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Review Article

A Survey on Just-Non-$\mathfrak{X}$ Groups

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Let $\mathfrak{X}$ be a class of groups. A group which does not belong to $\mathfrak{X}$ but all of whose proper quotient groups belong to $\mathfrak{X}$ is called just-non-$\mathfrak{X}$ group. The present note is a survey of recent results on the topic with a special attention to topological groups.

1. The Interest in the Literature

If $\mathfrak{X}$ is a class of groups, a group G which belongs to $\mathfrak{X}$ is said to be an *$\mathfrak{X}$-group*. A group G is said to be a *just-non-$\mathfrak{X}$ group*, or briefly a JNX group, if it is not an $\mathfrak{X}$-group but all of its proper quotients are $\mathfrak{X}$-groups.

By default, every simple group which is not an $\mathfrak{X}$-group is a JNX group, so the simple groups constitute an easy source of examples for JNX groups. Their structure was studied for several choices of the class $\mathfrak{X}$, so there is a well-developed theory about the topic. Some classic results can be found in [1–3] and recent contributions in [4–9]. Moreover the study of JNX groups has been investigated both in finite groups and infinite groups so that many techniques have general applications. Heineken's work [2] is typical for this line of research. In it a very special class of groups, namely, *lagrangian groups*, is treated. More recently, a team of authors treated in this fashion the class of *Dedekind groups* in [10].

H. Schunk was interested in studying JNX groups with respect to some problems of local theory of finite groups as [1, Chapter 3] exemplifies. JNX groups were called *groups of boundary* $\mathfrak{X}$ in the original works of H. Schunk and conditions of splitting were found (see [1, Chapters 6, 11]). Most of the time, the literature on JNX groups shows that their description overlaps the results of H. Schunk or a well-known splitting theorem of I. Schur and H. Zassenhaus (see [1, 18.1, 18.2]). Already in the context of locally finite groups, we may find generalizations, as described in [11, Chapter 6].

Some variations can be adapted to the context of topological groups and we will list only two recent contributions.

The first is in [5] and deals with JNL groups. Here a compact group is called a JNL group if it is not a Lie group, but all of its proper Hausdorff quotients are Lie groups. It is proved that a compact JNL group is profinite. This is another evidence that many techniques and methods have a general application in topics concerning JNX groups and that their topology is very special. We note that topological groups are treated in terms of classes and varieties of groups in [12–17], where restrictions, which are caused by the presence of topology, are investigated.

The second contribution is in [9] (and also in [4] under a different prospective) and deals with a topological group which is not compact, but all of its proper quotients are compact, that is, a *just-non-compact group*. Recall that a topological group G is a *pro-Lie group* if it is complete and the set $\mathcal{N}(G)$ of all closed normal subgroups N of G such that G/N is a Lie group is a filter basis converging to 1 (see [18] for feedback on pro-Lie groups). In [9] Theorem 2.3 states that a just-non-compact group which is a pro-Lie group is a Lie group. Furthermore, it can be written as the product of two suitable subgroups in which one of them is finite.

On the other hand, the knowledge of JNX groups is often accompanied by the following notion, which is dual in a certain sense.

A group G is called $\mathfrak{X}$*-critical group*, or *minimal non-*$\mathfrak{X}$*group*, or briefly MNX group, if G is not an $\mathfrak{X}$-group but all of its proper subgroups are $\mathfrak{X}$-groups. There is a long standing line of research on MNX groups, as we can see in [1, pages 59, 330, 402, 408, 480, 515, 525, 781] and in [19–31].

This literature shows that terminology and notations are not uniform and some results can be found independently with different approaches. For instance, the terminology minimal non-$\mathfrak{X}$ group is adopted by [32] while the terminology $\mathfrak{X}$-critical group is adopted by [1, 20].

The reason why JNX groups and MNX groups are related is due to an unexpected symmetry in their structure; this becomes clear once we compare [3, Theorems 11.1, 11.2, 12.26, 12.30, 14.1, 14.2, 14.8, 14.10, 14.18, 14.19, 15.4, 15.5, 15.11, 16.21, 16.24, 16.28, 16.30, 16.31, 16.32, 16.33, 17.5, 17.7, 17.8, 17.9, Corollaries 12.27, 12.28, 12.29] with [33, Theorem 9.1.9, Exercise 9.1.11, Theorem 10.3.3].

For instance, if $\mathfrak{A}$ is the class of the abelian groups, just-non-$\mathfrak{A}$ groups have been completely described by M. F. Newman in [3, Theorems 11.1, 11.2]. He proved that a just-non-$\mathfrak{A}$ group is characterized to be a homomorphic image of a direct product of an extra-special group by a quasicyclic group. Minimal non-$\mathfrak{A}$ groups have been completely described by O. Yu. Schmidt in [33, Theorem 9.1.9].

It is interesting to point out the great symmetry which pervades the result of M. F. Newman and that of O. Yu. Schmidt. The largest normal nilpotent subgroup $\mathrm{Fitt}(G)$ of a group G plays in the structure of a just-non-$\mathfrak{A}$ group the same role which is played by $G/\mathrm{Frat}(G)$ in the structure of a minimal non-$\mathfrak{A}$ group, where $\mathrm{Frat}(G)$ denotes the intersection of all maximal subgroups of G.

We continue to find these analogies for many choices of $\mathfrak{X}$ and not only for $\mathfrak{X} = \mathfrak{A}$. The quoted literature shows this fact in many situations.

The importance of just-non-$\mathfrak{X}$ groups and minimal non-$\mathfrak{X}$ groups becomes more relevant when we look at situations as in [32, Theorem 7.4.1]. Let $\mathfrak{P}$ be the class of polycyclic groups.

For instance, [32, Theorem 7.4.1] states that a finitely generated group G, which is not a polycyclic group, has a suitable homomorphic image which is a just-non-$\mathfrak{P}$ group.

Results of the type of [32, Theorem 7.4.1] holds for many choices of $\mathfrak{X}$ and not only for $\mathfrak{X} = \mathfrak{P}$. This shows that the knowledge of just-non-$\mathfrak{P}$ groups deals with the knowledge of all finitely generated groups and this emphasizes the importance of JNX groups from the point of view of the general theory. Unfortunately, many problems remain unsolved also for easy choices of $\mathfrak{X}$ as [3]; Open Questions shows.

We end this section with some easy observations due to the choice of locally compact groups, once we want to study JNX groups in the topological case. We know that there are some cautionary observations which are necessary to note, in order to have an approach as in [3] to topological groups. The existence of a topology in a group does not allow us to speak in the usual way either of formations or of varieties of groups (see [1, 34]). The literature on varieties of topological groups is relatively recent and most of the classical results of [3, Chapter 2] do not hold in the context of topological groups, because we may have a largest normal nilpotent closed subgroup which is not necessarily the Fitting subgroup (see, e.g., [16, 35] for the generalizations of the Fitting subgroup in profinite groups). Therefore we have to work in the category of Hausdorff topological groups with corresponding morphisms. In order to speak about quotients in a meaningful way in this category, we should refer to quotients modulo closed normal subgroups (see [15, Definition 1.7]). Many situations in this category show that we may not have any closed normal subgroups at all. This fact motivates us to pick a category consisting of Hausdorff topological groups for which the structure and the representation theory is highly developed such as the category of locally compact groups (see [15, 17]).

2. Some Open Questions

The present section deals with some open questions in the context of topological groups when we want to investigate JNX groups and MNX groups. There is literature in the abstract case as we mentioned in Section 1 of the present survey. It seems reasonable that a line of research as in [1, Chapters 6, 11] could be opened up in the context of topological groups, considering varieties of topological groups in the sense of [34].

To the best of our knowledge, a systematic study in such a direction of research should be new. Looking at similar situations for abstract groups, we can formulate some open questions, but we need the following notions.

Definition 2.1. Let Ω be a class of topological groups and $\mathfrak{V}(\Omega)$ a variety of Hausdorff groups generated by Ω. We will consider locally compact groups G in $\mathfrak{V}(\Omega)$, normal closed subgroups $N \neq \{1\}$ of G, and normal closed subgroups $M \neq G$ of G. Define

$$\begin{gathered}\mathcal{M}_{\mathfrak{V}(\Omega)}(G) = \left\{ N \lhd G : \frac{G}{N} \in \mathfrak{V}(\Omega) \right\}, \\ \widehat{\mathcal{M}}_{\mathfrak{V}(\Omega)}(G) = \{ M \lhd G : M \in \mathfrak{V}(\Omega) \}.\end{gathered} \tag{2.1}$$

(i) G is a just-non-$\mathfrak{V}(\Omega)$ group if it does not belong to $\mathfrak{V}(\Omega)$, but all of its closed normal subgroups belong to $\mathcal{M}_{\mathfrak{V}(\Omega)}(G)$.

(ii) G is a minimal non-$\mathfrak{V}(\Omega)$ group if it does not belong to $\mathfrak{V}(\Omega)$, but all of its closed normal subgroups belong to $\widehat{\mathcal{M}}_{\mathfrak{V}(\Omega)}(G)$.

In Definition 2.1, if G is simple and does not belong to $\mathfrak{V}(\Omega)$, then G satisfies both (i) and (ii). This is a source of examples for Definition 2.1.

Another source of examples for Definition 2.1(i) is given by the JNL groups in [5], where $\mathfrak{V}(\Omega) = \mathfrak{L}$ is the variety of Lie groups and G is a compact group. Similarly, the just-non-compact groups in [4, 8] are examples for Definition 2.1(i), when $\mathfrak{V}(\Omega) = \mathfrak{C}$ is the variety of compact groups. At this point, the following question is natural.

Open Question 1. What is the structure of a just-non-$\mathfrak{V}(\Omega)$ group in Definition 2.1(i)? And the structure of a minimal non-$\mathfrak{V}(\Omega)$ group in Definition 2.1(ii)?

Unfortunately, the next two results show that it is impossible to find a compact minimal non-Lie group.

Lemma 2.2. *Let G be a compact group and N a closed normal subgroup such that both N and G/N are Lie groups. Then G is a Lie group.*

Proof. See [13, Theorem 3.1] (or [15, Theorem 6.7]). □

Theorem 2.3. *There are no compact minimal non-Lie groups.*

Proof. Suppose that G is a compact minimal non-Lie group. Then $G \neq \{1\}$, since $\{1\}$ is a Lie group. Hence there is an element $g \in G \setminus \{1\}$. By [15, Lemma 9.1(ii)], there is a closed normal subgroup N such that $g \notin N$ and G/N is a Lie group. Since N is a proper closed subgroup and G is a compact minimal non-Lie group, N is a Lie group. From Lemma 2.2 it now follows that G is a Lie group contrary to the definition. □

Before of the next open question, we need to recall that the *nilradical* $N(G)$ of a topological group G is the subgroup generated by all normal closed nilpotent subgroups in G. In case of profinite groups the reader may refer to [35, page 146]. In case of pro-Lie groups the reader may refer to [18].

Theorem 2.4 (see [5, Proposition 2.4]). *A compact JNL group with a nonsingleton center is a central extension of a group $\mathbb{Z}_p$ of p-adic integers for some prime p by a finite group.*

Generalizations of Theorem 2.4 are recently obtained in [36]. However some problems remain still open. In [5, Example 2.6 (b)] a JNL group G is constructed, which is the semidirect product of its nilradical $N(G)$ by a finite group acting by automorphisms. It is reasonable to expect that all compact centerfree JNL groups can be constructed in this way, but there is not a proof of this fact.

A similar idea is behind the variation of the Schur-Zassenhaus theorem, quoted in Section 1. Now, for the symmetric group on 3 elements S_3, we know that $S_3 = C_3 \rtimes C_2$ and S_3 is minimal non-Abelian. Still a semidirect product is involved in the structure of an MNX group for a suitable choice of $\mathfrak{X}$, but here the topology is trivial, since S_3 is finite. This allows us to formulate the following questions.

Open Question 2. (i) Is it possible to treat JNX groups and MNX groups in terms of categories? And for which $\mathfrak{X}$?

(ii) Is it possible to treat the groups of Definition 2.1(i) and (ii) in terms of categories? And for which Ω?

We end this survey by illustrating some connections with geometric group theory.

A central issue in geometric group theory is to study classes of discrete groups with various properties of a geometrical and topological nature. These properties come from the setting of infinite complexes by means of the following idea, which was first used on large scale by Gromov. One can say that a finitely presented group G has a certain property $\mathcal{P}$ if the universal covering of some finite complex with G as fundamental group has the property $\mathcal{P}$.

Among these properties we are interested in the so-called *quasisimple filtration* (see [37]). This property should be compared to a tameness condition at infinity which is central in noncompact manifold theory, namely, the *simple connectivity at infinity*. Roughly speaking, the simple connectivity at infinity expresses the fact that loops at the infinity bound disks which are also near the infinity (see [38] fore more details on these topics). This topological property has been used by Siebenmann, Stallings, and Freedman for characterizing Euclidean spaces as being the contractible manifolds that are simply connected at infinity. Actually, it has been conjectured for a long time that contractible universal coverings of compact manifolds were homeomorphic to $\mathbb{R}^n$ (or, equivalently, simply connected at infinity). In the 1980s M. Davis came up with examples refuting the conjecture in any dimension $n \geq 4$. Then it is meaningful to ask if there exists a topological property which characterizes contractible universal coverings of compact manifolds. A possible candidate comes from the work of A. Casson and V. Poenaru on the previous conjecture in dimension 3 (see [38, 39]). More precisely, Casson and Poenaru studied some geometric conditions on the Cayley graph of a finitely presented group implying that the universal covering of a compact 3-manifold with given fundamental group is $\mathbb{R}^3$. In the proof of this result, the authors approximate the universal covering by compact, simply connected three-manifolds. Poenaru's main ingredients are the notions of *geometric simple connectivity* (i.e., handlebody decomposition without 1-handles) and *Dehn-exhaustibility* for open manifolds (see [38, 39]). The latter condition was then slightly modified and adapted to finitely presented groups by Brick and Mihalik in [37] as follows.

Definition 2.5. The simply connected noncompact PL space X is qsf (i.e., *quasisimply filtered*) if for any compact $C \subset X$ there exists a simply connected polyhedron K and a PL map $f : K \to X$ so that $C \subset f(K)$ and $f|_{f^{-1}(C)} : f^{-1}(C) \to C$ is a PL homeomorphism.

A finitely presented group G is qsf if the universal covering of the presentation 2-complex associated to one of its presentations is qsf.

Looking at [37–39] we note that the notion in Definition 2.5 influences strongly the structure of finitely presented groups. A classic result is the following.

Theorem 2.6 (Brick-Mihalik, see [37]). *Let G be a finitely presented group.*

(i) *G is qsf if and only if some finite index subgroup H of G is qsf.*

(ii) *Automatic, CAT(0), combable, hyperbolic, or one-relator groups are qsf.*

(iii) *Assume that $1 \to A \to G \to B \to 1$ is a short exact sequence of infinite finitely presented groups. Then G is qsf.*

It follows from Theorem 2.6 that the infinite dihedral group

$$D_\infty = \mathbb{Z} \rtimes C_2 = \left\langle a, x \mid a^x = a^{-1},\ x^2 = 1 \right\rangle \tag{2.2}$$

is qsf. At the same time, D_∞ is a just-nonfinite group, since it has by construction a unique normal subgroup $\mathbb{Z}$ and consequently a unique quotient of order 2. This easy example shows that there are finitely presented groups which are both qsf and just-non-finite. Therefore the following question is natural.

Open Question 3. In a finitely presented group G, for which choices of $\mathfrak{X}$ do we have that G is both qsf and JNX? Are there deeper connections between qsf and JNX?

Finally, we want to list some recent problems, which originate from [4, 9, 40, 41]. Recall that a topological space is called a k_ω-*space* if it is the direct limit of an ascending sequence of compact (Hausdorff) subspaces. A topological group G is k_ω it is k_ω as topological space. The class of k_ω-spaces is quite general and comprises, for example, all countable CW-complexes and countable direct limits of σ-compact locally compact groups (see [40]).

It would be interesting to replace the assumption of the group being locally compact by a milder condition such as being a (locally) k_ω group. There is a rich theory of k_ω topological spaces and a wealth of well-understood examples arising from the theory of Kac-Moody groups over locally compact field. More precisely the following problem is open.

Open Question 4. Is it possible to classify compactly generated just noncompact (locally) k_ω groups?

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