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Research Article

Asymmetric Information and Quantization in Financial Economics

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We show how a quantum formulation of financial economics can be derived from asymmetries with respect to Fisher information. Our approach leverages statistical derivations of quantum mechanics which provide a natural basis for interpreting quantum formulations of social sciences generally and of economics in particular. We illustrate the utility of this approach by deriving arbitrage-free derivative-security dynamics.

1. Introduction

Asymmetric information lies at the heart of capital markets and how it induces information flow and economic dynamics is a key element to understanding the structure and function of economic systems generally and of price discovery in particular [1–3]. The information-theoretic underpinnings of economics also provide a common framework through which economics can leverage results in other fields, an example being the well-known use of statistical mechanics in financial economics (see, e.g., [44, 45] and references therein). In addition to the statistical-mechanics representation of financial economics, however, a quantum-mechanics representation has also emerged (see [5, 18–30, 42, 46–48]) and the purpose of this paper is to show that this quantum framework too can be derived from asymmetric information, thus providing a more comprehensive information-theoretic basis for financial economics.

Financial economics is unique among economic disciplines in the extent to which stochastic processes are employed as an explanatory framework, a ubiquity epitomized in the modeling of financial derivatives.[1] Building on a history of shared metaphor between classical physics and neoclassical economics [4] and the initial use of simple diffusion processes, financial economics and statistical mechanics found a common language in stochastic dynamics with which statistical mechanics could be applied across a wide range of economics including finance, macroeconomics, and risk management (see, e.g., [44, 45, 52–56]). Underlying that common language is a fundamental information-theoretic basis, a basis with which financial economics can be expressed as probability theory with constraints.[2]

It is from the perspective of financial economics as probability theory with constraints that we propose to show how and why financial economics can be expressed as a quantum theory. Financial economics as quantum theory has developed in a manner similar to that taken by statistical mechanics, exploiting formal similarities (see [5, 18–30, 42, 46–48]). Financial economics as a quantum theory, however, lacks the history of common metaphor that enabled statistical mechanics to achieve its reach as an explanatory framework for financial economics. The resulting lack of common language has proved a challenge to the quantum representation of financial economics on questions ranging from the interpretation of coefficients (e.g., Planck's constant) to the ontological and epistemological content of the theory. We propose to develop a common language through the common structure of information theory in general and of Fisher information in particular. At a general level this paper follows naturally from research demonstrating the ubiquity of quantum theory generally [5] and the statistical origins of quantum mechanics in particular.[3] At a specific level this paper shows how constrained Fisher information can be used to quantize financial economics.

To this end we continue in Section 2 with a derivation of quantized financial economics. In this section we extend our prior work on the principle of minimum Fisher information as the fundamental expression of the concept of asymmetric information in economics (see [10–15]) via a financial-economic interpretation and adaptation of Reginatto's Fisher-information-based derivation of the time-dependent Schrödinger equation [6]. In Section 3 we demonstrate the utility of this approach by deriving the equilibrium and nonequilibrium probability densities associated with some canonical financial structures: forwards and options. Finally, we close in Section 4 with a discussion and summary.

2. Theory

It is our view that all things economic are information-theoretic in origin: economies are participatory, observer participancy gives rise to information, and information gives rise to economics. Dynamical laws follow from a perturbation of information flow which arises from the asymmetry between J, the information that is intrinsic to the system, and I, the measured Fisher information of the system: a natural consequence of the notion that any observation is a result of the $J \to I$ information-flow process. In this manner financial-economic dynamics are a natural consequence of our information-theoretic approach.

2.1. Fisher Information I

We consider the price of an asset, liability, or more generally of a security which, for a given instant in time t_0, we write as x_0. At each point in time, however, the measured price is x_{obs} which is necessarily imperfect due to fluctuations x, or

$$x_{\text{obs}} = x_0 + x. \tag{2.1}$$

This fluctuation arises from both the inevitable uncertainty in the *effective* present time t_0 and the fact that the economy undergoes persistent random change. The greater is the fluctuation x the greater is one's ignorance of the price. To represent this fluctuation we introduce the probability amplitude $\psi(x)$ for the price fluctuation x, with the associated probability $P(x,t)$ given by

$$P(x,t) \equiv \psi(x,t)\psi^*(x,t) = \left|\psi(x,t)\right|^2, \tag{2.2}$$

where the asterisk denotes the complex conjugate. Our use of probability amplitudes is at this stage independent of that seen in quantum theory: Fisher and Mather [7], for example, introduced them as a vehicle for simplifying calculations. Assuming that the statistics of x_{obs} are independent of the price level x_0, the likelihood law for the process obeys

$$P(x_{\text{obs}}|\, x_0, t) \equiv P(x_{\text{obs}} - x_0, t) = P(x,t) \tag{2.3}$$

by (2.1).

The classical Fisher information I for this one-dimensional problem is

$$I \equiv \left\langle \left[\frac{\partial \ln(p(x_{\text{obs}}|\, x_0, t))}{\partial x_0} \right]^2 \right\rangle, \tag{2.4}$$

where the angle bracket denotes an expectation over all possible data x_{obs} [8, 9]. This is a universal form that applies to all data acquisition problems and measures the information in the data *irrespective of the intrinsic nature of that data*. For our problem this can be reduced to

$$I = \int \frac{1}{P}\left(\frac{\partial P}{\partial t}\right)^2 dx\, dt = \int \left|\psi'(x)\right|^2 dx\, dt, \tag{2.5}$$

where $|\psi'|^2 = \psi'\psi'^*$ and $\psi' = d\psi/dx$. Fisher information corresponds well to our intuition regarding price discovery given in (2.1): if the price fluctuation x is small then the level of information in the observation should be high, whereas if x is large the information should be low. This notion is expressed in (2.5) by the shape of the probability density P and since for small price fluctuations P must be narrowly peaked about $x = 0$ implying high gradient dP/dx, a consequent high gradient $d\psi/dx$, and a large value for I. Similarly, if x has many large values then P will be broad; it and ψ will have low gradients and there will be a small value for I.

2.2. Intrinsic Information J

The intrinsic information J is the present, most complete and perfectly knowable collection of information concerning the system that is relevant to the measurement exercise [8, 9]. One example of this information is exact knowledge characterized by a unitary transform between the observation space and some conjugate space, a situation we exploited in our derivation of Tobin's Q-theory [10, 11]. Another example of J is empirical price data which we have employed in the application of Fisher-information-based statistical mechanics to economics [10–15]. To empirical data one can also add assumptions such as the conservation law for probability [6] and it is this approach that we will employ in this paper.

To develop J for financial economics we employ three well-known assumptions: the first being that the price of a cash flow is the probability-weighted, discounted (or present) value of that cash flow and that the price of any security is the sum of the price of the constituent cash flows. This assumption is the basis for valuation in financial economics [16].[4] Future cash flows are contingent on the future state of the economy, either implicitly (e.g., the ability of a corporation to make future bond coupon and principal payments) or explicitly (e.g., insurance coverage). Discounted state-contingent payments are also known as derivative securities. Thus, derivatives, the fundamental securities of an economy, will be our focus below.

Our second assumption is that a cash-flow price can be represented by a probability distribution $P(x,t)$. This assumption is central to the literature of derivative securities in which all underlying-asset prices are represented by time-dependent probability distributions. Our final assumption is closely related to our second assumption and is that a set of cash-flow price trajectories forms a coherent system [6, 17].

These three assumptions coalesce in the information J associated with empirical observations of prices in an economy as observed derivative-security prices $d_1,\ldots,d_N = \{d_n\}$ are averages of the discounted payoff functions $\{f_n(x,t)\}$,

$$d_n = \int f_n(x,t)P(x,t)dx\,dt \quad n = 1,\ldots,N. \tag{2.6}$$

Our focus on derivative securities complements Haven's [18–23] analysis of derivatives from a de Broglie-Bohm perspective by providing (as we shall see presently) an information-theoretic basis for the de Broglie-Bohm approach in financial economics and generalizes the work of Khrennikov and Choustova [5, 24–30] on equity prices in the de Broglie-Bohm framework as equity can be viewed as a derivative security, namely, a call option on the assets of the issuing firm [31–33].

The last two of our three assumptions imply that the velocity v of a cash-flow at price point x can be related to a real function $S(x,t)$ by an expression of the form [6]:

$$v = \frac{1}{m_e}\frac{\partial S}{\partial x}, \tag{2.7}$$

where m_e is the effective mass of the price represented by market turnover.[5] It follows that the probability distribution must satisfy a conservation law of the form

$$\frac{\partial P}{\partial t} + \frac{1}{m_e}\frac{\partial}{\partial x}\left(P\frac{\partial S}{\partial x}\right) = 0, \tag{2.8}$$

and, as discussed by Reginatto [6], (2.8) can be derived from a variational principle, by minimization of the expression

$$\int P\left(\frac{\partial S}{\partial t} + \frac{1}{2m_e}\left(\frac{\partial S}{\partial x}\right)^2\right)dx\,dt, \tag{2.9}$$

with respect to S.

2.3. Information Asymmetry and Dynamics

To minimize the Fisher information I in a manner consistent with our information J one can form the information asymmetry Lagrangian $\mathcal{L} = I - J$ [6]:

$$\mathcal{L} = \frac{\lambda_0}{m_e}\int \frac{1}{P}\left(\frac{\partial P}{\partial t}\right)^2 dx\,dt - \int P\left(\frac{\partial S}{\partial t} + \frac{1}{2m_e}\left(\frac{\partial S}{\partial x}\right)^2\right)dx\,dt - \sum_{n=1}^{N}\lambda_n\left[d_n - \int f_n P\;dx\,dt\right]. \tag{2.10}$$

Variation of the information asymmetry $\mathcal{L}$ with respect to S and P yields

$$\frac{\partial P}{\partial t} + \frac{1}{m_e}\frac{\partial}{\partial x}\left(P\frac{\partial S}{\partial x}\right) = 0, \tag{2.11}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m_e}\left(\frac{\partial S}{\partial x}\right)^2 + V(x,t) - \frac{\lambda_0}{m_e}\left(\frac{2}{P}\frac{\partial^2 P}{\partial x^2} - \frac{1}{P^2}\left(\frac{\partial P}{\partial x}\right)^2\right) = 0, \tag{2.12}$$

where

$$V(x,t) = \sum_{n=1}^{N}\lambda_n f_n(x,t). \tag{2.13}$$

Equations (2.11) and (2.12) are the Madelung hydrodynamic equations [34] that, via the Madelung transform $\psi = \sqrt{P}\exp(iS/\sqrt{8\lambda_0})$ where $i \equiv \sqrt{-1}$, are the real and imaginary parts of

$$i\frac{\partial\psi}{\partial t} = -\frac{\sqrt{2\lambda_0}}{m_e}\left[\frac{\partial^2\psi}{\partial x^2} - V(x,t)\right]\psi; \tag{2.14}$$

a Schrödinger-like wave equation with a potential function V that is a linear combination of the payoff functions of the derivative securities in the economy.

To complete the model we need Lagrange multipliers that are consistent with our information J. The Lagrange multiplier λ_0 can be resolved in a manner that relates this quantum approach to the traditional stochastic representation of financial economics through the use of Nelson's stochastic mechanics [35, 36]:

$$dx(t) = b(x(t),t)dt + dw(t), \tag{2.15}$$

where

$$b = \frac{\sqrt{8\lambda_0}}{m_e}\frac{\partial}{\partial x}\left(\frac{1}{2}\ln P + S\right). \tag{2.16}$$

$w(t)$ is a Wiener process, and $dw(t)$ is Gaussian with zero mean and product expectation $Edw_i(t)dw_j(t) = 2\nu\delta_{i,j}dt$ where $\nu = \sqrt{2\lambda_0}/m_e$ is the diffusion coefficient and $\delta_{i,j}$ is the

Kronecker delta function.[6] From this it follows that the Lagrange multiplier λ_0 is related to turnover and to the diffusivity of the economy by $\lambda_0 = m_e^2 \nu^2/2$.

Determining the Lagrange multipliers $\lambda_1, \ldots, \lambda_N$ is a straightforward exercise as the value of each is set by the requirement that the corresponding observed derivative price as expressed in (2.6) is recovered. A convenient consequence of this approach is that the calculated $P(x,t)$ is consistent with all observed security prices (i.e., is arbitrage free) by construction. A measure of the price uncertainty in the economy σ^2 can be had through the use of the Cramer-Rao inequality $\sigma^2 \geq 1/I$ [37, 38] with which a lower bound of the price variance, or the notion of implied volatility employed widely in derivatives trading, is seen to be the inverse of the Fisher information [9, 12].

3. Example: Derivative Securities

A particular advantage of our Fisher information approach to dynamics in financial economics is the natural way by which both time dependence and departures from equilibrium arise. Time dependence of the probability density and, by implication, security prices is expressed in (2.14). The simplest example of equilibrium and departures therefrom can be seen by considering the price of a forward d_{fwd} in a zero-rate environment which is the first moment:

$$d_{fwd} = \int_0^\infty xP(x,t)\delta(T-t)dx\,dt, \tag{3.1}$$

where T is the expiration date of the forward contract and $\delta(x)$ is the Dirac delta function. The solution of (2.14) that vanishes at the zero-price boundary for this linear payoff function is known to be a linear combination of the the Airy function $\mathrm{Ai}(x)$ [39]:

$$\psi_n(x) \propto \mathrm{Ai}(\xi_n(x)), \tag{3.2}$$

where

$$\xi_n(x) = \left[\left(\frac{m_e\lambda_1}{4\lambda_0}\right)^{1/3} x - a_n\right], \tag{3.3}$$

where a_n is the nth zero of the Airy function. The equilibrium solution for the forward price corresponds to the first zero of the Airy function $a_1 = -2.33811$, with the value of λ_1 chosen to reproduce the observed forward price. Disequilibrium forward prices can be represented as linear combinations of the solutions corresponding to the zeros of the Airy function for $n \geq 1$. This complete description of the temporal evolution of the forward price in and about the equilibrium state in terms of a collection of eigenfunctions is the information-theoretic first quantization of financial economics.

Extending this to call and put options which are the partial moments,

$$\begin{aligned} d_{\text{call}} &= \int_0^\infty \max(x-k,0)P(x,t)\delta(T-t)\,dx\,dt, \\ d_{\text{put}} &= \int_0^\infty \max(k-x,0)P(x,t)\delta(T-t)\,dx\,dt, \end{aligned} \tag{3.4}$$

where k is the strike price is straightforward. The simple linear potential function of the forward shown in (3.1) is replaced by a piecewise linear potential resulting from the linear combination of the payoff functions of each derivative weighted by their corresponding Lagrange multiplier.

4. Discussion and Summary

From the perspective of information theory, the quantum representation of financial economics is a natural outcome of the process of inference using Fisher information. This approach also complements the work of Haven, Choustova and Khrennikov [5, 18–30] concerning the quantum potential introduced by Bohm [40, 41]. Bohm interpreted (2.12) as a Hamilton-Jacobi equation and the final two terms on the left-hand side of this equation as a potential that acts, in our case, on the security. The first of these two terms V is the usual physical potential that in financial economics is the potential associated with the contractual state-contingent payoff structure of a derivative security (cf. (2.13)). The other potential term in (2.12), identified by Bohm [40, 41] as the quantum potential, was later reassessed by Reginatto [6] in light of its dependence on the probability to reflect the inferential nature of the quantum formalism: specifically, he noted that the average of the quantum potential is proportional to Fisher information. Thus, the potential in financial economics naturally splits into one component that originates in the specifics of the economy and another component that arises from the manner by which the market infers prices. This derivation also provides a clear distinction between the ontological and epistemological content of the quantum theory of financial economics. The epistemological content of this theory is the use of the minimization of Fisher information to choose the probability distribution that describes the price of securities. The economic content of the theory is the assumption that the price-space motion of securities is a coherent structure and the existence of observed security prices.[7]

Quantum theory is an attractive formalism to use in the treatment of economics generally and of financial economics in particular due to the manner in which uncertainty arises [5, 18–30, 42, 43]. In this paper we have adapted an information-theoretic approach—the use of the principle of minimum Fisher information—to the derivation of quantum mechanics to illuminate aspects of this formalism that have proved challenging to the use of the quantum formalism in financial economics, challenges due in part to the lack of a historic common language. This approach is consistent with existing foundational assumptions in financial economics, incorporates time-dependence as a natural consequence of conservation of probability, and yields both equilibrium and disequilibrium solutions for security prices. Finally, the clarity of expression and formal link with the statistical origins of quantum mechanics recommends this quantum formalism as a useful approach to the treatment of problems in financial economics.

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Endnotes

1. The physical basis for employing stochastic dynamics, uncertainty, has been a part of economics generally for some time since the pioneering work of Keynes and Knight [49–51].
2. The information-theoretic basis of statistical mechanics is discussed in [57–61]. The relationship between information theory and economics is discussed in detail in [11] and references therein. In recent communications we have shown that the dynamics of economic systems can be derived from information asymmetry with respect to Fisher information and that this form of asymmetric information yields a powerful explanatory statistical mechanical framework for financial economics [10–15]. With a common information-theoretic basis for both statistical mechanics and financial economics the notion of statistical mechanics as probability theory with constraints (see, e.g., [62]) suggests that financial economics can be expressed as probability theory with constraints.
3. For a review of the statistical origins of quantum mechanics, see [63].
4. A discrete-time illustration of this assumption expresses observed price data d as [64]

$$d = \sum_{t=1}^{T} \frac{(P_t + P_{t-1} D_t \mu) C_t}{(1+r)^t}, \tag{$*$}$$

   where T is the number of cash flows expected of the security, P_t is the probability that the tth cash flow C_t is received, D_t is the probability of default at time t, μ is the fraction of the cash flow C_t that is received in the event of default, r is the interest rate, and $(1+r)^{-t}$ is the discount factor associated with cash flow C_t. If the cash flows are dividends then this is the dividend-discount model for stock prices. If $D_t = 0$, the cash flows C_t for $t < T$ are coupon payments, and the final cash flow C_T is a coupon and principal payment; this is the model for a government bond: with $D_t > 0$ this becomes the model for a corporate bond. The price of a security is also shown in ($*$) to be the sum of the probability weighted P_t discounted by $(1+r)^{-t}$ state-contingent future payments C_t or $D_t \mu C_t$.
5. Turnover is the ratio of the amount traded (or volume) to the average amount traded during a given time period [65]. As a financial-economic representation of the notion of effective mass, it represents the ease with which a security traverses price space within a market and is a function of the trading environment in the market. The use of turnover in this context of price momentum vm_e is suggested in the economics literature [66–68] and found formally in the econophysics literature with the amount traded introduced by

Khrennikov and Choustova [5, 24–30] and turnover introduced by Ausloos and Ivanova [65].

6. The identity of the Lagrange multiplier λ_0 depends on the physical or economic nature of the associated derivation. In a quantum mechanics, Reginatto identified λ_0 in terms of Plank's constant as $\hbar = \sqrt{8\lambda_0}$ [6]. Derivations in optics or acoustics often identify λ_0 in terms of a wavelength (see, e.g., [69] and references therein). In the current paper we have identified it as a function of economic variables.

7. These last two sentences paraphrase and adapt the original observations of Reginatto regarding the ontological and epistemological content of quantum theory [6].

References

[1] J. E. Stiglitz, "The contributions of the economics of information to twentieth century economics," *Quarterly Journal of Economics*, vol. 115, no. 4, pp. 1441–1478, 2000.

[2] J. E. Stiglitz, "Information and the change in the paradigm in economics: part 1," *The American Economist*, vol. 47, pp. 6–26, 2003.

[3] J. E. Stiglitz, "Information and the change in the paradigm in economics: part 2," *The American Economist*, vol. 48, pp. 17–49, 2004.

[4] P. Mirowski, *More Heat Than Light: Economics as Social Physics, Physics as Nature's Economics. Historical Perspectives on Modern Economics*, Cambridge University Press, New York, NY, USA, 1991.

[5] A. Yu. Khrennikov, *Ubiquitous Quantum Structure*, Springer, Berlin, Germany, 2010.

[6] M. Reginatto, "Derivation of the equations of nonrelativistic quantum mechanics using the principle of minimum Fisher information," *Physical Review A*, vol. 58, no. 3, pp. 1775–1778, 1998.

[7] R. A. Fisher and K. Mather, "The inheritance of style length in Lythrum salicaria," *Annals of Eugenics*, vol. 12, no. 1, pp. 1–32, 1943.

[8] B. R. Frieden, *Physics from Fisher information: A Unification*, Cambridge University Press, Cambridge, UK, 1998.

[9] B. R. Frieden, *Science from Fisher Information: A Unification*, Cambridge University Press, Cambridge, UK, 2004.

[10] B. R. Frieden, R. J. Hawkins, and J. L. D'Anna, "Financial economics from Fisher information," in *Exploratory Data Analysis Using Fisher Information*, B. R. Frieden and R. A. Gatenby, Eds., pp. 42–73, Springer, London, UK, 2007.

[11] B. R. Frieden and R. J. Hawkins, "Asymmetric information and economics," *Physica A*, vol. 389, no. 2, pp. 287–295, 2010.

[12] R. J. Hawkins and B. R. Frieden, "Fisher information and equilibrium distributions in econophysics," *Physics Letters A*, vol. 322, no. 1-2, pp. 126–130, 2004.

[13] R. J. Hawkins, B. R. Frieden, and J. L. D'Anna, "Ab initio yield curve dynamics," *Physics Letters Section A*, vol. 344, no. 5, pp. 317–323, 2005.

[14] R. J. Hawkins, M. Aoki, and B. R. Frieden, "Asymmetric information and macroeconomic dynamics," *Physica A*, vol. 389, no. 17, pp. 3565–3571, 2010.

[15] R. J. Hawkins and B. R. Frieden, "Econophysics," in *Science from Fisher Information: A Unification*, B. R. Frieden, Ed., chapter 3, Cambridge University Press, Cambridge, UK, 2004.

[16] A. Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, Wiley Finance. John Wiley & Sons, New York, NY , USA, 2nd edition, 2002.

[17] J. L. Synge, "Classical dynamics," in *Handbuch der Physik*, pp. 1–225, Springer, Berlin, Germany, 1960.

[18] H. Ishio and E. Haven, "Information in asset pricing: a wave function approach," *Annalen der Physik*, vol. 18, no. 1, pp. 33–44, 2009.

[19] E. Haven, "Pilot-wave Theory and financial option pricing," *International Journal of Theoretical Physics*, vol. 44, no. 11, pp. 1957–1962, 2005.

[20] E. Haven, "Elementary quantum mechanical principles and social science: is there a connection?" *Romanian Journal of Economic Forecasting*, vol. 9, no. 1, pp. 41–58, 2008.

[21] E. Haven, "The variation of financial arbitrage via the use of an information wave function," *International Journal of Theoretical Physics*, vol. 47, no. 1, pp. 193–199, 2008.

[22] E. Haven, "Private information and the "information function": a survey of possible uses," *Theory and Decision*, vol. 64, no. 2-3, pp. 193–228, 2008.

[23] E. Haven, "The Blackwell and Dubins theorem and Rényi's amount of information measure: some applications," *Acta Applicandae Mathematicae*, vol. 109, no. 3, pp. 743–757, 2010.

[24] O. Al. Choustova, "Quantum Bohmian model for financial market," *Physica A*, vol. 374, no. 1, pp. 304–314, 2007.

[25] O. A. Shustova, "Quantum modeling of the nonlinear dynamics of stock prices: the Bohmian approach," *Rossiĭskaya Akademiya Nauk*, vol. 152, no. 2, pp. 405–415, 2007.

[26] O. Choustova, "Application of Bohmian mechanics to dynamics of prices of shares: stochastic model of Bohm-Vigier from properties of price trajectories," *International Journal of Theoretical Physics*, vol. 47, no. 1, pp. 252–260, 2008.

[27] O. Choustova, "Quantum probability and financial market," *Information Sciences*, vol. 179, no. 5, pp. 478–484, 2009.

[28] O. Choustova, "Quantum-like viewpoint on the complexity and randomness of the financial market," in *Coping With the Complexity of Economics*, *New Economic Windows*, F. Petri and F. Hahn, Eds., Springer, Milan, Italy, 2009.

[29] A. Khrennivov, "Classical and quantum mechanics on information spaces with applications to cognitive, psychological, social, and anomalous phenomena," *Foundations of Physics*, vol. 29, no. 7, pp. 1065–1098, 1999.

[30] A. Yu. Khrennikov, "Quantum-psychological model of the stock market," *Problems and Perspectives of Management*, vol. 1, pp. 136–148, 2003.

[31] F. Black and M. Scholes, "The pricing of options and corporate liabilities," *Journal of Political Economy*, vol. 81, pp. 637–654, 1973.

[32] R. C. Merton, "On the pricing of corporate debt: the risk structure of interest rates," *Journal of Finance*, vol. 29, no. 2, pp. 449–470, 1974.

[33] F. Black and J. C. Cox, "Valuing corporate securities: some effects of bond indenture provisions," *Journal of Finance*, vol. 31, no. 2, pp. 351–367, 1976.

[34] E. Madelung, "Quantentheorie in hydrodynamischer form," *Zeitschrift für Physik*, vol. 40, no. 3-4, pp. 322–326, 1927.

[35] E. Nelson, "Derivation of the Schrödinger equation from Newtonian mechanics," *Physical Review*, vol. 150, no. 4, pp. 1079–1085, 1966.

[36] E. Nelson, *Dynamical Theories of Brownian Motion*, Princeton University Press, Princeton, NJ, USA, 1967.

[37] H. Cramér, *Mathematical Methods of Statistics*, vol. 9 of *Princeton Mathematical Series*, Princeton University Press, Princeton, NJ, USA, 1946.

[38] C. Radhakrishna Rao, "Information and the accuracy attainable in the estimation of statistical parameters," *Bulletin of the Calcutta Mathematical Society*, vol. 37, pp. 81–91, 1945.

[39] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, vol. 3 of *Course of Theoretical Physics*, Pergamon Press, New York, NY, USA, 3rd edition, 1977.

[40] D. Bohm, "A suggested interpretation of the quantum theory in terms of "hidden" variables. I," *Physical Review*, vol. 85, pp. 166–179, 1952.

[41] D. Bohm, "A suggested interpretation of the quantum theory in terms of "hidden" variables. II," *Physical Review*, vol. 85, pp. 180–193, 1952.

[42] H. Kleinert, *Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets*, World Scientific, River Edge, NJ, USA, 5th edition, 2009.

[43] R. Wright, "Statistical structures underlying quantum mechanics and social science," *International Journal of Theoretical Physics*, vol. 46, no. 8, pp. 2026–2045, 2007.

[44] J.-P. Bouchaud and M. Potters, *Theory of Financial Risks*, Cambridge University Press, Cambridge, UK, 2nd edition, 2003.

[45] J. Voit, *The Statistical Mechanics of Financial Markets*, Texts and Monographs in Physics, Springer, Berlin, Germany, 3rd edition, 2005.

[46] B. E. Baaquie, *Quantum Finance*, Cambridge University Press, Cambridge, UK, 2004.

[47] B. E. Baaquie, *Interest Rates and Coupon Bonds in Quantum Finance*, Cambridge University Press, Cambridge, UK, 2010.

[48] K. N. Ilinski, *Physics of Finance: Gauge Modelling in Non-Equilibrium Pricing*, John Wiley & Sons, Chichester, UK, 2001.

[49] J. M. Keynes, *The General Theory of Employment, Interest, and Money*, Harvest/Harcourt, San Diego, Calif, USA, 1936.

[50] J. M. Keynes, "The general Theory of employment," *Quarterly Journal of Economics*, vol. 51, pp. 209–223, 1937.

[51] F. H. Knight, *Risk, Uncertainty and Profit. Reprints of Economic Classics*, Augustus M. Kelley, New York, NY, USA, 1921.

[52] M. Aoki and H. Yoshikawa, *Reconstructing Macroeconomics: A Perspective from Statistical Physics and Combinatorial Stochastic Processes. Japan-U.S. Center UFJ Bank Monographs on International Financial Markets*, Cambridge University Press, New York, NY, USA, 2007.

[53] D. K. Foley, "A statistical equilibrium Theory of markets," *Journal of Economic Theory*, vol. 62, pp. 321–345, 1994.

[54] D. K. Foley, "Statistical equilibrium in economics: method, interpretation, and an example," in *General Equilibrium: Problems and Prospects, Routledge Siena Studies in Political Economy*, F. Petri and F. Hahn, Eds., chapter 4, Taylor & Francis, London, UK, 2002.

[55] T. Lux, "Applications of statistical physics in finance and economics," in *Handbook of Research on Complexity*, J. B. Rosser Jr. and K. L. Cramer, Eds., chapter 9, Edward Elgar, Cheltenham, UK, 2009.

[56] V. M. Yakovenko and J. B. Rosser Jr., "Colloquium: statistical mechanics of money, wealth, and income," *Reviews of Modern Physics*, vol. 81, no. 4, pp. 1703–1725, 2009.

[57] R. Balian, "Information Theory and statistical entropy," in *From Microphysics to Macrophysics: Methods and Applications of Statistical Physics*, vol. 1, chapter 3, Springer, New York, NY, USA, 1982.

[58] A. Ben-Naim, *A Farewell to Entropy: Statistical Thermodynamics Based on Information*, World Scientific, Singapore, 2008.

[59] H. Haken, *Information and Self-Organization*, Springer Series in Synergetics, Springer, Berlin, Germany, 2nd edition, 2000.

[60] E. T. Jaynes, *Papers on Probability, Statistics and Statistical Physics*, vol. 158 of *Synthese Library, edited volume of Jaynes' work edited by R. D. Rosenkrantz*, D. Reidel, Dordrecht, The Netherlands, 1983.

[61] A. Katz, *Principles of Statistical Mechanics: The Information Theory Approach*, W. H. Freeman, San Francisco, Calif, USA, 1967.

[62] D. Sornette, *Critical Phenomena in Natural Sciences*, Springer Series in Synergetics, Springer, Berlin, Germany, 2000.

[63] U. Klein, "The statistical origins of quantum mechanics," *Physics Research International*, vol. 2010, Article ID 808424, 18 pages, 2010.

[64] J. S. Fons, "Using default rates to model the term structure of credit risk," *Financial Analysts Journal*, vol. 50, pp. 25–32, 1994.

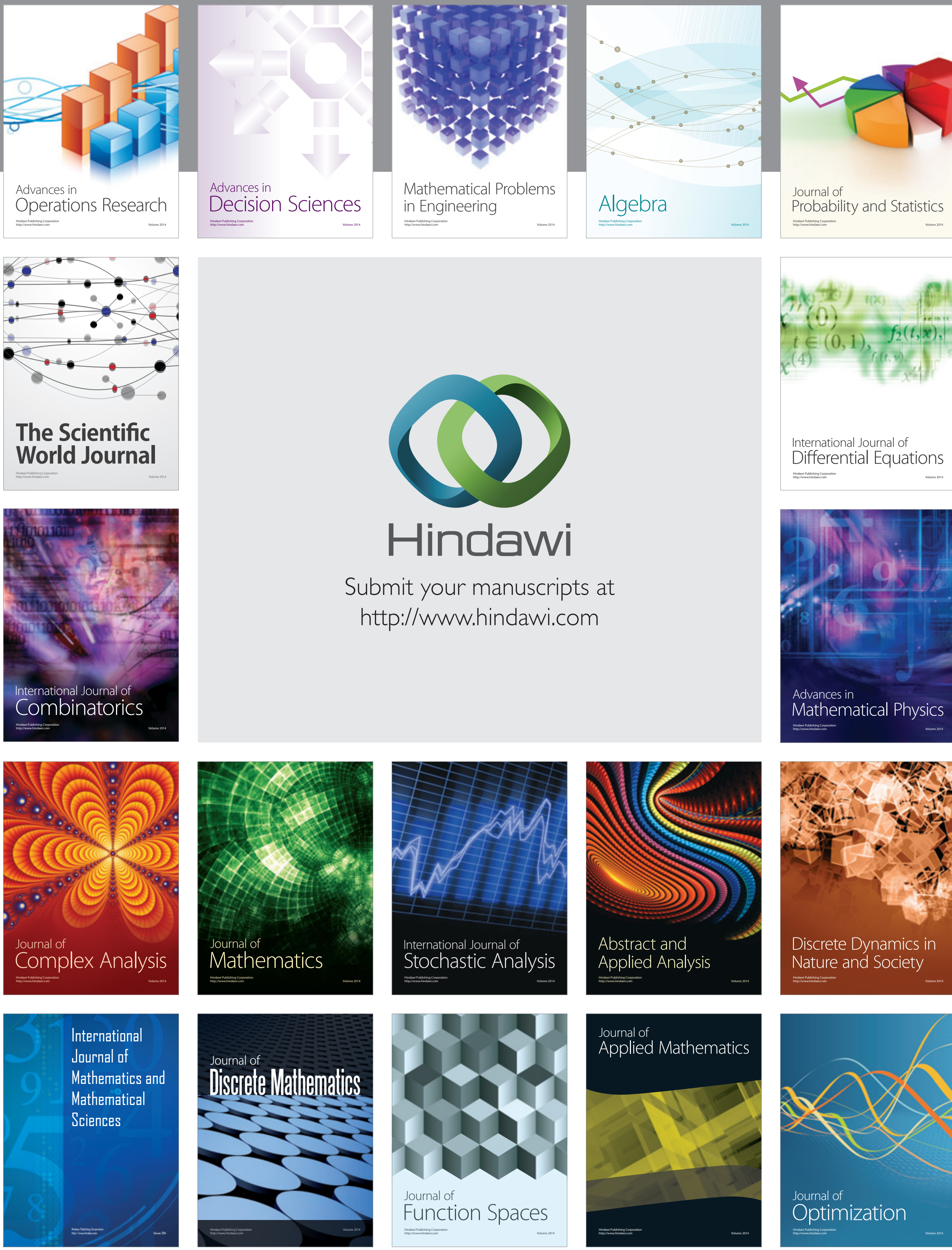
[65] M. Ausloos and K. Ivanova, "Mechanistic approach to generalized technical analysis of share prices and stock market indices," *European Physical Journal B*, vol. 27, no. 2, pp. 177–187, 2002.

[66] J. Karpoff, "The relation between price changes and trading volume: a survey," *Journal of Financial and Quantitative Analysis*, vol. 22, no. 1, pp. 109–126, 1987.

[67] L. Blume, D. Easley, and M. O'Hara, "Market statistics and technical analysis: the role of volume," *Journal of Finance*, vol. 49, no. 1, pp. 153–181, 1994.

[68] C. M. C. Lee and B. Swaminathan, "Price momentum and trading volume," *Journal of Finance*, vol. 55, no. 5, pp. 2017–2069, 2000.

[69] L. S. Schulman, *Techniques and Applications of Path Integration*, John Wiley & Sons, New York, NY, USA, 1981.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