ERRATUM TO "HYPERFINITE AND STANDARD UNIFICATIONS FOR PHYSICAL THEORIES"

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This corrects the major theorem on product consequence operators.

In [1], Definition 5.2, and Theorem 5.3 and its proof are stated incorrectly. The following is the correct definition, theorem, and proof.

Definition 5.2. Suppose one has a nonempty finite set $\mathscr{C} = \{C_1,\ldots,C_m\}$ of general consequence operators, each defined on a nonempty language L_i, $1 \leq i \leq m$. Define the operator ΠC_m as follows: for any $X \subset L_1 \times \cdots \times L_m$, using the projection pr_i, $1 \leq i \leq m$, define $\Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X))$.

Theorem 5.3. *The operator ΠC_m defined on the subsets of $L_1 \times \cdots \times L_m$ is a general consequence operator and if, at least, one member of $\mathscr{C}$ is axiomless, then ΠC_m is axiomless. If each member of $\mathscr{C}$ is finitary and axiomless, then ΠC_m is finitary.*

Proof. (a) Let $X \subset L_1 \times \cdots \times L_m$. Then for each i, $1 \leq i \leq m$, $pr_i(X) \subset C_i(pr_i(X)) \subset L_i$. But, $X \subset pr_1(X) \times \cdots \times pr_m(X) \subset C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X) \subset L_1 \times \cdots \times L_m$. Suppose that $X \neq \varnothing$. Then $\varnothing \neq \Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset L_1 \times \cdots \times L_m$. Hence, $\varnothing \neq pr_i(\Pi C_m(X)) = C_i(pr_i(X))$, $1 \leq i \leq m$, implies that $C_i(pr_i(\Pi C_m(X))) = C_i(C_i(pr_i(X))) = C_i(pr_i(X))$, $1 \leq i \leq m$. Hence, $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Let $X = \varnothing$ and assume that no member of $\mathscr{C}$ is axiomless. Then each $pr_i(X) = \varnothing$. But, each $C_i(pr_i(X)) \neq \varnothing$ implies that $\Pi C_m(X) \neq \varnothing$. By the previous method, it follows, in this case, that $\Pi C_m(\Pi C_m(X)) = \Pi C_m(X)$. Now suppose that there is some j such that C_j is axiomless. Hence, $C_j(pr_j(X)) = \varnothing$ implies that $\Pi C_m(X) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \varnothing$, which implies that $C_j(pr_j(\Pi C_m(X))) = \varnothing$. Consequently, $C_1(pr_1(\Pi C_m(X))) \times \cdots \times C_m(pr_m(\Pi C_m(X))) = \varnothing$. Thus, $\Pi C_m(\Pi C_m(X)) = \varnothing$ and axiom (1) holds. Also in the case where at least one member of $\mathscr{C}$ is axiomless, then ΠC_m is axiomless.

(b) Let $X \subset Y \subset L_1 \times \cdots \times L_m$. For each i, $1 \leq 1 \leq m$, $pr_i(X) \subset pr_i(Y)$, whether $pr_i(X)$ is the empty set or not. Hence, $C_i(pr_i(X)) \subset C_i(pr_i(Y))$. Therefore, $\Pi C_m(X) =$

$C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) \subset C_1(pr_1(Y)) \times \cdots \times C_m(pr_m(Y)) = \Pi C_m(Y))$ and axiom (2) holds. Thus, ΠC_m is, at least, a general consequence operator.

(c) Assume that each member of $\mathscr{C}$ is finitary and axiomless and let $x \in \Pi C_m(X)$ where, since ΠC_m is axiomless, X is nonempty. Then for each i, $pr_i(x) \in C_i(pr_i(X))$. Since each C_i is finitary and axiomless, then there is some nonempty finite $F_i \subset pr_i(X)$ such that $pr_i(x) \in C_i(F_i) \subset C_i(pr_i(X))$. Hence, nonempty and finite $F = F_1 \times \cdots \times F_m \subset pr_1(X) \times \cdots \times pr_m(X)$. Then for each i, $pr_i(F) = F_i$ implies that finite $F = F_1 \times \cdots \times F_m = pr_1(F) \times \cdots \times pr_m(F) \subset pr_1(X) \times \cdots \times pr_m(X)$. From axiom (2), $x \in \Pi C_m(F) = C_1(pr_1(F)) \times \cdots \times C_m(pr_m(F)) \subset \Pi C_m(pr_1(X) \times \cdots \times pr_m(X)) = C_1(pr_1(X)) \times \cdots \times C_m(pr_m(X)) = \Pi C_m(X)$. This completes the proof. $\square$

References

[1] R. A. Herrmann, *Hyperfinite and standard unifications for physical theories*, International Journal of Mathematics and Mathematical Sciences **28** (2001), no. 2, 93–102.

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