set of all $x \in X$ such that $0 \leq x$ (resp., $G(X) := \{x \in X \mid 0 * x = x\}$). Note that $G(X) \cap X_+ = \{0\}$ (see [22]). If $X_+ = \{0\}$, then X is called a *p-semisimple BCI-algebra*. In a p-semisimple *BCI*-algebra X, the following hold.

(a8) $(x * z) * (y * z) = x * y$.

(a9) $0 * (0 * x) = x$ for all $x \in X$.

(a10) $x * (0 * y) = y * (0 * x)$.

(a11) $x * y = 0$ implies $x = y$.

(a12) $x * a = x * b$ implies $a = b$.

(a13) $a * x = b * x$ implies $a = b$.

(a14) $a * (a * x) = x$.

(a15) $(x * y) * (w * z) = (x * w) * (y * z)$.

Let X be a p-semisimple *BCI*-algebra. We define addition "+" as $x + y = x * (0 * y)$ for all $x, y \in X$. Then $(X, +)$ is an abelian group with identity 0 and $x - y = x * y$. Conversely, let $(X, +)$ be an abelian group with identity 0, and let $x * y = x - y$. Then X is a p-semisimple *BCI*-algebra and $x + y = x * (0 * y)$ for all $x, y \in X$ (see [6]).

For a *BCI*-algebra X, we denote $x \wedge y = y * (y * x)$, in particular $0 * (0 * x) = a_x$, and $L_p(X) := \{a \in X \mid x * a = 0 \Rightarrow x = a, \forall x \in X\}$. We call the elements of $L_p(X)$ the *p-atoms* of X. For any $a \in X$, let $V(a) := \{x \in X \mid a * x = 0\}$, which is called the *branch* of X with respect to a. It follows that $x * y \in V(a * b)$ whenever $x \in V(a)$ and $y \in V(b)$ for all $x, y \in X$ and all $a, b \in L_p(X)$. Note that $L_p(X) = \{x \in X \mid a_x = x\}$,which is the p-semisimple part of X, and X is a p-semisimple *BCI*-algebra if and only if $L_p(X) = X$ (see [23, Proposition 3.2]). Note also that $a_x \in L_p(X)$; that is, $0 * (0 * a_x) = a_x$, which implies that $a_x * y \in L_p(X)$ for all $y \in X$. It is clear that $G(X) \subset L_p(X)$, and $x * (x * a) = a$ and $a * x \in L_p(X)$ for all $a \in L_p(X)$ and all $x \in X$. Let $D(\cdot,\cdot) : X \times X \to X$ be a symmetric mapping. Then for all $x \in X$, a mapping $d : X \to X$ defined by $d(x) = D(x, x)$ is called trace of D [13]. For more details, refer to [3, 4, 6, 11, 22, 23].

3. Symmetric Left Bi-Derivations

The following definition introduces the notion of *symmetric left bi-derivation* for a *BCI*-algebra X.

Definition 1. A symmetric mapping $F(\cdot,\cdot) : X \times X \to X$ is called a symmetric left bi-derivation of X if it satisfies the following identity:

$$(\forall x, y, z \in X) \quad (F(x * y, z) = (x * F(y, z)) \wedge (y * F(x, z))). \tag{1}$$

Example 2 (see [24]). Consider a p-semisimple *BCI*-algebra $X = \{0, 3, 4, 5\}$ with the following Cayley table:

$*$	0	3	4	5
0	0	3	4	5
3	3	0	5	4
4	4	5	0	3
5	5	4	3	0

(2)

Define a mapping $F(\cdot,\cdot): X \times X \to X$ by

$$\begin{aligned} F(0,0) = F(3,3) = F(4,4) = F(5,5) = 0, \\ F(0,3) = F(3,0) = 3, \\ F(0,4) = F(4,0) = 4, \\ F(0,5) = F(5,0) = 5, \\ F(3,4) = F(4,3) = 5, \\ F(3,5) = F(5,3) = 4, \\ F(4,5) = F(5,4) = 3. \end{aligned} \quad (3)$$

It is routine to verify that F is a *symmetric left bi-derivation* of X.

Theorem 3. *Let $F(\cdot,\cdot): X \times X \to X$ be a symmetric left bi-derivation of X. Then*

(1) $(\forall z \in X)$ $(a \in G(X) \Rightarrow F(a,z) \in G(X))$.
(2) $(\forall z \in X)$ $(a \in L_p(X) \Rightarrow F(a,z) \in L_p(X))$.
(3) $(\forall z \in X)$ $(a \in L_p(X) \Rightarrow F(a,z) = 0 + F(a,z))$.
(4) $(\forall z \in X)$ $(a \in L_p(X) \Rightarrow F(a,z) = a * F(0,z) = a + F(0,z))$.

Proof. (1) Let $a \in G(X)$. Then $0 * a = a$, and so

$$\begin{aligned} F(a,z) &= F(0 * a, z) \\ &= (0 * F(a,z)) \wedge (a * F(0,z)) \\ &= (a * F(0,z)) * ((a * F(0,z)) * (0 * F(a,z))) \\ &= 0 * F(a,z), \end{aligned} \quad (4)$$

since $0 * F(a,z) \in L_p(X)$. Hence $F(a,z) \in G(X)$.

(2) For any $a \in L_p(X)$ implies $a = 0 * (0 * a)$ and so

$$\begin{aligned} F(a,z) &= F(0 * (0 * a), z) \\ &= (0 * F(0 * a, z)) \wedge ((0 * a) * F(0,z)) \\ &= ((0 * a) * F(0,z)) \\ &\quad * (((0 * a) * F(0,z)) * (0 * F(0 * a, z))) \\ &= 0 * F(0 * a, z) \in L_p(X). \end{aligned} \quad (5)$$

(3) By (2), we have $F(a,z) \in L_p(X)$. Then

$$F(a,z) = 0 * (0 * F(a,z)) = 0 + F(a,z). \quad (6)$$

(4) For any $a \in L_p(X)$ and $z \in X$, we have

$$\begin{aligned} F(a,z) &= F(a * 0, z) \\ &= (a * F(0,z)) \wedge (0 * F(a,z)) \\ &= (0 * F(a,z)) * ((0 * F(a,z)) * (a * F(0,z))) \\ &= a * F(0,z) \\ &= a * (0 * F(0,z)) \\ &= a + F(0,z). \end{aligned} \quad (7)$$

This completes the proof. □

Using Theorem 3, we have the following corollary.

Corollary 4. *Let $F(\cdot,\cdot): X \times X \to X$ be a symmetric left bi-derivation and $d: X \to X$ be the trace of F. Then*

(1) $(\forall a \in G(X))$ $(d(a) \in G(X))$.
(2) $(\forall a \in L_p(X))$ $(d(a) \in L_p(X))$.

Theorem 5. *Let F be a symmetric left bi-derivation of X. Then*

(1) $(\forall z \in X)$ $(a, b \in L_p(X) \Rightarrow F(a+b,z) = a + F(b,z))$.
(2) $(\forall z \in X)$ $(a \in L_p(X) \Rightarrow F(a,z) = a$ *if and only if* $F(0,z) = 0)$.
(3) $(\forall x, y, z \in X)$ $(F(x * y, z) \leq x * F(y,z))$.
(4) $(\forall x, y, z \in X)$ $(x * F(x,z) = y * F(y,z))$.

Proof. (1) Let $a, b \in L_p(X)$. Then

$$\begin{aligned} F(a+b,z) &= F(a * (0 * b), z) \\ &= a * F(0 * b, z) \wedge (0 * b) * F(a,z) \\ &= a * F(0 * b, z) \\ &= a * (0 * F(b,z) \wedge b * F(0,z)) \\ &= a * (0 * F(b,z)) = a + F(b,z). \end{aligned} \quad (8)$$

(2) Suppose $F(a,z) = a$ for all $a \in L_p(X)$, $z \in X$. It is clear that, for $0 \in L_p(X)$, we have $F(0,z) = 0$. Conversely let us assume that $F(0,z) = 0$; then by using Theorem 3(4), we have $F(a,z) = a + F(0,z) = a + 0 = a$.

(3) For any $x, y, z \in X$, we have

$$\begin{aligned} F(x * y, z) &= (x * F(y,z)) \wedge (y * F(x,z)) \\ &= (y * F(x,z)) \\ &\quad * ((y * F(x,z)) * (x * F(y,z))) \\ &\leq x * F(y,z). \end{aligned} \quad (9)$$

(4) For any $x, z \in X$, we have

$$\begin{aligned} F(0,z) &= F(x * x, z) = (x * F(x,z)) \wedge (x * F(x,z)) \\ &= x * F(x,z). \end{aligned} \quad (10)$$

Thus, we can write $F(0,z) = x * F(x,z) = y * F(y,z)$ for any $y \in X$. This completes the proof. □

Definition 6. A *symmetric left bi-derivation* $F(\cdot,\cdot): X \times X \to X$ of a BCI-algebra X is said to be *componentwise regular* if $F(0,z) = 0$ for all $z \in X$. In particular, F is called d-regular if $F(0,0) = d(0) = 0$.

Theorem 7. *Let F be a symmetric left bi-derivation of BCI-algebra X. Then X is a BCK-algebra if and only if F is componentwise regular symmetric left bi-derivation.*

Proof. Suppose X is a BCK-algebra. Then for any $x, z \in X$, we have

$$\begin{aligned} F(0,z) &= F(0 * x, z) \\ &= (0 * F(x,z)) \wedge (x * F(0,z)) \\ &= 0 \wedge (x * F(0,z)) = 0. \end{aligned} \tag{11}$$

Hence F is *componentwise regular*.

Conversely, let F be a *componentwise regular symmetric left bi-derivation*. Let for any $a \in L_p(X)$ be such that $a \neq 0$. Then

$$F(a * 0, 0) = F(a, 0) = 0. \tag{12}$$

But it is clear that

$$\begin{aligned} a * F(0,0) \wedge 0 * F(a,0) &= a * 0 \wedge 0 * 0 \\ &= a \wedge 0 = 0 * (0 * a) \\ &= a \neq 0, \end{aligned} \tag{13}$$

which is not possible as F is a *componentwise regular symmetric left bi-derivation*. Thus 0 is the unique p-atom. Assume that for some $m \in X$, we have $0 * m \neq 0$, then $a_{0*m} = 0 * (0 * (0 * m)) = 0$, so $0 * m \in L_p(X)$, which is a contradiction. Henceforth, for all $m \in X$, we have $0 * m = 0$ implying thereby, X is a BCK-algebra.

This completes the proof. □

Theorem 8. *Let F be a componentwise regular symmetric left bi-derivation of a BCI-algebra X. Then*

(1) *Both x and $F(x,z)$ belong to the same branch for all $x, z \in X$.*

(2) $(\forall x, z \in X)\ (F(x,z) \leq x)$.

(3) $(\forall x, y, z \in X)\ (F(x,z) * y \leq x * F(y,z))$.

Proof. (1) For any $x, z \in X$, we get

$$\begin{aligned} 0 = F(0,z) &= F(a_x * x, z) \\ &= (a_x * F(x,z)) \wedge (x * F(a_x, z)) \\ &= (x * F(a_x, z)) * ((x * F(a_x, z)) * (a_x * F(x,z))) \\ &= a_x * F(x,z), \end{aligned} \tag{14}$$

since $a_x * F(x,z) \in L_p(X)$. Hence $a_x \leq F(x,z)$, and so $F(x,z) \in V(a_x)$. Obviously, $x \in V(a_x)$.

(2) Since F is componentwise regular, $F(0,z) = 0$. Then

$$\begin{aligned} F(x,z) &= F(x * 0, z) \\ &= (x * F(0,z)) \wedge (0 * F(x,z)) \\ &= (x * 0) \wedge (0 * F(x,z)) \\ &= (0 * F(x,z)) * ((0 * F(x,z)) * x) \\ &\leq x. \end{aligned} \tag{15}$$

(3) Since $F(x,z) \leq x$ for all $x, z \in X$ by (2), using (a3) we obtain

$$F(x,z) * y \leq x * y \leq x * F(y,z). \tag{16}$$

This completes the proof. □

Next, we prove some results in a p-semisimple BCI-algebra.

Theorem 9. *Let F be a symmetric left bi-derivation of a p-semisimple BCI-algebra X; one has the following assertions.*

(1) $(\forall x, y, z \in X)\ (F(x * y, z) = x * F(y,z))$.

(2) $(\forall x, y, z \in X)\ (F(x,z) * x = F(y,z) * y)$.

(3) $(\forall x, y, z \in X)\ (F(x,z) * x = y * F(y,z))$.

Proof. (1) Let X be a p-semisimple BCI-algebra. Then for any $x, y, z \in X$, we have

$$F(x * y, z) = (x * F(y,z)) \wedge (y * F(x,z)) = x * F(y,z). \tag{17}$$

(2) Let $x, y, z \in X$. Using (I), we have

$$\begin{aligned} (x * y) * (x * F(y,z)) &\leq F(y,z) * y, \\ (y * x) * (y * F(x,z)) &\leq F(x,z) * x. \end{aligned} \tag{18}$$

These above inequalities can be rewritten as

$$\begin{aligned} ((x * y) * (x * F(y,z))) * (F(y,z) * y) &= 0, \\ ((y * x) * (y * F(x,z))) * (F(x,z) * x) &= 0. \end{aligned} \tag{19}$$

Consequently, we get

$$\begin{aligned} &((x * y) * (x * F(y,z))) * (F(y,z) * y) \\ &\quad = ((y * x) * (y * F(x,z))) * (F(x,z) * x) \end{aligned} \tag{20}$$

Also, using Theorem 5(4) and (1), we obtain

$$\begin{aligned} &(x * y) * F(x * y, z) = (y * x) * F(y * x, z) \\ &\Longrightarrow (x * y) * (x * F(y,z)) = (y * x) * (y * F(x,z)). \end{aligned} \tag{21}$$

Since X is a p-semisimple BCI-algebra, hence, by using (21) and (a12), the above (20) yields $F(x,z) * x = F(y,z) * y$.

(3) We have $F(0,z) = x * F(x,z)$ by Theorem 5(4). Further, on letting $x = 0$, we get that $F(0,z) * 0 = F(y,z) * y$ implies $F(0,z) = F(y,z) * y$. Henceforth $F(y,z) * y = x * F(x,z)$, which amounts to say that $F(x,z) * x = y * F(y,z)$.

This completes the proof. □

Theorem 10. *Let X be a p-semisimple BCI-algebra. Then F is a symmetric left bi-derivation if and only if it is a symmetric bi-derivation on X.*

Proof. Suppose that F is a *symmetric left bi-derivation* on X. First, we show that F is a *(r,l)-symmetric bi-derivation* on X. Let $x, y, z \in X$. Using Theorem 9(1) and (a14), we have

$$\begin{aligned} F(x * y, z) &= x * F(y, z) \\ &= (F(x, z) * y) \\ &\quad * ((F(x, z) * y) * (x * F(y, z))) \\ &= (x * F(y, z)) \wedge (F(x, z) * y). \end{aligned} \tag{22}$$

Hence F is a (r, l)-*symmetric bi-derivation* on X.

Again, we show that F is a *(l,r)-symmetric bi-derivation* on X. Let $x, y, z \in X$. Using Theorem 9(1), (3) and (a15), we have

$$\begin{aligned} F(x * y, z) &= x * F(y, z) \\ &= (x * 0) * F(y, z) \\ &= (x * (F(0, z) * F(0, z))) * F(y, z) \\ &= (x * ((x * F(x, z)) * (F(y, z) * y))) \\ &\quad * F(y, z) \\ &= (x * F(y, z)) \\ &\quad * ((x * F(x, z)) * (F(y, z) * y)) \\ &= (x * F(y, z)) \\ &\quad * ((x * F(y, z)) * (F(x, z) * y)) \\ &= (F(x, z) * y) \wedge (x * F(y, z)). \end{aligned} \tag{23}$$

Conversely, suppose that F is a *symmetric bi-derivation* of X. As F is a *(r,l)-symmetric bi-derivation* on X, then for any $x, y, z \in X$ and using (a14), we have

$$\begin{aligned} F(x * y, z) &= (x * F(y, z)) \wedge (F(x, z) * y) \\ &= (F(x, z) * y) \\ &\quad * ((F(x, z) * y) * (x * F(y, z))) \\ &= x * F(y, z) \\ &= (y * F(x, z)) \\ &\quad * ((y * F(x, z)) * (x * F(y, z))) \\ &= (x * F(y, z)) \wedge (y * F(x, z)). \end{aligned} \tag{24}$$

Hence F is a *symmetric left bi-derivation*. This completes the proof. □

Acknowledgments

The authors are grateful to the anonymous referee(s) for a careful checking of the details and for helpful comments that improved the present paper. G. Muhiuddin and Abdullah M. Al-roqi were partially supported by the Deanship of Scientific Research, University of Tabuk, Ministry of Higher Education, Saudi Arabia.

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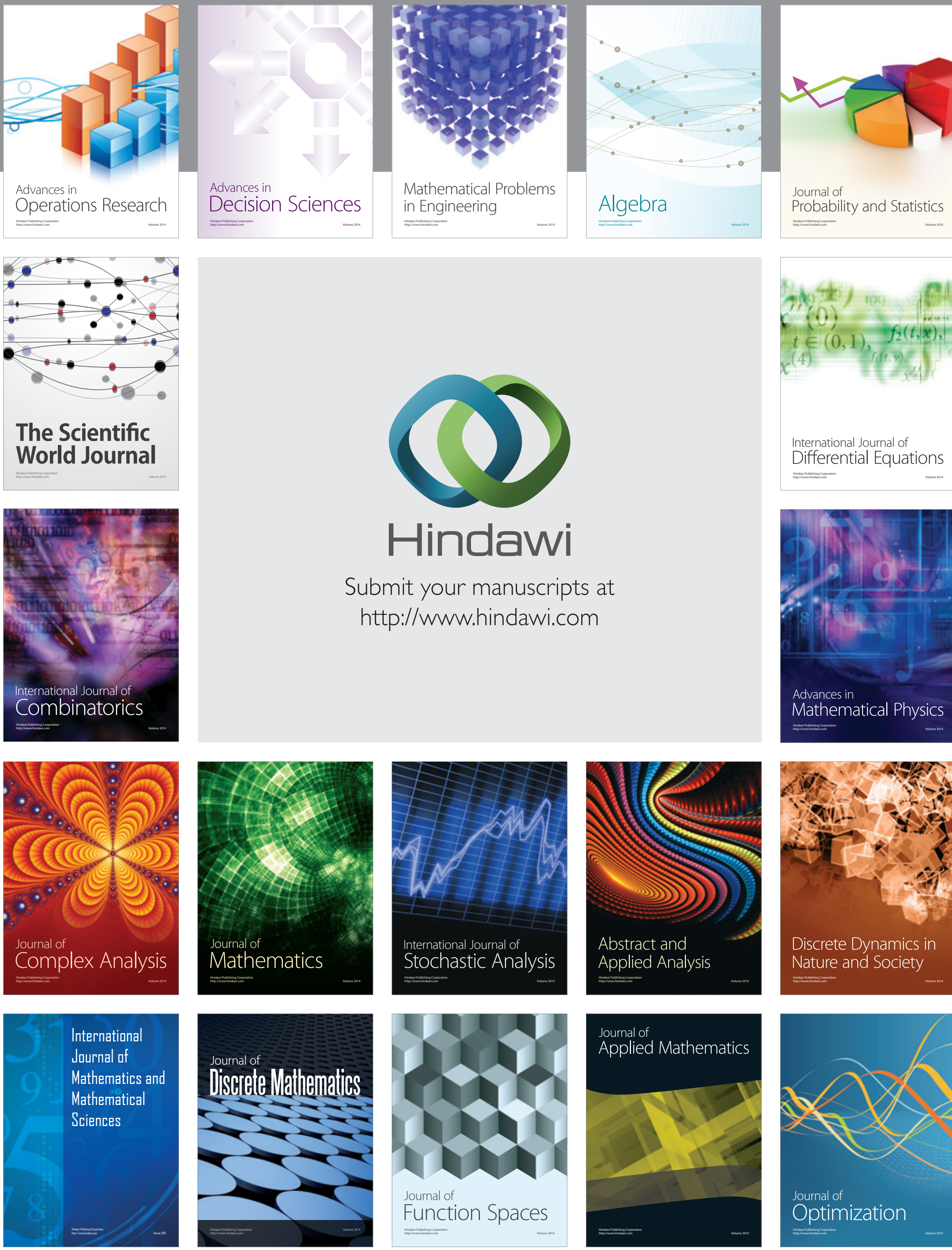
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