

EFFECT OF MAGNETIC FIELD ON THERMAL INSTABILITY OF A ROTATING RIVLIN-ERICKSEN VISCOELASTIC FLUID

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The thermal instability of a rotating Rivlin-Ericksen viscoelastic fluid in the presence of uniform vertical magnetic field is considered. For the case of stationary convection, Rivlin-Ericksen viscoelastic fluid behaves like a Newtonian fluid. It is found that rotation has a stabilizing effect, whereas the magnetic field has both stabilizing and destabilizing effects. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. The rotation and magnetic field are found to introduce oscillatory modes in the system which were nonexistent in their absence.

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1. Introduction

The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in geophysics, interior of the Earth, oceanography, and the atmospheric physics, and so forth, and has been investigated by several authors (e.g., Bénard [1], Rayleigh [8], Jeffreys [6]) under different conditions.

A detailed account of the theoretical and experimental study of thermal instability (Bénard convection) in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [4]. The use of Boussinesq approximation has been made throughout, which states that the density may be treated as a constant in all the terms in the equations of motion except the external force term. Bhatia and Steiner [2] have considered the effect of a uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and have found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner [3]. They have found that the magnetic field stabilizes a viscoelastic (Maxwell) fluid just as the Newtonian fluid. Sharma [10] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted on by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. In another study, Sharma [9] has studied the stability

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of a layer of an electrically conducting Oldroyd fluid [7] in the presence of a magnetic field and has found that the magnetic field has a stabilizing influence.

There are many elasto-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's [7] constitutive relations. One such class of elasto-viscous fluids is Rivlin-Ericksen fluid. Srivastava and Singh [13] have studied the unsteady flow of a dusty elasto-viscous Rivlin-Ericksen fluid through channels of different cross-sections in the presence of a time-dependent pressure gradient. In another study, Garg et al. [5] have studied the rectilinear oscillations of a sphere along its diameter in a conducting dusty Rivlin-Ericksen fluid in the presence of a uniform magnetic field. Sharma and Kumar [11] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elasto-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. A layer of such fluid heated from below under the action of magnetic field and rotation may find applications in geophysics, interior of the Earth, oceanography, and the atmospheric physics.

Keeping in mind the importance of non-Newtonian fluids, convection in fluid layer heated from below, magnetic field, and rotation, the present paper attempts to study the effect of uniform vertical magnetic field on Rivlin-Ericksen viscoelastic fluid heated from below in the presence of a uniform rotation.

2. Formulation of the problem and perturbation equations

Consider an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer of thickness d , heated from below so that the temperatures and densities at the bottom surface $z = 0$ are T_0 and ρ_0 and at the upper surface $z = d$ are T_d and ρ_d , respectively, and that a uniform temperature gradient $\beta (= |dT/dz|)$ is maintained. The gravity field $\vec{g}(0,0,-g)$, a uniform vertical magnetic field $\vec{H}(0,0,H)$, and a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$ act on the system.

The equations of motion, continuity, heat conduction, and Maxwell's equations governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of magnetic field and rotation are

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\nabla \left(\frac{p}{\rho_0} - \frac{1}{2} |\vec{\Omega} \times \vec{r}|^2 \right) + \vec{g} \left(1 + \frac{\delta \rho}{\rho_0} \right) + \left(v + v' \frac{\partial}{\partial t} \right) \nabla^2 \vec{v} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \vec{H}) \times \vec{H} + 2(\vec{v} \times \vec{\Omega}), \quad (2.1)$$

$$\nabla \cdot \vec{v} = 0, \quad (2.2)$$

$$\frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla) T = \chi \nabla^2 T, \quad (2.3)$$

$$\nabla \cdot \vec{H} = 0, \quad (2.4)$$

$$\frac{\partial \vec{H}}{\partial t} = (\vec{H} \cdot \nabla) \vec{v} + \eta \nabla^2 \vec{H}, \quad (2.5)$$

where $\vec{v}(u, v, w)$, p , ρ , T , v , and v' denote the velocity, pressure, density, temperature, kinematic viscosity, and kinematic viscoelasticity, respectively, and $\vec{r}(x, y, z)$.

The equation of state for the fluid is

$$\rho = \rho_0[1 - \alpha(T - T_0)], \quad (2.6)$$

where ρ_0 , T_0 are, respectively, the density and temperature of the fluid at the reference level $z = 0$ and α is the coefficient of thermal expansion. In writing (2.1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equations of motion except the external force term. The magnetic permeability μ_e , thermal diffusivity χ , and electrical resistivity η are all assumed to be constant.

The initial state is one in which the velocity, density, pressure, and temperature at any point in the fluid are, respectively, given by

$$\vec{v} = (0, 0, 0), \quad \rho = \rho(z), \quad p = p(z), \quad T = T(z). \quad (2.7)$$

Let $\vec{v}(u, v, w)$, δp , $\delta\rho$, θ , and $\vec{h}(h_x, h_y, h_z)$ denote, respectively, the perturbations in velocity \vec{v} (initially zero), pressure p , density ρ , temperature T , and the magnetic field $\vec{H}(0, 0, H)$. The change in density $\delta\rho$, caused by the perturbation θ in temperature, is given by

$$\rho + \delta\rho = \rho_0[1 - \alpha(T + \theta - T_0)] = \rho - \alpha\rho_0\theta, \quad \text{i.e., } \delta\rho = -\alpha\rho_0\theta. \quad (2.8)$$

Then the linearized perturbation equations are

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} &= -\frac{1}{\rho_0}(\nabla \delta p) - \vec{g}\alpha\theta + \left(v + v' \frac{\partial}{\partial t}\right) \nabla^2 \vec{v} + \frac{\mu_e}{4\pi\rho_0} (\nabla \times \vec{h}) \times \vec{H} + 2(\vec{v} \times \vec{\Omega}), \\ \nabla \cdot \vec{v} &= 0, \\ \frac{\partial \theta}{\partial t} &= \beta w + \chi \nabla^2 \theta, \\ \nabla \cdot \vec{h} &= 0, \\ \frac{\partial \vec{h}}{\partial t} &= (\vec{H} \cdot \nabla) \vec{v} + \eta \nabla^2 \vec{h}. \end{aligned} \quad (2.9)$$

Within the framework of Boussinesq approximation, (2.9) become

$$\begin{aligned} \frac{\partial}{\partial t} \nabla^2 w &= \left(v + v' \frac{\partial}{\partial t}\right) \nabla^4 w + \frac{\mu_e H}{4\pi\rho_0} \nabla^2 \left(\frac{\partial h_z}{\partial z}\right) + g\alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) - 2\Omega \frac{\partial \zeta}{\partial z}, \\ \frac{\partial \zeta}{\partial t} &= \left(v + v' \frac{\partial}{\partial t}\right) \nabla^2 \zeta + 2\Omega \frac{\partial w}{\partial z} - \frac{\mu_e H}{4\pi\rho_0} \frac{\partial \xi}{\partial z}, \\ \left[\frac{\partial}{\partial t} - \chi \nabla^2\right] \theta &= \beta w, \\ \left[\frac{\partial}{\partial t} - \eta \nabla^2\right] h_z &= H \frac{\partial w}{\partial z}, \\ \left[\frac{\partial}{\partial t} - \eta \nabla^2\right] \xi &= H \frac{\partial \zeta}{\partial z}, \end{aligned} \quad (2.10)$$

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where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ and $\zeta = \partial v/\partial x - \partial u/\partial y$; $\xi = \partial h_y/\partial x - \partial h_x/\partial y$ stand for the z -components of vorticity and current density, respectively.

3. Dispersion relation

We now analyze the disturbances into normal modes, assuming that the perturbation quantities are of the form

$$[w, \theta, h_z, \zeta, \xi] = [W(z), \Theta(z), K(z), Z(z), X(z)] \exp(ik_x x + ik_y y + nt), \quad (3.1)$$

where k_x, k_y are the wave numbers along x - and y -directions, respectively, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number, and n is the growth rate which is, in general, a complex constant.

Using expression (3.1), (2.10) in nondimensional form transform to

$$\left[\sigma(D^2 - a^2)W + \left(\frac{g\alpha d^2}{v}\right)a^2\Theta + \frac{2\Omega d^3}{v}DZ - \frac{\mu_e H d}{4\pi\rho_0 v}(D^2 - a^2)DK \right] \\ = [1 + F\sigma](D^2 - a^2)^2 W, \quad (3.2)$$

$$[\{1 + F\sigma\}(D^2 - a^2) - \sigma]Z = -\left(\frac{2\Omega d}{v}\right)DW - \left(\frac{\mu_e H d}{4\pi\rho_0 v}\right)DX, \quad (3.3)$$

$$[D^2 - a^2 - p_1\sigma]\Theta = -\left(\frac{\beta d^2}{\chi}\right)W, \quad (3.4)$$

$$[D^2 - a^2 - p_2\sigma]K = -\left(\frac{H d}{\eta}\right)DW, \quad (3.5)$$

$$[D^2 - a^2 - p_2\sigma]X = -\left(\frac{H d}{\eta}\right)DZ, \quad (3.6)$$

where we have introduced new coordinates $(x', y', z') = (x/d, y/d, z/d)$ in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have put $a = kd$, $\sigma = nd^2/v$, $F = v'/d^2$; $p_1 = v/\chi$ is the Prandtl number and $p_2 = v/\eta$ is the magnetic Prandtl number.

We now consider the case where both the boundaries are free as well as perfect conductors of heat, while the adjoining medium is also perfectly conducting. The case of two free boundaries is slightly artificial, except in stellar atmospheres (see Spiegel [12]) and in certain geophysical situations where it is most appropriate. However, the case of two free boundaries allows us to obtain analytical solution without affecting the essential features of the problem. The appropriate boundary conditions, with respect to which (3.2)–(3.6) must be solved, are

$$W = D^2 W = 0, \quad DZ = 0, \quad \Theta = 0 \quad \text{at } z = 0, z = 1, \\ DX = 0, \quad K = 0, \quad (3.7)$$

on a perfectly conducting boundary.

Using the above boundary conditions, it can be shown that all the even-order derivatives of W must vanish for $z = 0$ and $z = 1$, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (3.8)$$

where W_0 is a constant.

Eliminating Θ , K , Z , and X between (3.2)–(3.6) and substituting (3.8) in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x} \right) \frac{[\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1][1+x+i\sigma_1 p_1]}{(1+x+i\sigma_1 p_2)} \quad (3.9)$$

$$+ \frac{T_1(1+x+i\sigma_1 p_2)(1+x+i\sigma_1 p_1)}{x[\{(1+iF_1\sigma_1\pi^2)(1+x)+i\sigma_1\}\{1+x+i\sigma_1 p_2\}+Q_1]},$$

where $R = g\alpha\beta d^4/v\chi$, $Q = \mu_e H^2 d^2/4\pi\rho_0 v\eta$, $T_A = 4\Omega^2 d^4/v^2$ stand for the Rayleigh-number, the Chandrasekhar number, the Taylor number, respectively, and we have also put

$$x = \frac{a^2}{\pi^2}, \quad R_1 = \frac{R}{\pi^4}, \quad i\sigma_1 = \frac{\sigma}{\pi^2}, \quad F_1 = \pi^2 F, \quad (3.10)$$

$$T_1 = \frac{T_A}{\pi^4}, \quad Q_1 = \frac{Q}{\pi^2}, \quad i = \sqrt{-1}.$$

4. The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (3.9) reduces to

$$R_1 = \left(\frac{1+x}{x} \right) [(1+x)^2 + Q_1] + \frac{T_1(1+x)^2}{x[(1+x)^2 + Q_1]}, \quad (4.1)$$

a result given by Chandrasekhar [4, equation (59), page 202].

We thus find that for the stationary convection, the viscoelasticity parameter F vanishes with σ and Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid.

To study the effects of rotation and magnetic field, we examine the natures of dR_1/dT_1 and dR_1/dQ_1 analytically.

Equation (4.1) yields

$$\frac{dR_1}{dT_1} = \frac{(1+x)^2}{x[(1+x)^2 + Q_1]}, \quad (4.2)$$

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x} - \frac{T_1(1+x)^2}{x[(1+x)^2 + Q_1]^2}. \quad (4.3)$$

It is evident from (4.2) that for a stationary convection, dR_1/dT_1 is always positive, thus, the rotation has a stabilizing effect on the system. It is also clear from (4.3) that for

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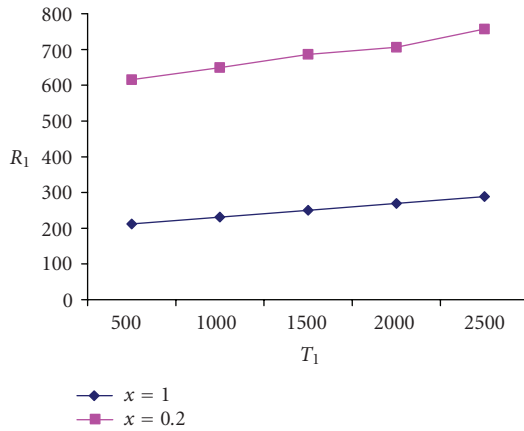


Figure 4.1. The variation of R_1 with T_1 for fixed values of $Q_1 = 100$ and $x = 0.2, 1$.

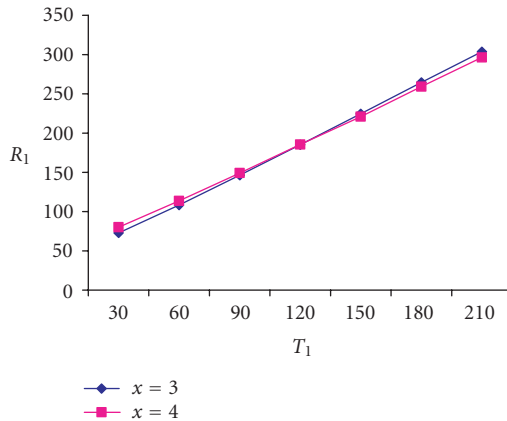


Figure 4.2. The variation of R_1 with Q_1 for fixed values of $T_1 = 100$ and $x = 3, 4$.

a stationary convection, dR_1/dQ_1 may be positive as well as negative, thus, the magnetic field has both stabilizing and destabilizing effects on the system.

The dispersion relation (4.1) is also analyzed numerically. In Figure 4.1, R_1 is plotted against T_1 , for fixed value of $Q_1 = 100$ and wave numbers $x = 0.2, 1$. The Rayleigh number R_1 increases with increase in rotation parameter T_1 showing its stabilizing effect on the system. Figure 4.2 shows the variation of R_1 with respect to Q_1 , for fixed value of $T_1 = 100$ and wave numbers $x = 3, 4$. It clearly depicts both the stabilizing and destabilizing effects of the magnetic field on the system.

5. Stability of the system and oscillatory modes

Multiplying (3.2) by W^* , the complex conjugate of W , integrating the resulting equation over the range of z and using (3.3)–(3.6), together with the boundary conditions (3.7),

we obtain

$$\begin{aligned}
 & -\sigma I_1 + \frac{g\alpha\chi a^2}{v\beta} (I_2 + p_1\sigma^* I_3) - d^2(1 + F\sigma^*) I_4 - d^2\sigma^* I_5 \\
 & - \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} (I_6 + p_2\sigma I_7) - \frac{\mu_e \eta}{4\pi\rho_0 v} (I_8 + p_2\sigma^* I_9) = (1 + F\sigma) I_{10},
 \end{aligned} \tag{5.1}$$

where

$$\begin{aligned}
 I_1 &= \int_0^1 (|DW|^2 + a^2|W|^2) dz, & I_2 &= \int_0^1 (|D\Theta|^2 + a^2|\Theta|^2) dz, \\
 I_3 &= \int_0^1 (|\Theta|^2) dz, & I_4 &= \int_0^1 (|DZ|^2 + a^2|Z|^2) dz, \\
 I_5 &= \int_0^1 (|Z|^2) dz, & I_6 &= \int_0^1 (|DX|^2 + a^2|X|^2) dz, \\
 I_7 &= \int_0^1 (|X|^2) dz, & I_8 &= \int_0^1 (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2) dz, \\
 I_9 &= \int_0^1 (|DK|^2 + a^2|K|^2) dz, & I_{10} &= \int_0^1 (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2) dz,
 \end{aligned} \tag{5.2}$$

and σ^* is the complex conjugate of σ . The integrals I_1, \dots, I_{10} are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$, where σ_r, σ_i are real and equating the real and imaginary parts of (5.1), we obtain

$$\begin{aligned}
 \sigma_r & \left[-I_1 + \frac{g\alpha\chi a^2}{v\beta} p_1 I_3 - d^2 F I_4 - d^2 I_5 - \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} p_2 I_7 - \frac{\mu_e \eta}{4\pi\rho_0 v} p_2 I_9 - F I_{10} \right] \\
 & = -\frac{g\alpha\chi a^2}{v\beta} I_2 + d^2 I_4 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} I_6 + \frac{\mu_e \eta}{4\pi\rho_0 v} I_8 + I_{10},
 \end{aligned} \tag{5.3}$$

$$\sigma_i \left[I_1 + \frac{g\alpha\chi a^2}{v\beta} p_1 I_3 - d^2 F I_4 - d^2 I_5 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} p_2 I_7 - \frac{\mu_e \eta}{4\pi\rho_0 v} p_2 I_9 + F I_{10} \right] = 0. \tag{5.4}$$

6. Discussion

From (5.4), it is clear that σ_i is zero when the quantity multiplying it is not zero and arbitrary when this quantity is zero.

If $\sigma_i \neq 0$, then (5.4) gives

$$\frac{g\alpha\chi a^2}{v\beta} p_1 I_3 - d^2 F I_4 - d^2 I_5 - \frac{\mu_e \eta}{4\pi\rho_0 v} p_2 I_9 = -I_1 - \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} p_2 I_7 - F I_{10}. \tag{6.1}$$

Substituting in (5.3), we have

$$I_{10} + \frac{\mu_e \eta}{4\pi\rho_0 v} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} I_6 + d^2 I_4 + 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0 v} p_2 I_7 + F I_{10} \right] = \frac{g\alpha\chi a^2}{v\beta} I_2. \tag{6.2}$$

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Equation (6.2) on using Rayleigh-Ritz inequality gives

$$\begin{aligned} & \frac{(\pi^2 + a^2)^3}{a^2} \int_0^1 |W|^2 dz + \frac{(\pi^2 + a^2)}{a^2} \\ & \quad \times \left\{ \frac{\mu_e \eta}{4\pi\rho_0\nu} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\nu} I_6 + d^2 I_4 + 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\nu} p_2 I_7 + FI_{10} \right] \right\} \\ & \leq \frac{g\alpha\chi}{\nu\beta} \int_0^1 |W|^2 dz. \end{aligned} \quad (6.3)$$

Therefore, it follows from (6.3) that

$$\begin{aligned} & \left[\frac{27\pi^4}{4} - \frac{g\alpha\chi}{\nu\beta} \right] \int_0^1 |W|^2 dz + \frac{(\pi^2 + a^2)}{a^2} \\ & \quad \times \left\{ \frac{\mu_e \eta}{4\pi\rho_0\nu} I_8 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\nu} I_6 + d^2 I_4 + 2\sigma_r \left[I_1 + \frac{\mu_e d^2 \eta}{4\pi\rho_0\nu} p_2 I_7 + FI_{10} \right] \right\} \leq 0, \end{aligned} \quad (6.4)$$

since minimum value of $(\pi^2 + a^2)^3/a^2$ with respect to a^2 is $27\pi^4/4$.

Now, let $\sigma_r \geq 0$, we necessarily have from (6.4) that

$$\frac{g\alpha\chi}{\nu\beta} > \frac{27\pi^4}{4}. \quad (6.5)$$

Hence, if

$$\frac{g\alpha\chi}{\nu\beta} \leq \frac{27\pi^4}{4}, \quad (6.6)$$

then $\sigma_r < 0$. Therefore, the system is stable.

Therefore, under condition (6.6), the system is stable and under condition (6.5) the system becomes unstable.

In the absence of rotation and magnetic field, (5.4) reduces to

$$\sigma_i \left[I_1 + \frac{g\alpha\chi a^2}{\nu\beta} p_1 I_3 + FI_{10} \right] = 0, \quad (6.7)$$

and the terms in brackets are positive definite. Thus, $\sigma_i = 0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for Rivlin-Ericksen viscoelastic fluid heated from below.

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Nomenclature

d	Depth of layer
T	Temperature
g	Acceleration due to gravity
\vec{g}	Gravity field
$\vec{H}(0, 0, H)$	Uniform vertical magnetic field
$\vec{\Omega}(0, 0, \Omega)$	Uniform vertical rotation field
\vec{v}	Filter velocity
p	Fluid pressure
$\vec{r}(x, y, z)$	Space coordinates
δp	Perturbation in pressure
$\vec{h}(h_x, h_y, h_z)$	Perturbation in magnetic field
k_x, k_y	Wave numbers in x - and y -directions
k	Resultant wave number
n	Growth rate
p_1	Prandtl number
p_2	Magnetic Prandtl number
R	Rayleigh number
Q	Chandrasekhar number
a	Dimensionless wave number
F	Dimensionless kinematic viscoelasticity

Greek letters

μ	Fluid viscosity
μ'	Fluid viscoelasticity
ρ	Density
β	Uniform temperature gradient
ν	Kinematic viscosity
ν'	Kinematic viscoelasticity
μ_e	Magnetic permeability
α	Coefficient of thermal expansion
χ	Thermal diffusivity
η	Electrical resistivity
$\delta\rho$	Perturbation in density
θ	Perturbation in temperature
ζ	z -component of vorticity
ξ	z -component of current density

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