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RESEARCH NOTES

RETRACTS IN EQUICONNECTED SPACES

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ABSTRACT. The object of this paper is to show that the concepts "retract" and "strong deformation retract" coincide for a subset of an equiconnected space. Also, we have a similar local version in a locally equiconnected space.

KEY WORDS AND PHRASES. Retract, strong deformation retract, and equiconnected.
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1. INTRODUCTION.

Let X be a topological space, let A be a subset of the product space $X \times X$ and let λ be a map from $A \times I$ (where I is the interval) to X. The map λ is said to have the connecting property on A if $\lambda(x,y,0) = x$, $\lambda(x,y,1) = y$ for all $(x,y) \in A$ and $\lambda(x,x,t) = x$ for all $t \in I$, $(x,x) \in A$. A topological space X is equiconnected (EC) if there is a map λ from $X \times X \times I$ to X which has the connecting property on $X \times X$. A space X is locally equiconnected (LEC) if there is a neighborhood $\Delta(X)$ of the diagonal in $X \times X$ and a map λ from $\Delta(X) \times I$ to X which has the connecting property on $\Delta(X)$. A subset S of X is called a (strong) neighborhood deformation retract of X if there is a neighborhood N of S such that S is a (strong) deformation retract of N over X.

Now we state a main theorem.

THEOREM 1. The following three statements are equivalent for a subset S of a LEC space X.

i) S is a neighborhood retract of X,

ii) S is a neighborhood deformation retract of X,

iii) S is a strong neighborhood deformation retract of X.

It is well known (c.f., XV 8.1 in [2]) that a subset of a metrizable space X is a strong deformation retract of X if it is both a deformation retract of X and a strong neighborhood deformation retract of X. We therefore have

COROLLARY 1. If S is a subset of a metrizable LEC space X, then the following are equivalent.

i) S is a deformation retract of X,

ii) S is a strong deformation retract of X.

Furthermore, if X is contractible, the above statements are equivalent to

iii) S is a retract of X.

PROOF. We need only to show the implication *i)* $\Rightarrow$ *iii)*. By assuming *i)*, there is an open neighborhood N of S such that S is a retract of N with retraction $r : N \to S$. Since every open subset of an LEC space is LEC, N is LEC. Let $\lambda : \Delta(N) \times I \to N$ be a local equiconnection, where $\Delta(N)$ is an open neighborhood of the diagonal in $N \times N$. We define a map $F : N \to N \times N$ by $F(x) = (x, r(x))$ for all $x \in N$. Then, F is continuous and $F^{-1}(\Delta(N))$ is a neighborhood of S in N, and the map $G : F^{-1}(\Delta(N)) \times I \to X$ defined by $G(x,t) = \lambda(F(x),t)$ for all $(x,t) \in F^{-1}(\Delta(N)) \times I$ is the desired homotopy to complete the proof.

A similar argument gives

THEOREM 2. If S is a subset of an EC space X, then the following are equivalent:

i) S is a retract of X,

ii) S is a deformation retract of X,

iii) S is a strong deformation retract of X.

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