# WEAKLY CLOSED MULTIFUNCTIONS AND PARACOMPACTNESS

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(Received on December 12, 1990 and in revised form June 6, 1991)

ABSTRACT. The purpose of the present paper is to investigate weakly closed multifunctions between topological spaces. We discuss basic properties of weakly closed multifunctions and some results of Rose and Janković are extended. Our main theorems are concerning some properties of paracompactness under weakly closed multifunctions. These theorems are generalizations of some results due to Kovaćević and Singal.

KEY WORDS AND PHRASES. Almost closed multifunction, weakly closed multifunction, paracompact, almost paracompact, nearly paracompact, $\mathfrak{M}$-paracompact, $\alpha$-paracompact, $\alpha$-$\mathfrak{M}$-paracompact.
1991 AMS SUBJECT CLASSIFICATION CODES. 54C60, 54D18.

## 1. INTRODUCTION.

Singal and Singal [1] introduced the almost closedness for arbitrary functions between topological spaces. Almost closed functions were also called regular closed by some authors in literature, for example, Noiri [2]. Another important generalization of closedness is the star closedness by Noiri [3] which implies almost closedness. Recently, Rose and Janković [4] introduced the weak closedness for functions which is a more general concept. Basic properties of weakly closed functions were also investigated in [4]. Rose and Janković [5] discussed conditions on weakly closed functions which assure a Hausdorff range. These results are improvements of Noiri's results in [2]. In this paper, weak closedness for multifunctions is studied. Firstly, we give a characterization of weakly closed multifunctions, then some conditions are given such that a weakly closed mutlifunction has a closed graph or strongly closed graph, and related results of [4, 5] are extended. Finally, we investigate some properties of paracompactness under weakly closed multifunctions. Some results of [6-8] are generalized. In particular, it is shown that the image of an $\alpha$-paracompact (resp. $\alpha$-almost paracompact, $\alpha$-$\mathfrak{M}$-paracompact, $\alpha$-almost $\mathfrak{M}$-paracompact) subset under a weakly closed multifunction is also $\alpha$-paracompact (resp. $\alpha$-almost paracompact, $\alpha$-$\mathfrak{M}$-paracompact, $\alpha$-almost $\mathfrak{M}$-paracompact) under certain conditions.

## 2. DEFINITIONS AND PRELIMINARIES.

Let $(X,T)$ be a topoligical space. $A \subseteq X$, $intA$ and $clA$ denote the interior and closure of $A$ respectively. $\Sigma(A)$ $(\Gamma(A))$ denotes the system of open (closed) neighborhoods of $A$. $x \in X$ is in the $\Theta$-closure of $A$, i.e., $x \in cl_\Theta A$, if each $V \in \Sigma(x)$ satisfies $A \cap clV \neq \phi$; $A$ is $\Theta$-closed if $A = cl_\Theta A$. $A$ is regular open (closed) if $A = int\ clA(A = cl\ intA)$. The family of all regular open subsets of $(X,T)$ is a base for a topology $T^*$ on $X$ which is called the semiregularization of $T$. $A \subseteq X$ is said to be star closed if $A$ is closed in $(X,T^*)$. $\mathcal{A}(X)$ is the family of all nonempty subsets of $X$. For $\mathcal{U} \subseteq \mathcal{A}(X)$, $\mathcal{U}^{\#} = \cup\{U:U \in \mathcal{U}\}$, and $|\mathcal{U}|$ denotes the cardinality of $\mathcal{U}$.

Recall that a space $X$ is said to be paracompact if every open covering of $X$ has a locally finite open covering refinement. $X$ is said to be $\mathfrak{M}$-paracompact if every open covering of cardinality $\leq \mathfrak{M}$ has a locally finite open refinement.

DEFINITION 1. ([9]) A subset $A$ of a space $X$ is said to be $\alpha$-paracompact iff for every $X$-open covering $\mathcal{U}$ of $A$ there exists an $X$-locally finite $X$ open covering $\mathcal{V}$ of $A$ which refines $\mathcal{U}$.

DEFINITION 2. ([3]) A space $X$ is nearly paracompact iff every regular open cover of $X$ has a locally finite open covering refinement.

DEFINITION 3. ([8]) A subset $A$ of a space $X$ is said to be $\alpha$-nearly paracompact iff every $X$-regular open cover of $A$ has an $X$-open $X$-locally finite refinement which covers $A$.

DEFINITION 4. ([6, 8]) A space $X$ is almost paracompact (almost $\mathfrak{M}$-paracompact) iff every open covering of $X$ (every open covering of $X$ of cardinality $\leq \mathfrak{M}$) has an open locally finite refinement whose union is dense in $X$.

DEFINITION 5. ([8, 10]) A subset $A$ of a space $X$ is said to be $\alpha$-almost paracompact ($\alpha$-almost $\mathfrak{M}$-paracompact) iff for every $X$-open cover $\mathcal{U}$ ($X$-open cover $\mathcal{U}$ and $|\mathcal{U}| \leq \mathfrak{M}$) of $A$ there exists an $X$-locally finite family of $X$-open subsets $\mathcal{V}$ which refines $\mathcal{U}$ and is that the family of $X$-closures of members of $\mathcal{V}$ forms a cover of $A$.

LEMMA 1. ([10]) A space $X$ is almost paracompact if and only if each regular open cover of $X$ has an open locally finite refinement whose union is dense in $X$.

COROLLARY. Paracompactness implies near paracompactness; near paracompactness implies almost paracompactness.

DEFINITION 6. ([11]) A subset $A$ of a space $X$ is said to be $\alpha$-nearly compact iff every $X$-regular open cover of $A$ has a finite subcover.

DEFINITION 7. A function $f:X \to Y$ is

(1) almost closed if $f(C)$ is closed for each regular closed set $C \subseteq X$ [ see 1];

(2) star closed if $f(C)$ is closed for every star closed $C \subseteq X$ [see 3];

(3) weakly closed if $clf(intC) \subseteq f(C)$ for every closed $C \subseteq X$ [see 4, 5].

Clearly we have the following implications: *closed→star closed→almost closed→weakly closed.*

A multifunction $F:X \to Y$ is a mapping from $X$ into $\mathcal{A}(Y)$. If $A \subseteq X, B \subseteq Y$, let $F(A) = \cup\{F(x):x \in A\}$, $F_+(A) = \{y \in Y: y \in F(x)$ for some $x \in A$ and $y \notin F(x)$ if $x \notin A\}$, $F^{-1}(B) = \{x \in X: F(x) \cap B \neq \phi\}$, and $F^{+1}(B) = \{x \in X: F(x) \subseteq B\}$. Recall that $F:X \to Y$ is upper semicontinuous at $x \in X$ (u.s.c. at $x \in X$) if $F^{+1}(V) \in \Sigma(x)$ for every $V \in \Sigma(F(x))$, and $F$ is lower semicontinuous at $x \in X$ (l.s.c. at $x \in X$) if $F^{-1}(V) \in \Sigma(x)$ for each open set $V$ with $V \cap F(x) \neq \phi$. $F$ is u.s.c. (l.s.c.) on $X$ if it is u.s.c. (l.s.c.) at each $x \in X$; and $F$ is continuous if it is both u.s.c. and l.s.c.

DEFINITION 8. ([12]) A multifunction $F:X \to Y$ is almost lower semicontinuous at $x \in X$ (a.l.s.c. at $x \in X$) iff $clF^{-1}(V) \in \Gamma(x)$ for each open set $V$ with $F(x) \cap V \neq \phi$.

$F$ is a.l.s.c. on $X$ if it is a.l.s.c. at each $x \in X$. We can prove the following lemma easily.

LEMMA 2. The following statements are equivalent for any multifunction $F:X \to Y$.

(1) $F$ is a.l.s.c.

(2) $F^{-1}(V) \subseteq int\ clF^{-1}(V)$ for all open $V \subseteq Y$.

(3) $F(clU) \subseteq clF(U)$ for all open $U \subseteq X$.

We say that a multifunction $F: X \to Y$ is $^*$-open if for each open $U \subseteq X$, $clF(U) = cl_\Theta F(U)$; $F$ is irreducible if $F(C) \neq Y$ for each proper closed subset $C$ of $X$; and $F$ has a closed graph if its graph $G(F) = \{(x,y): x \in X, y \in F(x)\}$ is closed in $X \times Y$.

DEFINITION 9. ([13]) A multifunction $F: X \to Y$ is said to have a strongly closed graph iff for each pair $(x,y) \in X \times Y - G(F)$ there are $U \in \Sigma(x)$ and $V \in \Sigma(y)$ such that $(U \times clV) \cap G(F) = \phi$.

Similarly to Definition 7, a multifunction $F: X \to Y$ is said to be almost closed (star closed) if $F(C)$ is closed for each regular closed (star closed) subset $C$ of $X$. $F$ is weakly closed if for each closed $C \subseteq X$, $clF(intC) \subseteq F(C)$.

Other basic concepts and terminilogy about topological spaces are referred to Engelking [14].

3. SOME BASIC RESULTS.

Let $X$ and $Y$ be topological spaces. Now we shall give a useful characterization of weakly closed multifunctions which is a generalization of a result in [4].

THEOREM 1. The following statements are equivalent for any multifunction $F: X \to Y$.

(1) $F$ is weakly closed.

(2) For each open $U \subseteq X$ and any $B \subseteq Y$ with $F^{-1}(B) \subseteq U$, there is an open $V \subseteq Y$ such that $B \subseteq V$ and $F^{-1}(V) \subseteq clU$.

(3) For each $y \in Y$ and each open set $U \subseteq X$ with $F^{-1}(y) \subseteq U$, there is $V \in \Sigma(y)$ such that $F^{-1}(V) \subseteq clU$.

PROOF. (1) implies (2). Let $U \subseteq X$ be open and $B \subseteq Y$ with $F^{-1}(B) \subseteq U$. Since $F$ is weakly closed, we have $F(X) = clF(X)$ and $clF(int(X-U)) \subseteq F(X-U) \subseteq F(X)$. Let $V = Y - clF(int(X-U))$, then $V$ is open and $B - F(X) \subseteq Y - F(X) \subseteq V$, so that we have $B = (B \cap F(X)) \cup (B - F(X)) \subseteq F_+(U) \cup V \subseteq V$ and $F^{-1}(V) = X - F^{+1}(clF(int(X-U))) \subseteq X - F^{+1}(F(int(X-U))) \subseteq X - int(X-U) \subseteq clU$.

(2) implies (3). It is obvious.

(3) implies (1). Let $C$ be any closed subset of $X$, $B = Y - F(C)$, then $F^{-1}(y) \subseteq X - C$ for each $y \in B$. Hence there is a $V_y \in \Sigma(y)$ such that $F^{-1}(V_y) \subseteq cl(X-C)$. Then $V = \cup\{V_y: y \in B\} \subseteq Y$ is open, and $F^{-1}(V) = \cup\{F^{-1}(Vy): y \in B\} \subseteq cl(X-C)$, $intC = X - cl(X-C) \subseteq X - F^{-1}(V)$. Therefore it follows $clF(intC) \subseteq clF(X - F^{-1}(V)) \subseteq Y - V \subseteq Y - B = F(C)$.

THEOREM 2. If a weakly closed multifunction $F: X \to Y$ is a.l.s.c., then $F$ is almost closed.

PROOF. Let $C$ be a regular closed subset of $X$. By the weak closedness of $F$, $clF(intC) \subseteq F(C)$. Since $F$ is a.l.s.c., by Lemma 2, $F^{-1}(Y - clF(intC)) \subseteq intclF^{-1}(Y - clF(intC)) \subseteq int\ cl(X - intC) = X - C$. Thus $clF(intC) = F(C)$. Therefore $F$ is almost closed.

COROLLARY ([5]). If $f: X \to Y$ is a weakly closed and almost continuous function, then $f$ is almost closed.

Now we shall consider conditions under which a weakly closed multifunction has a closed or strongly closed graph.

THEOREM 3. Let $F: X \to Y$ be a weakly closed multifunction of a space $X$ into a space $Y$ such that $F^1(y)$ is $\Theta$-closed for each $y \in Y$. Then $F$ has a closed graph.

PROOF. Similar to the proof of Theorem 4 of [5].

COROLLARY 3.1. ([5]) If $f: X \to Y$ is a weakly closed function that $f^{-1}(y)$ is $\Theta$-closed for each $y \in Y$, then $f$ has a closed graph.

COROLLARY 3.2. Let $F: X \to Y$ be a weakly closed multifunction of a regular space $X$ into a compact space $Y$ such that $F^{-1}(y)$ is closed for each $y \in Y$. Then $F$ is u.s.c. on $X$.

PROOF. Since $X$ is regular, then $F^{-1}(y)$ is $\Theta$-closed for each $y \in Y$. Hence, by Theorem 3, $F$ has a closed graph. Now suppose $F$ is not u.s.c. at some point $x_0 \in X$, there must exist $V \in \Sigma(F(x_0))$ such that $F(U) \nsubseteq V$ for each $U \in \Sigma(x_0)$. Therefore we can choose nets $\{x_\alpha : \alpha \in D\} \subseteq X$ and $\{y_\alpha : \alpha \in D\} \subseteq Y - V$ such that $x_\alpha \rightarrow x_0$ and $y_\alpha \in F(x_\alpha)$ for each $\alpha \in D$. By the compactness of $Y$, $\{y_\alpha : \alpha \in D\}$ must have a cluster point $y_0 \in Y - V$. Since $G(F)$ is closed, then $y_0 \in F(x_0)$. This is a contradiction.

THEOREM 4. Let $F: X \rightarrow Y$ be a $^*$-open multifunction of a space $X$ into a space $Y$. Then $F$ has a strongly closed graph if and only if it has a closed graph.

PROOF. Necessity is obvious. Sufficiency is similar to the proof of Theorem 3 of [5].

COROLLARY 4.1. ([5]) Let $f: X \rightarrow Y$ be a $^*$-open function. Then $f$ has a strongly closed graph if and only if it has a closed graph.

COROLLARY 4.2. Let $F: X \rightarrow Y$ be a $^*$-open and weakly closed multifunction such that $F^{-1}(y)$ is $\Theta$-closed for each $y \in Y$. Then $F$ has a strongly closed graph.

## 4. MAIN THEOREMS

In this section, we will investigate some properties of paracompactness under weakly closed multifunctions.

THEOREM 5. Let $F: X \rightarrow Y$ be a weakly closed, u.s.c., and open multifunction of a space $X$ into a space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$ and $F(x)$ is $\alpha$-paracompact for each $x \in X$. Then $F(K)$ is $\alpha$-paracompact if $K$ is an $\alpha$-paracompact set of $X$.

PROOF. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ be any $Y$-open cover of $F(K)$. Since $F(x)$ is $\alpha$-paracompact for each $x \in K$, then there is a $Y$-locally finite family of $Y$-open sets $\mathcal{V}_x$ which covers $F(x)$ and refines $\mathcal{U}$ and since $F$ is u.s.c., $\{F^{+1}(\mathcal{V}_x{}^{\#}) : x \in K\}$ is an open cover of $K$. By the $\alpha$-paracompactness of $K$, there is an $X$-locally finite and $X$-open cover $\mathcal{W} = \{W_\beta : \beta \in \nabla\}$ of $K$ which refines $\{F^{+1}(\mathcal{V}_x{}^{\#}) : x \in K\}$. Consequently, for each $\beta \in \nabla$, there is $x_\beta \in K$ such that $F(W_\beta) \subseteq \mathcal{V}_{x_\beta}{}^{\#}$. We set $\zeta_\beta = \{F(W_\beta) \cap V : V \in \mathcal{V}_{x_\beta}$ for each $\beta \in \nabla\}$, and $\zeta = \cup \zeta_\beta$. It is not difficult to see that $\zeta$ is a $Y$-open cover of $F(K)$ which refines $\mathcal{U}$. Now we shall show that $\zeta$ is $Y$-locally finite. For each $y \in Y$, since $\mathcal{W}$ is $X$-locally finite, then there exists an $X$-open set $H_x$ containing $x$ such that $clH_x$ intersects at most finite members of $\mathcal{W}$ for each $x \in F^{-1}(y)$. Thus $\{intclH_x : x \in F^{-1}(y)\}$ is an $X$-regular open cover of $F^{-1}(y)$ for each $y \in Y$. Since $F^{-1}(y)$ is $\alpha$-nearly compact, there exists $\{x_1, x_2, \cdots, x_n\} \subseteq F^{-1}(y)$ such that $F^{-1}(y) \subseteq \cup(int\ clH_{x_i}) \subseteq int\ cl(\cup H_{x_i}) = H_y$. Consequently, $H_y$ is $X$-open and intersects at most finite members of $\mathcal{W}$. By the weak closedness of $F$ and Theorem 1 there exists $Q_y \in \Sigma(y)$ such that $F^{-1}(y) \subseteq F^{-1}(Q_y) \subseteq clH_y \subseteq cl(\cup H_{x_i})$. It follows that $Q_y$ must meet only with finite members of $\zeta$. Hence $F(K)$ is $\alpha$-paracompact.

COROLLARY 5.1. If $F: X \rightarrow Y$ is a weakly closed, u.s.c., and open multifunction of a paracompact space $X$ onto a space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$, and $F(x)$ is $\alpha$-paracompact for each $x \in X$, then $Y$ is also paracompact.

COROLLARY 5.2. ([7]) If $F: X \rightarrow Y$ is an almost closed, open and u.s.c. multifunction of a paracompact space $X$ onto a space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$ and $F(x)$ is $\alpha$-paracompact for each $x \in X$, then $Y$ is paracompact.

COROLLARY 5.3. ([8]) If $f: X \rightarrow Y$ is an almost closed, open and continuous function of a paracompact space $X$ onto a space $Y$ such that $f^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$, then $Y$ is also paracompact.

Similarly we can obtain the following series of results.

THEOREM 6. Let $F: X \to Y$ be a weakly closed, u.s.c., and open multifunction of a space $X$ into a space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$ and $F(x)$ is $\alpha$-$\mathfrak{M}$-paracompact for each $x \in X$. Then $F(K)$ is $\alpha$-$\mathfrak{M}$-paracompact if $K$ is an $\alpha$-$\mathfrak{M}$-paracompact subset of $X$.

COROLLARY. If $F: X \to Y$ is a weakly closed, u.s.c. and open multifunction of an $\mathfrak{M}$-paracompact space $X$ onto a space $Y$ such that $F(x)$ is $\alpha$-$\mathfrak{M}$-paracompact for each $x \in X$ and $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$, then $Y$ is $\mathfrak{M}$-paracompact.

THEOREM 7. Let $F: X \to Y$ be a weakly closed, u.s.c., a.l.s.c. and open multifunction of a space $X$ into a space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$ and $F(x)$ is $\alpha$-paracompact ($\alpha$-$\mathfrak{M}$-paracompact) for each $x \in X$. Then $F(K)$ is $\alpha$-almost paracompact ($\alpha$-almost $\mathfrak{M}$-paracompact) if $K$ is an $\alpha$-almost paracompact ($\alpha$-almost $\mathfrak{M}$-paracompact) subset of $X$.

COROLLARY ([6]). If $f: X \to Y$ is a closed, continuous and open function of an almost $\mathfrak{M}$-paracompact space $X$ onto a space $Y$ such that $f^{-1}(y)$ is compact for each $y \in Y$, then $Y$ is also almost $\mathfrak{M}$-paracompact.

THEOREM 8. Let $F: X \to Y$ be a weakly closed, continuous and irreducible multifunction of a space $X$ onto an almost paracompact space $Y$ such that $F^{-1}(y)$ is $\alpha$-nearly paracompact for each $y \in Y$ and $F(x)$ is $\alpha$-nearly compact for each $x \in X$. Then $X$ is almost paracompact.

PROOF. Let $\mathfrak{U} = \{U_\alpha : \alpha \in \Delta\}$ be any regular open cover of $X$. Since $F^{-1}(y)$ is $\alpha$-nearly paracompact for each $y \in Y$, there is an $X$-locally finite family of $X$-open sets $\mathfrak{V}_y$ which covers $F^{-1}(y)$ and refines $\mathfrak{U}$. By the weak closedness of $F$ and Theorem 1, there is $W_y \in \Sigma(y)$ such that $F^{-1}(y) \subseteq F^{-1}(W_y) \subseteq cl(\mathfrak{V}_y{}^{\#})$ for each $y \in Y$. Thus we obtain an open cover $\mathcal{W} = \{W_y : y \in Y\}$ of $Y$. Since $Y$ is almost paracompact, there is a locally finite family of open subsets $\mathfrak{D} = \{D_\beta : \beta \in \nabla\}$ refining $\mathcal{W}$ such that $\{clD_\beta : \beta \in \nabla\}$ covers $Y$. Hence for each $\beta \in \nabla$, there must be $y_\beta \in Y$ such that $D_\beta \subseteq W_{y_\beta}$ and then $F^{-1}(D_\beta) \subseteq F^{-1}(W_{y_\beta}) \subseteq cl(\mathfrak{V}_{y_\beta}{}^{\#})$. Since $F$ is l.s.c., $\{F^{-1}(D_\beta) : \beta \in \nabla\}$ is a family of open subsets of $X$, and then for each $\beta \in \nabla$, $F^{-1}(D_\beta) = F^{-1}(D_\beta) \cap cl\mathfrak{V}_{y_\beta}{}^{\#}$ $= cl(F^{-1}(D_\beta) \cap \mathfrak{V}_{y_\beta}{}^{\#}) \cap F^{-1}(D_\beta) \subseteq cl(F^{-1}(D_\beta) \cap \mathfrak{V}_{y_\beta}{}^{\#})$. Consequently, we have $clF^{-1}(D_\beta)$ $\subseteq cl(F^{-1}(D_\beta) \cap \mathfrak{V}_{y_\beta}{}^{\#})$ for each $\beta \in \nabla$. Now let $\zeta_\beta = \{F^{-1}(D_\beta) \cap V : V \in \mathfrak{V}_{y_\beta}\}$ for each $\beta \in \nabla$, and $\zeta = \cup \zeta_\beta$. It is evident that $\zeta$ is a family of open subsets of $X$ which refines $\mathfrak{U}$. Firstly, we shall show that $\zeta$ is locally finite. To do this, for each $x \in X$, since $F(x)$ is $\alpha$-nearly compact and $\mathfrak{D}$ is locally finite, there is a regular open $G \subseteq Y$ with $F(x) \subseteq G$ and $G$ meets at most finite members of $\mathfrak{D}$. Then $F^{+1}(G)$ intersects at most finite members of the family $\{F^{-1}(D_\beta) : \beta \in \nabla\}$ and since $F$ is u.s.c., $F^{+1}(G) \in \Sigma(x)$. This shows that $\zeta$ is locally finite. Now we shall prove that the closures of members of $\zeta$ form a covering of $X$. Since $F$ is weakly closed, we have $clD_\beta \subseteq clF(F^{-1}(D_\beta))$ $\subseteq clF(int\ clF^{-1}(D_\beta)) \subseteq F(clF^{-1}(D_\beta))$ for each $\beta \in \nabla$. Therefore $Y = \cup clD_\beta \subseteq \cup F(clF^{-1}(D_\beta))$ $= F(\cup clF^{-1}(D_\beta))$. Since $F$ is irreducible and $\{F^{-1}(D_\beta) : \beta \in \nabla\}$ is closure-preserving, being locally finite, then $\cup clF^{-1}(D_\beta) = X$ and since each $\zeta_\beta (\beta \in \nabla)$ is closure-preserving, being locally finite, $X = \cup clF^{-1}(D_\beta) \subseteq \cup cl(F^{-1}(D_\beta) \cap \mathfrak{V}_{y_\beta}{}^{\#}) \subseteq \cup cl(\zeta_\beta{}^{\#}) = \{clS : S \in \zeta\}^{\#}$. Therefore, by Lemma 1, $X$ is an almost paracompact space.

COROLLARY ([8]). Let $f: X \to Y$ be a star closed, continuous and irreducible function of a space $X$ onto an almost paracompact space $Y$ such that $f^{-1}(y)$ is $\alpha$-nearly compact for each $y \in Y$. Then $X$ is almost paracompact.

By the similar proof of Theorem 8, we can get the following results.

THEOREM 9. Let $F: X \to Y$ be a continuous, weakly closed and irreducible multifunction of a space $X$ onto a space $Y$ such that $F^{-1}(y)$ is $\alpha$-$\mathfrak{M}$-paracompact for each $y \in Y$ and $F(x)$ is $\alpha$-nearly compact for each $x \in X$. Then $X$ is almost $\mathfrak{M}$-paracompact if $Y$ is almost $\mathfrak{M}$-paracompact.

COROLLARY ([6]). If $f: X \to Y$ is a closed, continuous and irreducible function of a space $X$ onto an almost $\mathfrak{M}$-paracompact space $Y$ such that $f^{-1}(y)$ is $\mathfrak{M}$-compact for each $y \in Y$. Then $X$ is also almost $\mathfrak{M}$-paracompact.

ACKNOWLEDGEMENT. The author would like to thank the referee for his valuable suggestions, Prof. D.A. Rose and Prof. D.S. Janković for their kind of help.

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