IJMMS 2004:26, 1379–1391
PII. S0161171204212078
http://ijmms.hindawi.com

MAGILL-TYPE THEOREMS FOR MAPPINGS

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Received 9 December 2002

Magill's and Rayburn's theorems on the homeomorphism of Stone-Čech remainders and some of their generalizations to the remainders of arbitrary Hausdorff compactifications of Tychonoff spaces are extended to some class of mappings.

2000 Mathematics Subject Classification: 54C05, 54C20, 54D40, 54D50.

1. Introduction. In 1968, Magill [8] proved the following theorem.

Theorem 1.1. *For two locally compact Tychonoff spaces X and Y, the Stone-Čech remainders (i.e., the remainders of the Stone-Čech compactifications) $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic if and only if the posets $K(X)$ and $K(Y)$ of all Hausdorff compactifications of X and Y are isomorphic.*

In 1973, Rayburn [12] gave the following definition.

Definition 1.2. A Tychonoff space X is called k-absolute if $\beta X \setminus X$ is a k-space.

It is proved in [12] that a *Tychonoff space X is k-absolute if and only if $cX \setminus X$ is a k-space for some $cX \in K(X)$ and if and only if $cX \setminus X$ is a k-space for any $cX \in K(X)$.*

All locally compact Tychonoff spaces are k-absolute because their Stone-Čech remainders are compact. Consequently, the following Rayburn's theorem generalizes one half of Theorem 1.1.

Theorem 1.3. *For any pair of k-absolute spaces X, Y, if the posets $K(X)$ and $K(Y)$ are isomorphic, then the Stone-Čech remainders $\beta X \setminus X$ and $\beta Y \setminus Y$ are homeomorphic.*

We note that the second half of Theorem 1.1 cannot be generalized to k-absolute spaces (see [12, Example (B)]).

In [2], both Theorems 1.1 and 1.3 are generalized to arbitrary compactifications of two Tychonoff spaces.

This paper is devoted to an extension of Theorems 1.1 and 1.3 and their generalizations (given in [2]) to the class of WZ-mappings (in particular, closed mappings) from a locally compact Tychonoff space or a k-absolute space to a compact Hausdorff space. Although, even for this rather narrow class of mappings, the formulations of corresponding theorems look rather complicated, the examples presented in Section 4 show that more simple approaches are not sufficient.

We note that results concerning the extension of Theorem 1.1 to mappings are also contained in [5], but they are different from ours (see the remark at the end of this paper).

2. Preliminaries. Throughout this paper, space will mean a topological space and mapping will mean a continuous function. Terms and undefined concepts are used as in [4]. In this section, we recall some definitions and results from [2]. Some additional notions concerning fibrewise general topology (FGT) can be found in [9, 10].

Definition 2.1. Let X, Y, Z be spaces and $\lambda : X \to Y$, $\mu : X \to Z$ mappings. We say that λ is equivalent to μ and we will write $\lambda \equiv \mu$ if there exists some homeomorphism $h : Y \to Z$ such that $\mu = h \circ \lambda$.

Evidently, the homeomorphism h is unique.

We will identify equivalent mappings, and so we can consider the set $\mathscr{C}(X)$ of all the continuous maps from a fixed space X onto other spaces.

Definition 2.2. Let $\lambda, \mu \in \mathscr{C}(X)$. We say that λ *follows* μ and we will write $\lambda \geq \mu$ if there exists some continuous mapping $h : Y \to Z$ such that $\mu = h \circ \lambda$.

It is evident that $(\mathscr{C}(X), \geq)$ is a poset.

Definition 2.3. Let $\lambda \in \mathscr{C}(X)$. We say that

(i) λ is *simple* if there exists a unique point $t_\lambda \in \lambda(X)$ such that $|\lambda^{-1}(\{t_\lambda\})| > 1$ and $|\lambda^{-1}(\{t\})| = 1$ for every $t \in \lambda(X) \setminus \{t_\lambda\}$;

(ii) λ is *finite simple* if there exists a nonempty finite set $T \subset \lambda(X)$ of points such that $|\lambda^{-1}(\{t\})| > 1$ for every $t \in T$ and $|\lambda^{-1}(\{t\})| = 1$ for every $t \in \lambda(X) \setminus T$.

We will suppose from this moment that X is a Hausdorff space and that $\mathscr{P}(X)$ denotes the poset (as a subposet of $\mathscr{C}(X)$) of all perfect onto mappings of X. Clearly, $\lambda(X)$ is Hausdorff for any $\lambda \in \mathscr{P}(X)$.

Definition 2.4. A mapping $\lambda \in \mathscr{P}(X)$ is called a *dual point* if it is simple and $|\lambda^{-1}(\{t_\lambda\})| = 2$.

Let $\mathscr{D} = \mathscr{D}(X)$ denote the set of all dual points of $\mathscr{P}(X)$ and $\mathscr{F}\mathscr{S}(X) = \{\lambda \in \mathscr{P}(X) : \lambda \text{ is finite simple}\} \cup \{\mathrm{id}_X\}$.

Definition 2.5. A family $\mathscr{F} \subset \mathscr{D}$ is said to be a *3-vertex family* if for any distinct $\alpha, \beta \in \mathscr{F}$ there exists some $\gamma \in \mathscr{D} \setminus \mathscr{F}$ such that $\gamma > \inf\{\alpha, \beta\}$.

Definition 2.6. A 3-vertex family $\mathscr{F} \subset \mathscr{D}$ is called a *point family* if it is maximal (i.e., if there is no 3-vertex family properly containing $\mathscr{F}$).

Lemma 2.7. *If $\mathscr{F}$ is a 3-vertex family consisting of more than one element, then the set $X_{\mathscr{F}} = \bigcap\{\lambda^{-1}(\{t_\lambda\}) : \lambda \in \mathscr{F}\}$ is a single point (which will be denoted by $J_{\mathscr{I}}(\mathscr{F})$).*

Definition 2.8. Let $\mathscr{I} \subset \mathscr{P}(X)$ such that $\mathscr{F}\mathscr{S}(X) \subset \mathscr{I}$ and $x \in X$. We put $K_{\mathscr{I}}(x) = \{\delta \in \mathscr{D} : x \in \delta^{-1}(\{t_\delta\})\}$.

Lemma 2.9. *If $|X| > 2$, then $K_{\mathscr{I}}(x)$ is a point family.*

Evidently, for $|X| > 2$,

$$J_{\mathcal{I}}(K_{\mathcal{I}}(x)) = x \quad \text{for any } x \in X, \tag{2.1}$$

$$K_{\mathcal{I}}(J_{\mathcal{I}}(\mathcal{F})) = \mathcal{F} \quad \text{for any point family } \mathcal{F} \subset \mathcal{D}. \tag{2.2}$$

In [2], dual points are characterized only by means of the order in $\mathcal{I}$. It follows from this that if, for Hausdorff spaces X_j, we have $\mathcal{FS}(X_j) \subset \mathcal{I}_j \subset \mathcal{P}(X_j)$ with $j = 1,2$ and $i : \mathcal{I}_1 \to \mathcal{I}_2$ is a poset isomorphism, then $i(K_{\mathcal{I}_1}(x))$ is a point family in $\mathcal{I}_2$.

We recall [2] that a one-to-one mapping $f : X \to Y$ between Hausdorff spaces is called a k-homeomorphism if f is continuous on compact subsets of X and its inverse f^{-1} is continuous on compact subsets of Y. Clearly, a k-homeomorphism between k-spaces is a homeomorphism.

Theorem 2.10. *Let X_j be a Hausdorff space and $\mathcal{P}(X_j)$ the set of all perfect onto mappings of X_j (with $j = 1,2$). If X_1 and X_2 are k-homeomorphic and they are k-spaces, then $\mathcal{P}(X_1)$ and $\mathcal{P}(X_2)$ (and so $\mathcal{FS}(X_1)$ and $\mathcal{FS}(X_2)$) are isomorphic. Let $\mathcal{FS}(X_j) \subset \mathcal{I}_j \subset \mathcal{P}(X_j)$ for $j = 1,2$. If $\mathcal{I}_1$ and $\mathcal{I}_2$ are poset isomorphic, then X_1 and X_2 are k-homeomorphic, and if, additionally, X_1 and X_2 are k-spaces, then they are homeomorphic. More precisely, if $i : \mathcal{I}_1 \to \mathcal{I}_2$ is a poset isomorphism and $|X_1| > 2$, then the function $h_i : X_1 \to X_2$, such that $h_i(x) = J_{\mathcal{I}_2}(i(K_{\mathcal{I}_1}(x)))$ for any $x \in X_1$, is a k-homeomorphism.*

Now, let X be a Tychonoff space and let $K(X)$ denote the poset of all Hausdorff compactifications of X (see, e.g., [3]).

For any $cX, dX \in K(X)$ such that $cX < dX$, let

$$\pi_{dc} = \pi_{dcX} : dX \longrightarrow cX \tag{2.3}$$

be the canonical map (i.e., it is continuous and $\pi_{dc|X} = \mathrm{id}_X$).

Then $\pi_{dc}^{-1}(cX \setminus X) = dX \setminus X$ and the mapping

$$r\pi_{dc} = r\pi_{dcX} \overset{\text{def}}{=} \pi_{dc} : dX \setminus X \longrightarrow cXX \tag{2.4}$$

is perfect and onto, that is, $r\pi_{dc} \in \mathcal{P}(dX \setminus X)$.

Fix $eX \in K(X)$ and let

$$K(eX) = \{cX \in K(X) : cX \le eX\} \tag{2.5}$$

(in particular, $K(\beta X) = K(X)$).

In [2], the function

$$\sigma_{eX} : K(eX) \longrightarrow \mathcal{P}(eX \setminus X) \tag{2.6}$$

was defined by $\sigma_{eX}(cX) = r\pi_{ec}$ and the following lemma was proved.

Lemma 2.11. *σ_{eX} is an isomorphism of the posets $K(eX)$ and*

$$\mathcal{L}(eX) \overset{\text{def}}{=} \sigma_{eX}(K(eX)). \tag{2.7}$$

The following two lemmas were also proved in [2].

Lemma 2.12. *$\mathcal{FS}(eX\setminus X)\subset\mathcal{L}(eX)$.*

Lemma 2.13. *If X is a locally compact space, then $\mathcal{L}(eX)=\mathcal{P}(eX\setminus X)$.*

3. On the homeomorphisms of two pairs of spaces

Definition 3.1. A space X and a closed subset A are called a *pair of spaces* and denoted by (X,A).

Definition 3.2. Suppose (X,A) and (Y,B) are pairs of spaces, where X, Y are Hausdorff. Then a homeomorphism (a k-homeomorphism) $h:X\to Y$ is called a homeomorphism (a k-homeomorphism) of the pair (X,A) onto the pair (Y,B) if $h(A)=B$.

Let X be a Hausdorff space and A a closed subset of X.

For any $\lambda\in\mathcal{P}(X)$, let

$$\mathrm{res}_{XA}(\lambda)=\lambda:A\longrightarrow\lambda(A), \tag{3.1}$$

that is, $\mathrm{res}_{XA}(\lambda)$ is the corestriction of λ to A.

Evidently, $\mathrm{res}_{XA}(\mathcal{P}(X))\subset\mathcal{P}(A)$ and $\mathrm{res}_{XA}:\mathcal{P}(X)\to\mathcal{P}(A)$ is monotonous.

It is not difficult to prove the following lemma.

Lemma 3.3. $\mathrm{res}_{XA}(\mathcal{P}(X))=\mathcal{P}(A)$, $\mathrm{res}_{XA}(\mathcal{FS}(X))=\mathcal{FS}(A)$, *and* $\mathrm{res}_{XA}(\mathcal{D}(X)\cup\{\mathrm{id}_X\})=\mathcal{D}(A)\cup\{\mathrm{id}_A\}$.

Let $\mathcal{FS}(X)\subset\mathcal{I}(X)\subset\mathcal{P}(X)$ and $\mathcal{FS}(A)\subset\mathcal{I}(A)\subset\mathcal{P}(A)$. Then, clearly,

$$\mathrm{res}_{XA}\left(K_{\mathcal{I}(X)}(x)\right)\setminus\{\mathrm{id}_A\}=K_{\mathcal{I}(A)}(x)\quad\forall x\in A. \tag{3.2}$$

Theorem 3.4. *Let X and Y be Hausdorff spaces, let A be a closed subset of X, and let B be a closed subset of Y,*

$$\begin{aligned}\min\{|X|,|Y|\}\ge 3,&\qquad \min\{|A|,|B|\}\ge 2,\\ \mathcal{FS}(X)\subset\mathcal{I}(X)\subset\mathcal{P}(X),&\qquad \mathcal{FS}(A)\subset\mathcal{I}(A)\subset\mathcal{P}(A),\\ \mathcal{FS}(Y)\subset\mathcal{I}(Y)\subset\mathcal{P}(Y),&\qquad \mathcal{FS}(B)\subset\mathcal{I}(B)\subset\mathcal{P}(B),\end{aligned} \tag{3.3}$$

and let

$$i_{XY}:\mathcal{I}(X)\longrightarrow\mathcal{I}(Y),\qquad i_{AB}:\mathcal{I}(A)\longrightarrow\mathcal{I}(B) \tag{3.4}$$

be poset isomorphisms such that

$$i_{AB}\circ\mathrm{res}_{XA}=\mathrm{res}_{YB}\circ i_{XY}. \tag{3.5}$$

Then

(i) *in the case* $\min\{|A|,|B|\}\ge 3$ *for k-homeomorphisms $h_{i_{AB}}:A\to B$ and $h_{i_{XY}}:X\to Y$ (see Theorem 2.10),*

$$h_{i_{AB}}(x)=h_{i_{XY}}(x)\quad \textit{for every } x\in A; \tag{3.6}$$

(ii) *in the case* $\min\{|A|,|B|\} = 2$*, there exists a homeomorphism* $h_{i_{AB}}(x) : A \to B$ *such that (3.6) is also true.*

Thus

$$h_{i_{XY}}(A) \subset B, \qquad h_{i_{AB}} = h_{i_{XY}} : A \longrightarrow B, \tag{3.7}$$

and so $h_{i_{XY}}$ *is a* k*-homeomorphism of* (X,A) *onto* (Y,B)*.*

If, additionally, X *and* Y *are* k*-spaces, then* $h_{i_{XY}}$ *and* $h_{i_{AB}}$ *are homeomorphisms, and so* $h_{i_{XY}}$ *is a homeomorphism of* (X,A) *onto* (Y,B)*.*

Proof. First, let $\min\{|A|,|B|\} \geq 3$.

Let $x \in A$. Then, by Theorem 2.10, $h_{i_{XY}}$ and $h_{i_{AB}}$ are k-homeomorphisms and

$$\begin{aligned}
h_{i_{AB}}(x) &= J_{\mathcal{J}(B)}\big(i_{AB}(K_{\mathcal{J}(A)}(x))\big) \quad \text{(by (3.2))}\\
&= J_{\mathcal{J}(B)}\big(i_{AB}(\operatorname{res}_{XA}(K_{\mathcal{J}(X)}(x)) \setminus \{\mathrm{id}_A\})\big)\\
&\qquad \text{(by (3.5) and since } i_{AB} \text{ is a poset isomorphism)}\\
&= J_{\mathcal{J}(B)}\big(\operatorname{res}_{YB}(i_{XY}(K_{\mathcal{J}(X)}(x))) \setminus \{\mathrm{id}_B\}\big)\\
&\qquad \text{(by (2.2) and since } i_{XY} \text{ is a poset isomorphism)}\\
&= J_{\mathcal{J}(B)}\big(\operatorname{res}_{YB}(K_{\mathcal{J}(Y)}(J_{\mathcal{J}(Y)}(i_{XY}(K_{\mathcal{J}(X)}(x))))) \setminus \{\mathrm{id}_B\}\big)\\
&\qquad \text{(by the definition of } h_{i_{XY}})\\
&= J_{\mathcal{J}(B)}\big(\operatorname{res}_{YB}(K_{\mathcal{J}(Y)}(h_{i_{XY}}(x))) \setminus \{\mathrm{id}_B\}\big) \quad \text{(by (3.2))}\\
&= J_{\mathcal{J}(B)}\big(K_{\mathcal{J}(B)}(h_{i_{XY}}(x))\big) \quad \text{(by (2.1))}\\
&= h_{i_{XY}}(x).
\end{aligned} \tag{3.8}$$

Now, let $|A| = 2$. Since i_{AB} is a poset isomorphism, $|\mathcal{J}(B)| = |\mathcal{J}(A)| = 2$ and so $|B| = 2$.

There is a unique dual point $\lambda \in \mathcal{J}(X)$ such that $\lambda^{-1}(\{t_\lambda\}) = A$. Let $A = \{x_1,x_2\}$. Evidently, $K_{\mathcal{J}(X)}(x_1) \cap K_{\mathcal{J}(X)}(x_2) = \{\lambda\}$. Then, by (2.2),

$$\begin{aligned}
K_{\mathcal{J}(Y)}(h_{i_{XY}}(x_i)) &= K_{\mathcal{J}(Y)}(J_{\mathcal{J}(Y)}(i_{XY}(K_{\mathcal{J}(X)}(x_i))))\\
&= i_{XY}(K_{\mathcal{J}(X)}(x_i)) \quad \text{for } i = 1,2.
\end{aligned} \tag{3.9}$$

Hence

$$\begin{aligned}
i_{XY}(\{\lambda\}) &= i_{XY}(K_{\mathcal{J}(X)}(x_1) \cap K_{\mathcal{J}(X)}(x_2))\\
&= i_{XY}(K_{\mathcal{J}(X)}(x_1)) \cap i_{XY}(K_{\mathcal{J}(X)}(x_2))\\
&= K_{\mathcal{J}(Y)}(h_{i_{XY}}(x_1)) \cap K_{\mathcal{J}(Y)}(h_{i_{XY}}(x_2)).
\end{aligned} \tag{3.10}$$

Thus, for $\eta = i_{XY}(\lambda)$, $\eta^{-1}(\{t_\eta\}) = \{h_{i_{XY}}(x_1), h_{i_{XY}}(x_1)\} = h_{i_{XY}}(A)$. But, by (3.5), $\xi = \operatorname{res}_{YB}(\eta) = \operatorname{res}_{YB}(i_{XY}(\lambda)) = i_{AB}(\operatorname{res}_{XA}(\lambda))$. Since $\operatorname{res}_{XA}(\lambda)$ is a dual point in $\mathcal{J}(A)$ and i_{AB} is a poset isomorphism, ξ is also a dual point in $\mathcal{J}(B)$. Thus, $B = \xi^{-1}(\{t_\xi\}) = \eta^{-1}(\{t_\eta\}) = h_{i_{XY}}(A)$. □

4. Extensions of Magill's and Rayburn's theorems to mappings

Definition 4.1. For mappings $f_j : X_j \to Y$ of (Hausdorff) spaces X_j (with $j = 1,2$), a (k-) homeomorphism $h : X_1 \to X_2$ is called a (k-) homeomorphism of f_1 onto f_2

if $f_1 = f_2 \circ h$. The mappings $f_j : X_j \to Y$ (for $j = 1,2$) are (k-) homeomorphic if there exists a (k-) homeomorphism of f_1 onto f_2.

It is not difficult to prove the following lemma.

Lemma 4.2. *For mappings $f_j : X_j \to Y$ (with $j = 1,2$) and for (k-) homeomorphism $h : X_1 \to X_2$ of spaces X_1 and X_2, the following conditions are equivalent:*
- (i) *h is a homeomorphism of f_1 onto f_2;*
- (ii) *$h(f_1^{-1}(\{y\})) \subset f_2^{-1}(\{y\})$ for every $y \in Y$;*
- (iii) *there is a function $h_y : f_1^{-1}(\{y\}) \to f_2^{-1}(\{y\})$ such that $h_y = h : f_1^{-1}(\{y\}) \to f_2^{-1}(\{y\})$ for every $y \in Y$.*

Given a Tychonoff space X and a closed subset A of X, we may define a function

$$\text{kres}_{XA} : K(X) \longrightarrow K(A) \tag{4.1}$$

such that

$$\text{kres}_{XA}(cX) = cl_{cX}(A) \quad \text{for every } cX \in K(X). \tag{4.2}$$

If $eX, cX \in K(X)$ and $cX < eX$, then $\pi_{ecX}(cl_{eX}(A)) = cl_{cX}(A)$, $\pi_{ecX}(A) = A$, and $(\pi_{ecX} : A \to A) = \text{id}_A$.

Consequently, $(eA = \text{kres}_{XA}(eX)) > (cA = \text{kres}_{XA}(cX))$ and $\pi_{ecA} = \pi_{ecX} : eA \to cA$. Thus, kres_{XA} is monotone and $r\pi_{ecA} = r\pi_{ecX} : eA \setminus A \to cA \setminus A$, that is, $r\pi_{ecA} = \text{res}_{eX\setminus X, eA\setminus A}(r\pi_{ecX})$. It follows from this that $(\sigma_{eA} \circ \text{kres}_{XA})(cX) = \sigma_{eA}(cA) = r\pi_{ecA} = \text{res}_{eX\setminus X, eA\setminus A}(r\pi_{ecX}) = \text{res}_{eX\setminus X, eA\setminus A}(\sigma_{eX}(cX))$.

We have then proven the following lemma.

Lemma 4.3. *If $eX \in K(X)$ and $eA = \text{kres}_{XA}(eX)$, then*

$$\text{res}_{eX\setminus X, eA\setminus A} \circ \sigma_{eX} = \sigma_{eA} \circ \text{kres}_{XA}|_{K(eX)}. \tag{4.3}$$

Let $f : X \to Y$ be a mapping to a Tychonoff space Y and let $\beta f^+ : \beta X \to \beta Y$ be the (usual) continuous extension of f over the Stone-Cěch compactifications βX, $\beta_f X = (\beta f^+)^{-1}(Y)$, and $\beta f = \beta f^+ : \beta_f X \to Y$. Evidently, the mapping βf is perfect. We note that X is C^*-embedded in $\beta_f X$ because $X \subset \beta_f X \subset \beta X$. Recall that f is called a *WZ-mapping* [6] (resp., a *Z-mapping*) if $(\beta f)^{-1}(\{y\}) = cl_{\beta_f X}(f^{-1}(\{y\}))$ for every $y \in Y$ (resp., if $f(Z)$ is closed for any zero-set Z in X). It is clear that $(\beta f)^{-1}(\{y\}) = \beta(f^{-1}(\{y\}))$ if the space X is normal and f is a WZ-mapping. It is known [6] that every Z-mapping is a WZ-mapping.

Theorem 4.4. *Let X_j be a Tychonoff space, let Y be a compact Hausdorff space, let $f_j : X_j \to Y$ be a WZ-mapping, let eX_j be a Hausdorff compactification of X_j, and let $ef_j : eX_j \to Y$ be a continuous extension of f_j (thus, ef_j is a compactification of f_j) for $j = 1,2$. Let also $X_{jy} = f_j^{-1}(\{y\})$, $eX_{jy} = cl_{eX_j}(X_{jy})$ (i.e., $eX_{jy} = \text{kres}_{X_j X_{jy}}(eX_j)$) for $j = 1,2$, and suppose that there exist poset isomorphisms $i : K(eX_1) \to K(eX_2)$ and $i_y : K(eX_{1y}) \to K(eX_{2y})$ such that*

$$i_y \circ \text{kres}_{X_1 X_{1y}} = \text{kres}_{X_2 X_{2y}} \circ i \quad \textit{for every } y \in Y. \tag{4.4}$$

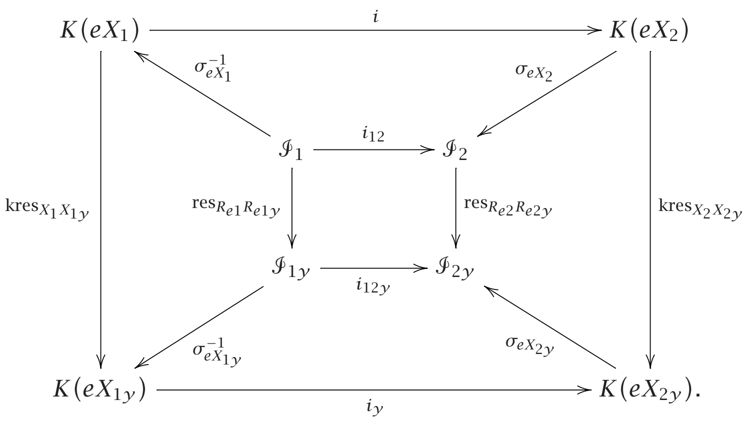
Then the remainders $ef_j \setminus f_j \overset{\text{def}}{=} ef_j : eX_j \setminus X_j \to Y$ *of* ef_j *for* $j = 1,2$ *are k-homeomorphic (more exactly, if* $\min\{|eX_1 \setminus X_1|, |eX_2 \setminus X_2|\} \geq 3$*, then the function* $h_{\sigma_{eX_2} \circ i \circ \sigma_{eX_1}^{-1}} : eX_1 \setminus X_1 \to eX_2 \setminus X_2$ *is a k-homeomorphism of* $ef_1 \setminus f_1$ *onto* $ef_2 \setminus f_2$*). If, additionally,* X_1 *and* X_2 *are k-absolute spaces, then the remainders* $ef_1 \setminus f_1$ *and* $ef_2 \setminus f_2$ *are homeomorphic.*

Proof. Fix $y \in Y$. Let $R_{ej} = eX_j \setminus X_j$ and $R_{ejy} = eX_{jy} \setminus X_{jy}$ for $j = 1,2$.

By Lemmas 2.11 and 2.12, the mappings $\sigma_{eX_j} : K(eX_j) \to (\mathscr{I}_j \overset{\text{def}}{=} \sigma_{eX_j}(K(eX_j)))$ and $\sigma_{eX_{jy}} : K(eX_{jy}) \to (\mathscr{I}_{jy} \overset{\text{def}}{=} \sigma_{eX_{jy}}(K(eX_{jy})))$ are poset isomorphisms and $\mathscr{F}\mathscr{S}(R_{ej}) \subset \mathscr{I}_j$, $\mathscr{F}\mathscr{S}(R_{ejy}) \subset \mathscr{I}_{jy}$ for $j = 1,2$.

Hence, $i_{12} = \sigma_{eX_2} \circ i \circ \sigma_{eX_1}^{-1} : \mathscr{I}_1 \to \mathscr{I}_2$ and $i_{12y} = \sigma_{eX_{2y}} \circ i_y \circ \sigma_{eX_{1y}}^{-1} : \mathscr{I}_{1y} \to \mathscr{I}_{2y}$ are poset isomorphisms too.

We consider the following diagram:



(4.5)

By (4.4), the external part of it is commutative. We prove that its internal part is commutative too, that is, that

$$\operatorname{res}_{R_{e2}R_{e2y}} \circ i_{12} = i_{12y} \circ \operatorname{res}_{R_{e1}R_{e1y}}. \tag{4.6}$$

By (4.3), for $j = 1,2$, we have (see the diagram)

$$\operatorname{res}_{R_{ej}R_{ejy}} \circ \sigma_{eX_j} = \sigma_{eX_{jy}} \circ \operatorname{kres}_{X_jX_{jy}}|_{K(eX_j)}. \tag{4.7}$$

Thus, since σ_{eX_j}, $\sigma_{eX_{jy}}$ are isomorphisms,

$$\begin{gathered}\operatorname{kres}_{X_jX_{jy}}|_{K(eX_j)} \circ \sigma_{eX_j}^{-1} = \sigma_{eX_{jy}}^{-1} \circ \operatorname{res}_{R_{ej}R_{ejy}}, \\ \operatorname{res}_{R_{ej}R_{ejy}}(\mathscr{I}_j) \subset \mathscr{I}_{jy}.\end{gathered} \tag{4.8}$$

Hence,

$$\begin{aligned}\mathrm{res}_{R_{e2}R_{e2y}}\circ i_{12} &= \mathrm{res}_{R_{e2}R_{e2y}}\circ\sigma_{eX_2}\circ i\circ\sigma_{eX_1}^{-1}\\ &= \sigma_{eX_{2y}}\circ\mathrm{kres}_{X_2X_{2y}}\circ i\circ\sigma_{eX_1}^{-1}\\ &= \sigma_{eX_{2y}}\circ i_y\circ\mathrm{kres}_{X_1X_{1y}}\circ\sigma_{eX_1}^{-1}\\ &= \sigma_{eX_{2y}}\circ i_y\circ\sigma_{eX_{1y}}^{-1}\circ\mathrm{res}_{R_{e1}R_{e1y}}\\ &= i_{12y}\circ\mathrm{res}_{R_{e1}R_{e1y}}.\end{aligned}\tag{4.9}$$

By Theorem 3.4, in the case $\min\{|R_{e1}|,|R_{e2}|,|R_{e1y}|,|R_{e2y}|\}\ge 3$, the homeomorphisms $h_{i_{12}}:R_{e1}\to R_{e2}$ and $h_{i_{12y}}:R_{e1y}\to R_{e2y}$ are such that

$$h_{i_{12y}}=h_{i_{12}}:R_{e1y}\longrightarrow R_{e2y},\tag{4.10}$$

and in the case $\min\{|R_{e1}|,|R_{e2}|\}\ge 3$, $\min\{|R_{e1y}|,|R_{e2y}|\}\ge 2$, there exists a homeomorphism $h_{i_{12y}}:R_{e1y}\to R_{e2y}$ such that (4.10) is also true.

Finally, in the case $\min\{|R_{e1}|,|R_{e2}|\}\ge 3$, $\min\{|R_{e1y}|,|R_{e2y}|\}\le 1$, the existence of a homeomorphism $h_{i_{12y}}:R_{e1y}\to R_{e2y}$ such that (4.10) holds is evident.

Thus, in the case $\min\{|R_{e1}|,|R_{e2}|\}\ge 3$, (4.10) holds for every $y\in Y$.

In the case $\min\{|R_{e1}|,|R_{e2}|\}\le 2$, the existence of homeomorphisms $h_{i_{12}}:R_{e1}\to R_{e2}$ and $h_{i_{12y}}:R_{e1y}\to R_{e2y}$ such that (4.10) holds is evident.

Since f_j is a WZ-mapping, $R_{ejy}=(ef_j)^{-1}(\{y\})\setminus f_j{}^{-1}(\{y\})$ for $j=1,2$, and so

$$h_{i_{12y}}=h_{i_{12}}:(ef_1)^{-1}(\{y\})\setminus f_1{}^{-1}(\{y\})\longrightarrow(ef_2)^{-1}(\{y\})\setminus f_2{}^{-1}(\{y\})\quad\text{for every } y\in Y.\tag{4.11}$$

By Lemma 4.2, $h_{i_{12}}$ is a k-homeomorphism of $ef_1\setminus f_1$ onto $ef_2\setminus f_2$. □

Corollary 4.5 [2]. *Let X_1,X_2 be Tychonoff spaces and let eX_1,eX_2 be their Hausdorff compactifications. If $K(eX_1)$ and $K(eX_2)$ are poset isomorphic, then the remainders $eX_1\setminus X_1$ and $eX_2\setminus X_2$ are k-homeomorphic, and they are homeomorphic if, additionally, X_1, X_2 are k-absolute spaces.*

Proof. It is sufficient to apply Theorem 4.4 to (the simplest) mappings f_j of X_j to the single point space Y for $j=1,2$. □

In particular, in Corollary 4.5, for $eX_j=\beta X_j$ $(j=1,2)$, we have Rayburn's Theorem 1.3. Thus, Theorem 4.4 is a generalization of this theorem to mappings.

Theorem 4.6. *Let X_j be a locally compact Tychonoff space, let Y be a compact Hausdorff space, let $f_j:X_j\to Y$ be a WZ-mapping, let eX_j be a Hausdorff compactification of X_j, and let $ef_j:eX_j\to Y$ be a continuous extension of f_j for $j=1,2$. Let also $X_{jy}=f_j^{-1}(\{y\})$, $eX_{jy}=cl_{eX_j}(X_{jy})$ (i.e., $eX_{jy}=\mathrm{kres}_{X_jX_{jy}}(eX_j)$) (for $j=1,2$). Then the remainders $ef_j\setminus f_j=ef_j:eX_j\setminus X_j\to Y$ of ef_j for $j=1,2$ are homeomorphic if and only* *if there exist poset isomorphisms* $i : K(eX_1) \to K(eX_2)$ *and* $i_y : K(eX_{1y}) \to K(eX_{2y})$ *such that (4.4) holds.*

Proof. One half of the theorem follows from Theorem 4.4.

Let $R_{ej} = eX_j \setminus X_j$ and $R_{ejy} = eX_{jy} \setminus X_{jy}$ for all $y \in Y$ and $j = 1,2$. Now, suppose that the remainders $ef_1 \setminus f_1$ and $ef_2 \setminus f_2$ are homeomorphic. Then there exists a homeomorphism $h : R_{e2} \to R_{e1}$ such that $ef_2 \setminus f_2 = (ef_1 \setminus f_1) \circ h$. Hence, the mappings $h_y = h : R_{e2y} \to R_{e1y}$ are homeomorphisms for all $y \in Y$. Since f_j is a WZ-mapping, $(ef_j)^{-1}(\{y\})$ is a compactification of $f_j^{-1}(\{y\})$, and so $eX_{jy} = (ef_j)^{-1}(\{y\})$ for all $y \in Y$ and $j = 1,2$. Evidently, $i_{12} : \mathcal{P}(R_{e1}) \to \mathcal{P}(R_{e2})$ and $i_{12y} : \mathcal{P}(R_{e1y}) \to \mathcal{P}(R_{e2y})$, such that $i_{12}(\lambda) = \lambda \circ h$ for $\lambda \in \mathcal{P}(X_1)$ and $i_{12y}(\lambda) = \lambda \circ h_y$ for $\lambda \in \mathcal{P}(X_{1y})$ and all $y \in Y$, are poset isomorphisms.

We prove that (4.4) holds for all $y \in Y$.

Indeed, for every $\lambda \in \mathcal{P}(R_{e1})$ and $y \in Y$, we have

$$\begin{aligned} i_{12y} \circ \operatorname{res}_{R_{e1}R_{e1y}}(\lambda) &= i_{12y}(\lambda|_{R_{e1y}}) = \lambda|_{R_{e1y}} \circ h_y \\ &= (\lambda \circ h)|_{R_{e2y}} = (i_{12}(\lambda))|_{R_{e2y}} \\ &= \operatorname{res}_{R_{e2}R_{e2y}} \circ i_{12}(\lambda). \end{aligned} \tag{4.12}$$

By Lemmas 2.11 and 2.13, $\sigma_{eX_j} : K(eX_j) \to \mathcal{P}(R_{ej})$ and $\sigma_{eX_{jy}} : K(eX_{jy}) \to \mathcal{P}(R_{ejy})$, for $j = 1,2$ and $y \in Y$, are poset isomorphisms. Hence, $i = \sigma_{eX_2}^{-1} \circ i_{12} \circ \sigma_{eX_1}$ and $i_y = \sigma_{eX_{2y}}^{-1} \circ i_{12y} \circ \sigma_{eX_{1y}}$ are poset isomorphisms for $y \in Y$ and $j = 1,2$.

We consider the diagram obtained from the previous one by replacing $\mathcal{I}_j$ and $\mathcal{I}_{jy}$ by $\mathcal{P}(R_{ej})$ and $\mathcal{P}(R_{ejy})$ for $j = 1,2$ and $\sigma_{eX_1}^{-1}$, $\sigma_{eX_{1y}}^{-1}$, σ_{eX_2}, $\sigma_{eX_{2y}}$ by σ_{eX_1}, $\sigma_{eX_{1y}}$, $\sigma_{eX_2}^{-1}$, $\sigma_{eX_{2y}}^{-1}$, respectively. By (4.12), its internal part is commutative. As above, in the proof of Theorem 4.4, we can prove that its external part is commutative too, that is, that (4.4) holds. □

Corollary 4.7 [2]. *Let* X_1, X_2 *be locally compact Tychonoff spaces and let* eX_1, eX_2 *be Hausdorff compactifications of* X_1 *and* X_2, *respectively. Then the remainders* $eX_1 \setminus X_1$ *and* $eX_2 \setminus X_2$ *are homeomorphic if and only if* $K(eX_1)$ *and* $K(eX_2)$ *are poset isomorphic.*

Proof. It is sufficient to apply Theorem 4.6 to the simplest mappings f_j of X_j and ef_j of eX_j to the single point space Y for $j = 1,2$. □

In particular, when in Corollary 4.7, $eX_j = \beta X_j$ for $j = 1,2$, we have Magill's theorem from [8]. Thus, Theorem 4.6 is a generalization of this theorem to mappings.

5. Reformulations of results obtained above and some examples. Some readers may find that Theorems 4.4 and 4.6 do not sound very natural. The reformulations, in the framework of FGT, sound better to us.

We will start with some definitions and results of FGT.

A mapping is called *compact* if it is perfect.

The following is evident.

Lemma 5.1. *For a compact Hausdorff space Y, a mapping $f : X \to Y$ is compact if and only if X is compact.*

Definition 5.2. A mapping $f : X \to Y$ is said to be T_0 [11] if for every $x, x' \in X$ such that $x \neq x'$ and $f(x) = f(x')$, there exists a neighbourhood of x in X which does not contain x' or a neighbourhood of x' in X not containing x.

Definition 5.3. A mapping $f : X \to Y$ is said to be *completely regular* [11] if for every closed set F of X and $x \in X \setminus F$, there exist a neighbourhood O of $f(x)$ in Y and a continuous function $\varphi : f^{-1}(O) \to [0,1]$ such that $\varphi(x) = 0$ and $\varphi(F \cap f^{-1}(O)) \subseteq \{1\}$. A completely regular T_0-mapping is called *Tychonoff* (or $T_{3(1/2)}$).

It is not difficult to prove the following lemma.

Lemma 5.4 [11]. *For a Tychonoff space Y, a mapping $f : X \to Y$ is Tychonoff if and only if X is Tychonoff.*

Definition 5.5. A compact Tychonoff mapping $ef : e_fX \to Y$ is called a *Tychonoff compactification* of a Tychonoff mapping $f : X \to Y$ if $X \subset e_fX$, X is dense in e_fX, and $e_fX|_X = f$ (more precisely, if some embedding $e : X \to e_fX$ is fixed so that $e(X)$ is dense in e_fX and $f = ef \circ e$, but usually, X and $e(X)$ are identified by means of e).

Throughout the rest of the paper, we fix a space Y and we will consider only Tychonoff mappings to Y and their Tychonoff compactifications.

Definition 5.6. A mapping $\lambda : d_fX \to c_fX$ between two compactifications $cf : c_fX \to Y$ and $df : d_fX \to Y$ is called *canonical* if $df = cf \circ \lambda$ and $\lambda_{|X} = \mathrm{id}_X$. In this case, one says that we have a canonical morphism $\lambda : df \to cf$ (and we write that $df > cf$).

It is not difficult to prove that df and cf are homeomorphic if and only if $df > cf$ and $cf > df$ (see, e.g., [1]).

It is proved in [11] (see also [1]) that all compactifications of a mapping to Y form a set up to canonical homeomorphisms. This set will be denoted by $TK(f)$. Evidently, with respect to the just defined relation $>$, $TK(f)$ is a poset.

In [11], it is also proved that there exists the maximal element $\beta f : \beta_f X \to Y$ in $TK(f)$ and that, if Y is Tychonoff, βf may be obtained in the following way.

By Lemma 5.4, X is Tychonoff. Hence, there exists the unique continuous extension $\beta f^+ : \beta X \to \beta Y$ of f. Then $\beta_f X = (\beta f^+)^{-1}(Y)$ and $\beta f = \beta f^+ : \beta_f X \to Y$.

For a compactification $ef : e_fX \to Y$ of a mapping $f : X \to Y$, the mapping $ef \setminus f = ef : e_fX \setminus X \to Y$ is called the *remainder* of ef.

A mapping $f : X \to Y$ is called *locally compact* [7] if for any point $x \in X$, there exists a neighbourhood O of x in X such that $f|_{cl_X(O)} : cl_X(O) \to Y$ is compact.

It is not difficult to prove that

(i) for a locally compact Tychonoff space Y, a mapping $f : X \to Y$ is locally compact if and only if X is locally compact;

(ii) a mapping $f : X \to Y$ is locally compact if and only if X is open in $\beta_f X$ or, equivalently, if X is open in $e_f X$ for any compactification $ef : e_f X \to Y$ of f.

Now, Theorem 4.4 can be reformulated in the following way.

Theorem 5.7. *Let Y be a compact Hausdorff space and $ef_j : e_{f_j} X_j \to Y$ a Tychonoff compactification of a locally compact Tychonoff WZ-mapping $f_j : X_j \to Y$ (for $j = 1,2$). Let also $X_{jy} = f_j^{-1}(\{y\})$, $eX_{jy} = cl_{eX_j}(X_{jy})$ for $j = 1,2$. Then the remainders $ef_1 \setminus f_1$ and $ef_2 \setminus f_2$ are homeomorphic if and only if there exist poset isomorphisms $i : K(e_{f_1} X_1) \to K(e_{f_2} X_2)$ and $i_y : K(eX_{1y}) \to K(eX_{2y})$ such that (4.4) holds.*

Theorem 3.4 may be reformulated analogously. But, even in the style of Theorem 5.7, the formulation of Theorem 4.4 (and Theorem 3.4) seems too complicated, but this complexity may not be avoided. In order to explain why, for any fixed space Y, consider the category $\mathbf{Top}_Y$, where

$$\mathrm{Ob}\,(\mathbf{Top}_Y) = \{f \in C(X,Y) : \ X \in \mathrm{Ob}\,(\mathbf{Top})\} \tag{5.1}$$

is the class of the *objects* and, for every pair $f : X \to Y$, $g : Z \to Y$ of objects,

$$M(f,g) = \{\lambda \in C(X,Z) : \ g \circ \lambda = f\} \tag{5.2}$$

is the class of the *morphisms* from f to g, whose generic representation is denoted in short by $\lambda : f \to g$.

So, the question is: may the passage from the category **Top** *to the category* $\mathbf{Top}_Y$ *allow us to give simpler variants of Theorems 4.4 and 4.6 which can generalize Magill's and Rayburn's theorems?*

In this connection, we will give two examples which demonstrate that in the framework of $\mathbf{Top}_Y$, such generalizations are impossible.

Example 5.8. Let $Y = [0,2]$, $X_1 = X_2 = \mathbb{N}$, and $f_j : X_j \to Y$ be such that $f_j(X_j) = \{j\}$ for $j = 1,2$. Then $\beta_{f_j} X_j = \beta\mathbb{N}$, $\beta f_j(\beta_{f_j} X_j) = \{j\}$, $\beta f_j \setminus f_j = \beta f_j : \beta_{f_j} X_j \setminus X_j \to Y$, and so $\beta f_j \setminus f_j = \beta f_j : \beta\mathbb{N} \setminus \mathbb{N} \to Y$, with $(\beta f_j \setminus f_j)(\beta\mathbb{N} \setminus \mathbb{N}) = \{j\}$ for $j = 1,2$. Thus, the remainders $\beta f_1 \setminus f_1$ and $\beta f_2 \setminus f_2$ are not homeomorphic, but $TK(f_1)$ and $TK(f_2)$ are poset isomorphic because they, in fact, coincide with $K(\mathbb{N})$.

This example shows that an extension of Magill's Theorem to the category $\mathbf{Top}_Y$ must take into consideration fibres of objects of $\mathbf{Top}_Y$.

Example 5.9. Let $I = \{(x,0) \in \mathbb{R}^2 : 0 \le x \le 1\}$, $L = \{(0,y) \in \mathbb{R}^2 : -1 \le y \le 1\}$, $J = \{(x,1) \in \mathbb{R}^2 : 0 \le x \le 1\}$, $Y = I$, $S_1 = I \cup L$, and $S_2 = J \cup L$. Let $j = 1,2$. Put $pr_j(x,y) = x$ for any $(x,y) \in S_j$. Let ω_1 be the space of all finite and countable ordinal numbers. Then $\beta\omega_1 = \omega_1 + 1$. Put $X_j = S_j \times \omega_1$ and let π_j be the projection of the product X_j onto its factor S_j. Take $f_j = pr_j \circ \pi_j$. Evidently, f_j is either closed or open and all its fibres are countable compact. Then $\beta X_j = S_j \times (\omega_1 + 1)$ and $R_j = \beta X_j \setminus X_j$ is homeomorphic to S_j. Let p_j be the projection of the product βX_j onto its factor S_j. Then $\beta f_j = pr_j \circ p_j$ and $\beta f_j \setminus f_j = \beta f_j : R_j \to Y$ is homeomorphic to pr_j.

Thus, *the remainders* $\beta f_1 \setminus f_1$ and $\beta f_2 \setminus f_2$ *are not homeomorphic*.

For any $t = (x, 0) \in Y$, $(\beta f_j)^{-1}(\{t\}) = \beta(f_j^{-1}(\{t\}))$, and, for any $t = (x, 0) \in Y$ with $x > 0$, the remainders $R_{jt} = (\beta f_j)^{-1}(\{t\}) \setminus f_j{}^{-1}(\{t\}) = \beta(f_j{}^{-1}(\{t\})) \setminus f_j{}^{-1}(\{t\})$ are single points. Hence, $K(f_j{}^{-1}(\{t\}))$ consists of only one element for $t = (x, 0) \in Y$ with $x > 0$. For $t = (0, 0)$, the remainder R_{jt} is homeomorphic to L. Evidently, $TK(f_j)$ and $K(f_j^{-1}(\{(0, 0)\}))$ are poset isomorphic to $\mathcal{P}(L)$.

Now, it is evident that *there exist poset isomorphisms* $i : TK(f_1) \to TK(f_2)$, $i_t : K(f_1{}^{-1}(\{t\})) \to K(f_2{}^{-1}(\{t\}))$ for $t \in Y$ *and monotone functions* $m_{tj} : TK(f_j) \to K(f_j{}^{-1}(\{t\}))$ *for* $t \in Y$ *and* $j = 1, 2$ *such that*

(i) *for any* $cf_j : c_{f_j} X_j \to Y$ *with* $cf_j \in TK(f_j)$, $m_{tj}(cf_j) = (cf_j)^{-1}(\{t\})$, *that is,* $m_{tj}(cf_j)$ *is the closure of* $f_j^{-1}(\{t\})$ *in* $c_{f_j} X_j$;

(ii) $m_{t2} \circ i = i_t \circ m_{t1}$ *for every* $t \in Y$.

Example 5.9 shows that the use of the posets $K(eX_1)$ and $K(eX_2)$ (in particular $K(X_1)$ and $K(X_2)$) instead of $TK(f_1)$ and $TK(f_2)$ is justified.

Remark 5.10. Let $f : X \to Y$ be a mapping between Tychonoff spaces $M(f) = \{y \in Y : f \text{ is not closed at } y \text{ or } f^{-1}(\{y\}) \text{ is not compact}\}$ and $M_1(f) = \{y \in Y : |(\beta f)^{-1}(\{y\}) \setminus f^{-1}(\{y\})| = 1\}$. Evidently, $M_1(f) \subset M(f)$ and $\beta f(\beta_f X \setminus X) = M(f)$. The following proposition is proved in [5].

Proposition 5.11. *Let* $f : X \to Y$ *and* $g : Z \to T$ *be locally compact mappings between Tychonoff spaces and* $|M_1(f)| = |M_1(g)| = 1$ *. Then there exists a one-to-one correspondence* $\Psi : M(f) \to M(g)$ *such that* $(\beta f \setminus f)^{-1}(\{y\})$ *and* $(\beta g \setminus g)^{-1}(\{\Psi(y)\})$ *are homeomorphic for any* $y \in Y$. *If, additionally,* $M(f)$ *and* $M(g)$ *are discrete, then* $\beta f \setminus f$ *and* $\beta g \setminus g$ *are homeomorphic in the sense of* [5]*, that is, there exist homeomorphisms* $\varphi : \beta_f X \setminus X \to \beta_g Z \setminus Z$ *and* $\psi : M(f) \to M(g)$ *such that* $\psi \circ (\beta f \setminus f) = (\beta g \setminus g) \circ \varphi$ *if and only if* $TK(f)$ *and* $TK(g)$ *are poset isomorphic.*

Acknowledgments. The authors would like to thank the anonymous referees for their very accurate reading and the valuable suggestions. This research was supported by MURST and CNR (GNSAGA). The second author was supported by KCFE and RFFI.

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