T-FUZZY IDEALS IN BCI-ALGEBRAS

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Using a t-norm T, we introduce the notions of T-fuzzy subalgebras and T-fuzzy H-ideals in BCI-algebras and investigate some of their properties.

1. Introduction

The notion of BCI-algebras was introduced by Iséki [3] which is a generalization of BCK-algebras [2]. This notion is originated from two different ways: one of the motivations is based on set theory, another motivation is from classical and nonclassical propositional calculi.

Zadeh [8] introduced the notion of fuzzy sets. Many researchers have applied this concept to mathematical branches, such as semigroup, loop, group, ring, semiring, field, near ring, vector spaces, topological spaces, functional analysis, automation. Jun et al. [4, 5] introduced the notions of fuzzy subalgebras and fuzzy ideals of BCK-algebras with respect to a t-norm T, and studied some of their properties. In this paper, we obtain some related results of T-fuzzy subalgebras and T-fuzzy H-ideals in BCI-algebras.

2. Preliminaries

In this section, we review some definitions that will be used in the sequel.

Definition 2.1. An algebra $(X;*,0)$ of type (2, 0) is called a BCI-*algebra* if, for all $x,y,z \in X$, the following axioms hold.

(1) $((x*y)*(x*z))*(z*y)=0$.
(2) $(x*(x*y))*y=0$.
(3) $x*x=0$.
(4) $x*y=0$ and $y*x=0 \Rightarrow x=y$.

In BCI-algebras, the following hold.

(5) $(x*0)=x$.
(6) $(x*y)*z=(x*z)*y$.
(7) $0*(y*x)=(0*y)*(0*x)$.

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Definition 2.2. Let S be a nonempty subset of a BCI-algebra X, then S is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$.

Definition 2.3. A subset A of a BCI-algebra $(X;*,0)$ is called an *ideal* of X if for any $x, y \in X$, the following conditions hold.

(i) $0 \in A$.

(ii) $x * y$ and $y \in A$ imply that $x \in A$.

Definition 2.4 [6]. A subset A of a BCI-algebra $(X;*,0)$ is called an *H-ideal* of X if for any $x, y, z \in X$, the following conditions hold.

(a) $0 \in A$.

(b) $(x * (y * z))$ and $y \in A \Rightarrow x * z \in A$.

Definition 2.5. A mapping $f : X \to Y$ of BCI-algebras is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

Note that if f is a homomorphism of BCI-algebras, then $f(0) = \acute{0}$.

Definition 2.6. Let X be a nonempty set. A *fuzzy* (sub)set μ of the set X is a mapping $\mu : X \to [0,1]$. The *complement* of a fuzzy set μ of a set X is denoted by $\overline{\mu}$ and defined by $\overline{\mu}(x) = 1 - \mu(x)$, for all $x \in X$.

Definition 2.7 [1]. A triangular norm (t-norm) is a function $T : [0,1] \times [0,1] \to [0,1]$ that satisfies the following conditions.

(T1) $T(x,1) = x$.

(T2) $T(x,y) = T(y,x)$.

(T3) $T(x,T(y,z)) = T(T(x,y),z)$.

(T4) $T(x,y) \leq T(x,z)$ whenever $y \leq z$, for all $x, y, z \in [0,1]$.

A simple example of such defined t-norm is a function $T(x,y) = \min(x,y)$. In a general case, $T(x,y) \leq \min(x,y)$ and $T(x,0) = 0$ for all $x, y \in [0,1]$.

3. *T*-fuzzy subalgebras

In what follows, let X denote a BCI-algebra unless otherwise specified.

Definition 3.1. A fuzzy set μ in X is called a subalgebra of X with respect to a t-norm T (briefly, a T-fuzzy subalgebra of X) if $\mu(x * y) \geq T(\mu(x), \mu(y))$ for all $x, y \in X$.

Example 3.2. Let $X := \{0,1,2\}$ be a BCI-algebra with the following Cayley table:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Define a fuzzy set $\mu : X \to [0,1]$ by $\mu(0) = 0.81$ and $\mu(x) = 0.25$ for all $x \neq 0$ and let $T_m : [0,1] \times [0,1] \to [0,1]$ be a function defined by $T_m(x,y) = \max(x + y - 1, 0)$ which is a t-norm for all $x, y \in [0,1]$. Then T_m is a t-norm [7]. By routine calculations, it is easy to check that μ satisfies $\mu(x * y) \geq T_m(\mu(x),\mu(y))$ for all $x, y \in X$. Hence, μ is a T_m-fuzzy subalgebra of X.

THEOREM 3.3. *Let μ be a T-fuzzy subalgebra of X and $\alpha \in [0,1]$.*
(i) *If $\alpha = 1$, then $U(\mu;\alpha)$ is either empty or a subalgebra of X.*
(ii) *If $T = \min$, then $U(\mu;\alpha)$ is either empty or a subalgebra of X.*
(iii) *$\mu(0) \geq \mu(x)$ for all $x \in X$.*

Proof. (i) Assume that $\alpha = 1$. If $x, y \in U(\mu;1)$, then $\mu(x) \geq 1$ and $\mu(y) \geq 1$. It follows from Definitions 2.7 and 3.1 that $\mu(x * y) \geq T(\mu(x),\mu(y)) \geq T(1,1) = 1$ so that $x * y \in U(\mu;1)$, that is, $U(\mu;1)$ is a subalgebra of X.

(ii) Assume that $T = \min$ and let $x, y \in U(\mu;\alpha)$. Then, $\mu(x * y) \geq T(\mu(x),\mu(y)) = \min(\mu(x),\mu(y)) \geq \min(\alpha,\alpha) = \alpha$, and so $x * y \in U(\mu;\alpha)$.

Hence $U(\mu;\alpha)$ is a subalgebra of X.

(iii) Since $x * x = 0$ for all $x \in X$, we have $\mu(0) = \mu(x * x) \geq T(\mu(x),\mu(x)) = \min(\mu(x),\mu(x)) = \mu(x)$. This completes the proof. □

THEOREM 3.4. *Let μ be a T-fuzzy subalgebra of X. If there is a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} T(\mu(x_n),\mu(x_n)) = 1$, then $\mu(0) = 1$.*

Proof. Let $x \in X$, then $\mu(0) = \mu(x * x) \geq T(\mu(x),\mu(x))$. Hence $\mu(0) \geq T(\mu(x_n),\mu(x_n))$ for any $n \in \mathbb{N}$. Since $1 \geq \mu(0) \geq \lim_{n\to\infty} T(\mu(x_n),\mu(x_n)) = 1$, it follows that $\mu(0) = 1$. This completes the proof. □

Definition 3.5. Let λ and μ be T-fuzzy subalgebras of X. Then *direct product* of T-fuzzy subalgebras is defined by $(\lambda \times \mu)(x, y) = T(\lambda(x),\mu(y))$, for all $x, y \in X$.

THEOREM 3.6. *If μ_1 and μ_2 are T-fuzzy subalgebras of X, then $\mu = \mu_1 \times \mu_2$ is a T-fuzzy subalgebra of $X \times X$.*

Proof. For any (x_1,x_2) and $(y_1, y_2) \in X \times X$, we have

$$\begin{aligned}
\mu((x_1,x_2) * (y_1,y_2)) &= \mu(x_1 * y_1, x_2 * y_2) \\
&= (\mu_1 \times \mu_2)(x_1 * y_1, x_2 * y_2) \\
&= T(\mu_1(x_1 * y_1),\mu_2(x_2 * y_2)) \\
&\geq T(T(\mu_1(x_1),\mu_1(y_1)),T(\mu_2(x_2),\mu_2(y_2))) \\
&= T(T(\mu_1(x_1),\mu_2(x_2)),T(\mu_1(y_1),\mu_2(y_2))) \\
&= T((\mu_1 \times \mu_2)(x_1,x_2),(\mu_1 \times \mu_2)(y_1,y_2)) \\
&= T(\mu(x_1,x_2),\mu(y_1,y_2)).
\end{aligned} \tag{3.1}$$

Hence, $\mu = \mu_1 \times \mu_2$ is a T-fuzzy subalgebra of $X \times X$. □

Definition 3.7. Let f be a mapping on X. If ν is a fuzzy set in $f(X)$, then fuzzy set $\mu = \nu \circ f$ (i.e., $(\nu \circ f)(x) = \nu(f(x))$) in X is called *preimage* of ν under f.

THEOREM 3.8. *An epimorphism preimage of a T-fuzzy subalgebra of X is a T-fuzzy subalgebra.*

Proof. Let $f : X \to Y$ be an epimorphism of BCI-algebras, let ν be a T-fuzzy subalgebra of Y, and let μ be the preimage of ν under f. Then for any $x, y \in X$, we have

$$\begin{aligned}\mu(x * y) &= (\nu \circ f)(x * y) \\ &= \nu(f(x * y)) = \nu(f(x) * f(y)) \\ &\geq T(\nu(f(x)), \nu(f(y))) \\ &= T((\nu \circ f)(x), (\nu \circ f)(y)) \\ &= T(\mu(x), \mu(y)).\end{aligned} \tag{3.2}$$

Hence, μ is a fuzzy subalgebra of X with respect to a t-norm T. □

Definition 3.9. If μ is a fuzzy set in a subalgebra X and f is a mapping defined on X, the fuzzy set μ^f in $f(X)$ defined by

$$\mu^f(y) = \operatorname*{Sup}_{x \in f^{-1}(y)} \mu(x) \quad \forall y \in f(X) \tag{3.3}$$

is called the *image* of μ under f.

Definition 3.10. A fuzzy set μ in X has the Sup *property* if for any subset $A \subseteq X$, there exists $a_0 \in A$ such that $\mu(a_0) = \operatorname{Sup}_{a \in A} \mu(a)$.

Theorem 3.11. *An epimorphism image of a fuzzy subalgebra with* Sup *property is a fuzzy subalgebra.*

Proof. Let $f : X \to Y$ be an epimorphism of X and let μ be a fuzzy subalgebra of X with Sup property. Let $f(x), f(y) \in f(X)$ and let $x_0, y_0 \in f^{-1}(f(x))$ be such that

$$\begin{aligned}\mu(x_0) &= \operatorname*{Sup}_{t \in f^{-1}(f(x))} \mu(t), \\ \mu(y_0) &= \operatorname*{Sup}_{t \in f^{-1}(f(y))} \mu(t),\end{aligned} \tag{3.4}$$

respectively. Then,

$$\begin{aligned}\mu^f(f(x) * f(y)) &= \operatorname*{Sup}_{z \in f^{-1}(f(x) * f(y))} \mu(z) \\ &\geq \min\{\mu(x_0), \mu(y_0)\} \\ &= \min\left\{\operatorname*{Sup}_{t \in f^{-1}(f(x))} \mu(t), \operatorname*{Sup}_{t \in f^{-1}(f(y))} \mu(t)\right\} \\ &= \min\{\mu^f(f(x)), \mu^f(f(y))\}.\end{aligned} \tag{3.5}$$

Hence μ^f is a fuzzy subalgebra of Y. □

Definition 3.12 [7]. A t-norm T on [0,1] is called a *continuous t-norm* if T is a continuous function from $[0,1]\times[0,1]$ to [0, 1] with respect to the usual topology. Note that the function min is a continuous t-norm.

THEOREM 3.13. *Let T be a continuous t-norm and let f be a homomorphism on X. If μ is a T-fuzzy subalgebra of X, then μ^f is a T-fuzzy subalgebra of $f(X)$.*

Proof. Let $Z_1 = f^{-1}(y_1)$, $Z_2 = f^{-1}(y_2)$, and $Z_{12} = f^{-1}(y_1 * y_2)$, where $y_1, y_2 \in f(X)$.

Consider the set $Z_1 * Z_2 = \{x \in X \mid x = z_1 * z_2 \text{ for some } z_1 \in Z_1 \text{ and } z_2 \in Z_2\}$. If $x \in Z_1 * Z_2$, then $x = x_1 * x_2$ for some $x_1 \in Z_1$ and $x_2 \in Z_2$.

Thus $f(x) = f(x_1 * x_2) = f(x_1) * f(x_2) = y_1 * y_2$, that is, $x \in f^{-1}(y_1 * y_2) = Z_{12}$. Hence $Z_1 * Z_2 \subseteq Z_{12}$. It follows that

$$\begin{aligned}\mu^f(y_1 * y_2) &= \operatorname*{Sup}_{x\in f^{-1}(y_1*y_2)} \mu(x)\\ &= \operatorname*{Sup}_{Z_{12}} \mu(x) \geq \operatorname*{Sup}_{x\in Z_1*Z_2} \mu(x)\\ &\geq \operatorname*{Sup}_{x_1\in Z_1, x_2\in Z_2} \mu(x_1 * x_2)\\ &\geq \operatorname*{Sup}_{x_1\in Z_1, x_2\in Z_2} T(\mu(x_1), \mu(x_2)).\end{aligned} \tag{3.6}$$

Since T is continuous, for every $\varepsilon > 0$, there exists a number $\delta > 0$ such that if $\operatorname{Sup}_{x_1\in Z_1}\mu(x_1) - x_1^* \leq \delta$ and $\operatorname{Sup}_{x_2\in Z_2}\mu(x_2) - x_2^* \leq \delta$, then

$$T\Big(\operatorname*{Sup}_{x_1\in Z_1}\mu(x_1), \operatorname*{Sup}_{x_2\in Z_2}\mu(x_2)\Big) - T(x_1^*, x_2^*) \leq \varepsilon. \tag{3.7}$$

Choose $z_1 \in Z_1$ and $z_2 \in Z_2$ such that $\operatorname{Sup}_{x_1\in Z_1}\mu(x_1) - \mu(z_1) \leq \delta$ and $\operatorname{Sup}_{x_2\in Z_2}\mu(x_2) - \mu(z_2) \leq \delta$, then $T(\operatorname{Sup}_{x_1\in Z_1}\mu(x_1), \operatorname{Sup}_{x_2\in Z_2}\mu(x_2))$ -$T(\mu(z_1) - \mu(z_2)) \leq \varepsilon$. Consequently, $\mu^f(y_1 * y_2) \geq \operatorname{Sup}_{x_1\in Z_1, x_2\in Z_2} T(\mu(x_1), \mu(x_2)) \geq T(\operatorname{Sup}_{x_1\in Z_1}\mu(x_1), \operatorname{Sup}_{x_2\in Z_2}\mu(x_2)) = T(\mu^f(y_1), \mu^f(y_2))$, which shows that μ^f is a T-fuzzy subalgebra of $f(X)$. □

4. T-fuzzy H-ideals

Definition 4.1. A fuzzy set μ in X is called *T-fuzzy ideals* of X if

(1) $\mu(0) \geq \mu(x)$ for all $x \in X$,

(2) $\mu(x) \geq T(\mu(x * y), \mu(y))$ for all $x, y \in X$.

Definition 4.2. A fuzzy set μ in X is called *T-fuzzy H-ideals* of X if

(TF1) $\mu(0) \geq \mu(x)$ for all $x \in X$,

(TF2) $\mu(x * z) \geq \mathrm{T}\ (\mu(x * (y * z)), \mu(y))$ for all $x, y, z \in X$.

Example 4.3. Let $X := \{0,a,b,c\}$ be a BCI-algebra with the following Cayley table:

*	0	a	b	c
0	0	c	0	a
a	a	0	a	c
b	b	c	0	a
c	c	a	c	0

Define a fuzzy set $\mu : X \to [0,1]$ by $\mu(0) = t_1$ and $\mu(x) = t_2$ for all $x \neq 0$, where $t_1 > t_2$ and let $T_m : [0,1] \times [0,1] \to [0,1]$ be a function defined by $T_m(x,y) = \max(x+y-1,0)$ which is a t-norm for all $x,y \in [0,1]$. By routine calculations, it is easy to check that μ is a T_m-fuzzy H-ideal of X.

Proposition 4.4. *Every T-fuzzy H-ideal in a* BCI-*algebra X is a T-fuzzy ideal of X.*

Proof. For $x,y,z \in X$, we have

$$\begin{aligned} &\mu_A(x*z) \geq T(\mu_A(x*(y*z)),\mu_A(y)), \\ &\mu_A(x*0) \geq T(\mu_A(x*(y*0)),\mu_A(y)) \quad (\text{putting } z=0) \\ &\mu_A(x) \geq T(\mu_A(x*y),\mu_A(y)) \quad (\text{using } (5)), \end{aligned} \tag{4.1}$$

which completes the proof. □

Proposition 4.5. *Every T-fuzzy H-ideal of a* BCI-*algebra X is a T-fuzzy subalgebra of X.*

Proof. For $x,y,z \in X$, we have

$$\begin{aligned} &\mu_A(x*z) \geq T(\mu_A(x*(y*z)),\mu_A(y)), \\ &\mu_A(x*y) \geq T(\mu_A(x*(y*y)),\mu_A(y)) \quad (\text{replacing } z \text{ by } y) \\ &\mu_A(x*y) \geq T(\mu_A(x*0),\mu_A(y)) \quad (\text{using } (3)) \\ &\mu_A(x*y) \geq T(\mu_A(x),\mu_A(y)) \quad (\text{using } (5)). \end{aligned} \tag{4.2}$$

This ends the proof. □

Theorem 4.6. *If μ is a T-fuzzy H-ideal of X, then each nonempty level subset $U(\mu;1)$ is H-ideal of X.*

Proof. Suppose that μ is a T-fuzzy H-ideal of X. Since $U(\mu,1)$ is nonempty, there exists $x \in U(\mu;1)$. It follows from (TF1) that $\mu(0) \geq \mu(x) \geq 1$, that is, $0 \in U(\mu;\alpha)$. Let $x,y,z \in X$ be such that $x*(y*z) \in U(\mu;1)$ and $y \in U(\mu;1)$. Then $\mu(x*z) \geq T(\mu(x*(y*z)),\mu(y)) \geq T(1,1) = 1$ so that $x*z \in U(\mu;1)$.

Hence $U(\mu;1)$is a H-ideal of X. □

Theorem 4.7. *If λ and μ are T-fuzzy H-ideals of a* BCI-*algebra X, then $\lambda\times\mu$ is a T-fuzzy H-ideal of $X\times X$.*

Proof. For any $(x,y)\in X\times X$, we have $(\lambda\times\mu)(0,0)=T(\lambda(0),\mu(0))\geq T(\lambda(x),\mu(y))=(\lambda\times\mu)(x,y)$. Let $x=(x_1,x_2)$, $y=(y_1,y_2)$, and $z=(z_1,z_2)\in X\times X$. Then

$$\begin{aligned}(\lambda\times\mu)(x*z)&=(\lambda\times\mu)((x_1,x_2)*(z_1,z_2))\\&=(\lambda\times\mu)(x_1*z_1,x_2*z_2)\\&=T(\lambda(x_1*z_1),\mu(x_2*z_2))\\&\geq T(T(\lambda(x_1*(y_1*z_1)),\lambda(y_1)),T(\mu(x_2*(y_2*z_2)),\mu(y_2)))\\&=T(T(\lambda(x_1*(y_1*z_1)),\mu(x_2*(y_2*z_2))),T(\lambda(y_1),\mu(y_2)))\\&=T((\lambda\times\mu)((x_1*(y_1*z_1),x_2*(y_2*z_2))),(\lambda\times\mu)((y_1,y_2)))\\&=T((\lambda\times\mu)((x_1,x_2)*((y_1,y_2)*(z_1,z_2)),(\lambda\times\mu)((y_1,y_2))))\\&=T((\lambda\times\mu)(x*(y*z)),(\lambda\times\mu)(y)).\end{aligned}\tag{4.3}$$

Hence $\lambda\times\mu$ is a T-fuzzy H-ideal of $X\times X$. □

Theorem 4.8. *Let $f:X\to Y$ be a homomorphism of* BCI-*algebras. If μ is a T-fuzzy H-ideal of Y, then μ^f is a T-fuzzy H-ideal of X.*

Proof. For any $x\in X$, we have $\mu^f(x)=\mu(f(x))\leq\mu(\acute{0})=\mu(f(0))=\mu^f(0)$. Thus, $\mu^f(x)\leq\mu^f(0)$, for all $x\in X$. Let $x,y,z\in X$. Then

$$\begin{aligned}T(\mu^f(x*(y*z)),\mu^f(y))&=T(\mu(f(x*(y*z))),\mu(f(y)))\\&=T(\mu(f(x)*(f(y)*f(z))),\mu(f(y)))\\&\leq\mu(f(x)*f(z))=\mu(f(x*z))=\mu^f(x*z).\end{aligned}\tag{4.4}$$

Hence μ^f is a T-fuzzy H-ideal of X. □

Theorem 4.9. *Let $f:X\to Y$ be an epimorphism of* BCI-*algebras. If μ^f is a T-fuzzy H-ideal of X, then μ is a T-fuzzy H-ideal of Y.*

Proof. Let $y\in Y$, there exists $x\in X$ such that $f(x)=y$. Then $\mu(y)=\mu(f(x))=\mu^f(x)\leq\mu^f(0)=\mu(f(0))=\mu(\acute{0})$. Let $x,y,z\in Y$. Then there exist $a,b,c\in X$ such that $f(a)=x$, $f(b)=y$, and $f(c)=z$. It follows that

$$\begin{aligned}\mu(x*z)=\mu(f(a)*f(c))=\mu(f(a*c))&=\mu^f(a*c)\\&\geq T(\mu^f(a*(b*c)),\mu^f(b))\\&=T(\mu(f(a*(b*c))),\mu(f(b)))\\&=T(\mu(f(a)*(f(b)*f(c))),\mu(f(b)))\\&=T(\mu(x*(y*z)),\mu(y)).\end{aligned}\tag{4.5}$$

Hence μ is a T-fuzzy H-ideal of Y. □

Definition 4.10. Let T be a t-norm and let λ and μ be two fuzzy sets in X. Then the *T-product* of λ and μ is denoted by $[\lambda\cdot\mu]_T$ and defined by $[\lambda\cdot\mu]_T(x)=T(\lambda(x),\mu(x))$, for all $x\in X$.

Note that
(i) $[\lambda\cdot\mu]_T=[\mu\cdot\lambda]_T$,
(ii) $[\lambda\cdot\mu]_T$ is a fuzzy set in X.

Theorem 4.11. *Let λ and μ be two T-fuzzy H-ideals of X. If a t-norm T^* dominates T, that is, if $T^*(T(\alpha,\gamma),T(\beta,\delta))\geq T(T^*(\alpha,\beta),T^*(\gamma,\delta))$ for all $\alpha,\beta,\gamma,\delta\in[0,1]$, then T^*-product $[\lambda\cdot\mu]_T^*$ is a T-fuzzy H-ideal of X.*

Proof. For any $x\in X$, we have $[\lambda\cdot\mu]_T^*(0)=T^*(\lambda(0),\mu(0))\geq T^*(\lambda(x),\mu(x))=[\lambda\cdot\mu]_T^*(x)$. Let $x,y,z\in X$. Then

$$\begin{aligned}[\lambda\cdot\mu]_T^*(x*z)&=T^*(\lambda(x*z),\mu(x*z))\\&\geq T^*(T(\lambda(x*(y*z)),\lambda(y)),T(\mu(x*(y*z)),\mu(y)))\\&\geq T(T^*(\lambda(x*(y*z)),\mu(x*(y*z))),T^*(\lambda(y),\mu(y)))\\&=T([\lambda\cdot\mu]_T^*(x*(y*z)),[\lambda\cdot\mu]_T^*(y)).\end{aligned}\tag{4.6}$$

This proves that $[\lambda\cdot\mu]_T^*$ is a T-fuzzy H-ideal of X. □

Corollary 4.12. *The T-product of two T-fuzzy H-ideals of X is a T-fuzzy ideal of the same* BCI-*algebra X.*

Theorem 4.13. *Let T and T^* be T-norms in which T^* dominates T. Let $f:X\to Y$ be an epimorphism of* BCI-*algebras. If λ and μ are T-fuzzy H-ideals of Y, then $f^{-1}([\lambda\cdot\mu]_T^*)=[f^{-1}(\lambda),f^{-1}(\mu)]_T^*$.*

Proof. For any $x\in X$, we have

$$\begin{aligned}[f^{-1}([\lambda\cdot\mu]_T^*)](x)&=[\lambda\cdot\mu]_T^*(f(x))\\&=T^*(\lambda(f(x)),\mu(f(x)))\\&=T^*([f^{-1}(\lambda)](x),[f^{-1}(\mu)](x))\\&=[f^{-1}(\lambda),f^{-1}(\mu)]_T^*(x).\end{aligned}\tag{4.7}$$

This completes the proof. □

Corollary 4.14. *If $f:X\to Y$ is an epimorphism of* BCI-*algebras, then $f^{-1}([\lambda\cdot\mu]_T)=[f^{-1}(\lambda),f^{-1}(\mu)]_T$ for any T-fuzzy H-ideals λ and μ of Y.*

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