Internat. J. Math. & Math. Sci.
VOL. 15 NO. 2 (1992) 371-378

ON G-FINITISTIC SPACES AND RELATED NOTIONS

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(Received September 7, 1988 and in revised form December 12, 1990)

ABSTRACT. Let X be a G-space where G is a topological group. We show that X is G-finitistic iff the orbit space X/G is finitistic. This result allows us to answer a question raised in [5] asking for an equivariant characterization of a non-finitistic G-space where G is a compact Lie group. For an arbitrary compact group G a simple characterization of G-finitistic spaces has been obtained in terms of new notions of G-compactness and G-dimension.

KEY WORDS AND PHRASES. G-space, finitistic space, G-finitistic spaces, compact Lie group and covering dimension.
1991 AMS SUBJECT CLASSIFICATION CODE. 54B17, 54F45.

1. INTRODUCTION.

Finitistic spaces provide the most natural setting for the cohomological theory of topological transformation groups ([1], [4], [6]). These spaces are sufficiently general and include all compact spaces as well as finite dimensional paracompact spaces. Finite dimensional normal spaces which are even metacompact, however, need not be finitistic [7]. On the other hand a finitistic space is always metacompact and so the finitisticness property forces a collectionwise normal space to become paracompact. In this context, the general question as to whether or not a finitistic normal space is paracompact remains unresolved [7].

Let G be a topological group and X be a G-space, i.e., X is a topological space with a continuous G-action. We shall denote by X/G the orbit space of X and use the symbol $q: X \to X/G$ to denote the quotient map. Note that q is always an open map; it is closed whenever G is a compact group. A subspace Y of X is called a G-subspace if Y is invariant under the action of G.

Recall [9] that an open cover $\mathfrak{U}$ of X is said to be of dimension (or of order) n, $n \geq 0$, if there exists a set of $n+1$ members of $\mathfrak{U}$ whose intersection is nonempty but the intersection of any $n+2$ members of $\mathfrak{U}$ is always empty. A topological space X is said to be finitistic if each open cover $\mathfrak{U}$

of X has an open refinement $\mathcal{V}$ which is n-dimensional for some $n \geq 0$. In other words the refinement $\mathcal{V}$ is finite dimensional. Let us have:

DEFINITION 1.1. A G-space X is said to be G-finitistic if each G-open cover, i.e., open cover consisting of invariant open sets of X, has a finite dimensional refinement by G-open sets.

A basic problem about the orbit spaces which remains unresolved is the following: Let G be a compact group acting on a finitistic space X. Under what general conditions on G and X can we say that the orbit space X/G is also finitistic? The only known answer is the following (cf. [4]):

THEOREM A. If G is a compact Lie group acting on a paracompact finitistic space X, then the orbit space X/G is also finitistic.

One of the objectives of this paper is to give an interesting equivalent formulation of the above mentioned problem in terms of the concept of G-finitisticness as given by Definition 1.1.

We introduce the concept of covering G-dimension and indicate how to introduce other equivalent topological notions. This helps us in obtaining a simple characterization theorem for G-finitistic spaces in terms of G-compactness and covering G-dimension. Completely G-finitistic spaces are also introduced and it is proved that such a space which is also G-normal must be finite G-dimension.

2. THE COVERING G-DIMENSION.

To start with, we want to separate two simple observations about the order of open coverings which will be repeatedly used in the whole paper.

B. Let $f: X \to Y$ be a continuous onto map and $\{V_\beta\}$ be an open cover of Y. Then $\{f^{-1}(V_\beta)\}$ is an open cover of X and

$$\dim\{f^{-1}(V_\beta)\} = \dim\{V_\beta\}$$

C. Let $f: X \to Y$ be a continuous onto open map and $\{U_\alpha\}$ be an open cover of X. Then $\{f(U_\alpha)\}$ is an open cover of Y and any of the following possibilities can occur:

(a) $\dim\{f(U_\alpha)\} < \dim\{U_\alpha\}$

(b) $\dim\{f(U_\alpha)\} > \dim\{U_\alpha\}$

(c) $\dim\{f(U_\alpha)\} = \dim\{U_\alpha\}$

However, if each U_α is a saturated open set, then only (c) can happen.

PROOF. Suppose $U_{\alpha_1}, U_{\alpha_2}, \ldots, U_{\alpha_n}$ intersect. Then there exists a point $x \in \bigcap_{i=1}^{n} U_{\alpha_i}$. Since each U_{α_i} is saturated, $f^{-1}(f(x)) \subseteq U_{\alpha_i} \; \forall \; i = 1, 2, \ldots, n$. This means $f(x) \in \bigcap_{i=1}^{n} f(U_{\alpha_i})$. Hence

$$\dim\{U_\alpha\} \leq \dim\{f(U_\alpha)\}.$$

For converse, suppose $f(U_{\alpha_1}), \ldots, f(U_{\alpha_n})$ intersect. Then, because U_{α_i} are saturated, $U_{\alpha_i} = f^{-1}(f(U_{\alpha_i}))$ for $i = 1, 2, \ldots, n$ also intersect. Hence

$$\dim\{f(U_\alpha)\} \leq \dim\{U_\alpha\}. \qquad Q.E.D.$$

Now let X be a G-space. We introduce the notion of covering G-dimension or equivariant dimension of X as follows:

DEFINITION 2.1. A G-space X is said to have covering G-dimension $\leq n$ if each finite G-open covering $\mathcal{U}$ of X has a refinement by G-open sets of order $\leq n$. As usual, we say that G-$\dim(X) = n$ if G-$\dim(X) \leq n$ and G-$\dim(X) \not\leq n-1$.

The following result gives a simple alternative form of looking at the G-dimension of a G-space X. It tells us that G-$\dim(X) = \dim(X/G)$.

PROPOSITION 2.2. A G-space X is of G-dimension $\leq m$ if and only if $\dim(X/G) \leq m$.

PROOF. Suppose $G\text{-}\dim X = n$. Let $\mathfrak{U} = \{U_i\}$ be a finite open cover of X/G. Then $\{q^{-1}(U_i)\}$ is a finite open cover of X by G-open sets. Therefore it has a refinement, say $\{V_j\}$ consisting of G-open sets whose order $\leq n$. Then $\{q(V_j)\}$ is an open cover of X/G which refines $\{U_i\}$, and by observation C, dim $\{q(V_j)\} = \dim\{V_j\} \leq n$. Hence $\dim X/G \leq n$.

Conversely, suppose $\dim X/G = n$. Let $\mathfrak{U} = \{U_i\}$ be a finite open cover of X by G-open sets. Then clearly $\{q(U_i)\}$ is a finite open cover of X/G. Hence it has an open refinement say $\{V_j\}$ of order $\leq n$. Then clearly $\{q^{-1}(V_j)\}$ refines $\{U_j\}$ and by observation B, dim $\{q^{-1}(V_j)\} = \dim\{V_j\} \leq n$. This gives us a refinement of $\{U_i\}$ by G-open sets of order $\leq n$ and so $G\text{-}\dim X \leq n$.

Proposition 2.2 also helps us in computing the G-dimension of a G-space X. Let us consider the following examples.

EXAMPLE 2.3. Let G be an infinite dimensional topological group, e.g., $G = \prod_{i=1}^{\infty} S_i^1$, where S_i^1 is the circle group for each i. Then G acts on (itself) $X = G$ by translations. In this case the only G-invariant open sets of X is X itself and so $G\text{-}\dim X = 0$. But clearly $\dim X = \infty$. This is, of course, the extreme situation.

EXAMPLE 2.4. Let $G = \prod_{i=1}^{n} S_i^1$ be the product of n copies of circles $S_i^1 = S^1$ for each i and let G act on $X = G$ by

$$(g_1, g_2, \ldots, g_n)\,(x_1, x_2, \ldots, x_n) = (g_1x_1, x_2, \ldots, x_n)\ .$$

Then $X/G \approx \prod_{i=1}^{n-1} S_i^1$ and so G-dim $X = n-1$ where as dim $X = n$. Clearly, we can likewise have a G-space X such that dim $X = n$ and G-dim $X = r$ for any $r, 0 \leq r \leq n$.

EXAMPLE 2.5. Consider the action of p-adic group A_p on the compact Hausdorff space X constructed in [9]. Then $\dim X = n$ but $A_p - \dim(X) = n+2$.

These examples combined with Proposition 2.2 tell us that, in general, $\dim X$ and $G\text{-}\dim X$ are not related. However, by ([4] Prop. 3.7) we have the following:

COROLLARY 2.6. If G is a compact Lie group and X is a paracompact Hausdorff G-space. Then

$$G\text{-}\dim X \leq \dim X\ ,$$

In case G acts on X freely and X is an n-manifold, then we know that

$$\dim X = \dim X/G + \dim G$$

and so in such a case

$$G\text{-}\dim X = \dim X\text{-}\dim G\ .$$

It is interesting to note that in view of Proposition 2.2., the question whether an orbit map can convert a finite dimensional space X into an infinite dimensional space is equivalent to asking whether there exists a G-space X such that $\dim X < \infty$ but $G\text{-}\dim X = \infty$.

EQUIVARIANT DEFINITIONS. Any topological concept can always be suitably defined in the equivariant category of G-spaces. For example, let us consider the concept of G-normality.

DEFINITION 2.7. We will call a G-space X to be equivariantly normal or G-normal if given any two disjoint G-closed subsets of X there exist two disjoint open G-subsets of X separating them.

With this definition one can easily verify that a G-space X is G-normal if and only if the orbit space X/G is normal. Likewise, we have

DEFINITION 2.8. A G-space X will be called G-metacompact if each G-open cover of X has an orbit-finite refinement by G-open sets.

This clearly makes the following statement true: A G-space X is G-metacompact if and only if X/G is metacompact. Now we can define G-Hausdorff spaces as follows:

DEFINITION 2.9. A G-space X is G-Hausdorff if any two distinct orbits of X can be separated by disjoint G-open sets.

Then, it is easily checked that, X is G-Hausdorff if and only if X/G is Hausdorff. Finally, let us define G-paracompactness as follows:

DEFINITION 2.10. A G space X will be called G-paracompact if each G-open cover of X has a locally finite open refinement by G-open sets.

With this definition X/G is paracompact certainly implies that X is G-paracompact, but the converse need not be true. The following result shows that the converse is true provided G is compact.

PROPOSITION 2.11. If G is compact and X is G-paracompact, then X/G is paracompact.

PROOF. Let $\{U_\alpha\}$ be an open cover of X/G. Then $\{q^{-1}(U_\alpha)\}$ is an open cover of X by G-open sets. Hence there is locally finite refinement $\{V_\beta\}$ of the above covering by G-open sets. Let $G(x)$ be the orbit representing $\tilde{x} \in X/G$. Then $G(x)$ is compact and each $x \in G(x)$ has an open neighborhood N_x which intersects only finitely many members of $\{V_\beta\}$. Select a finite subcover $N_{x_1}, \ldots, N_{x_k}$ of $G(x)$ and put $N = N_{x_1} \cup N_{x_2} \cup \ldots \cup N_{x_k}$. Then N is an open neighborhood of the orbit $G(x)$ and meets finitely many members of $\{V_\beta\}$. Since G is compact, we can find an invariant neighborhood M of $G(x)$ in G which is contained in N and so meets finitely many members of $\{V_\beta\}$. Then clearly $q(M)$ is a neighbourhood of x which meets only finitely many members of $\{q(V_\beta)\}$ and this last covering of X/G refines $\{U_\alpha\}$.

However, if we want the converse to be true for all groups G, we will have to modify Definition 2.10 as follows:

DEFINITION 2.12. A G-space X is G-paracompact if each G-open cover of X has a refinement by G-open sets which has the property that each orbit has a G-neighbourhood which meets only finitely many members of the refinement.

With this definition one can easily verify that for any group G a G-space is G-paracompact if and only if X/G is paracompact. Now the question arises as to what is the relationship between the paracompactness of G-space X and G-paracompactness of X?

The answer is the G-paracompactness always implies paracompactness of X for any G. The converse is also true if the group is compact because in that case $q\colon X \to X/G$ is closed and so X/G is paracompact and this means X is G-paracompact. A similar statement can be made for other concepts also. In contrast with this, note that a G-space X is compact implies X/G is compact and so X is G-compact. But the converse need not be true unless G is compact. For a concrete example, take $G = R$ acting on $X = R$ by translations. Then X is G-compact but evidently it is not compact.

The various forms of sum theorems for G-dimension can now easily be formulated for arbitrary group G. We have

PROPOSITION 2.13. Let X be a G-normal space. Then for the covering G-dimension all of the following sum theorems hold:

(i) countable sum theorem for G-closed sets;

(ii) locally finite sum theorem for G-subsets;

(iii) complementary sum theorem for G-subsets; and

(iv) disjoint sum theorem for G-subsets.

PROOF. We only prove (i), the proofs of the rest are just analogous. Let $\{A_n\}$ be a countable covering of X by closed G-subsets of X. We have to prove that

$$G\text{-}\dim X = \sup_n \{G\text{-}\dim A_n\}$$

X is G-normal implies X/G is normal and $\{A_n/G\}$ is obviously countable closed covering of X/G. Since for covering dimension, the countable sum theorem for normal spaces holds for closed sets, we find that

$$\dim X/G = \operatorname*{Sup}_n \{\dim A_n/G\}$$

Now the proof follows from Proposition 2.2.

As yet another application of Proposition 2.2 we have the next result which tells us that for compact groups and for paracompact G-spaces, the word "finite" in Definition 2.1 can be dropped.

PROPOSITION 2.14: Let X be a paracompact G-space where G is a compact group. Then the following statements are equivalent:

(i) Every finite G-open cover of X has a refinement by G-open sets of order $\leq n$.

(ii) Every G-open cover of X has a refinement by G-open sets of order $\leq n$.

PROOF. (ii)$\Rightarrow$(i) is trivial. Suppose (i) holds, i.e., $G\text{-}\dim X \leq n$. Then by Proposition 2.2, $\dim X/G \leq n$. Since G is compact, the quotient map $q\colon X \to X/G$ is closed and so X/G has an open refinement of order $\leq n$. This clearly implies that each G-open cover of X has a refinement by G-open sets of order $\leq n$.

3. G-FINITISTIC SPACES.

If we look at the definitions of finitistic and G-finitistic spaces, they do not appear at the first glance to be related for a given G-space X. This is indeed so, and we will give examples to support this later on. However, one thing is quite straightforward: if G is a finite group, then the two concepts are identical for any G-space X as shown below:

PROPOSITION 3.1. Let G be a finite group acting on a space X. Then X is finitistic if and only if X is G-finitistic.

PROOF. Suppose X is finitistic and let $\mathfrak{U} = \{\mathfrak{U}_\alpha\}$ be an open cover of X by G-open sets. Choose a finite dimensional open refinement $\mathcal{V} = \{V_\beta \mid \beta \in B\}$ of $\mathfrak{U}$. Then for each $g \in G$, $gV = gV_\beta \mid \beta \in B\}$ is also an open cover of X and hence the finite intersection viz $\bigcap_{g \in G} gV$ is an open refinement of V. Note that this intersection covering is a covering by G-open sets and

$$\dim\left(\bigcap_{g \in G} gV\right) \leq (\dim V)^{|G|} - 1 .$$

Hence we have found a finite dimensional refinement of $\mathfrak{U}$ by G-open sets and so X is G-finitistic.

Conversely suppose X is G-finitistic and let $\mathfrak{U} = \{U_\alpha\}$ be an open cover of X. As in the first part $\bigcap_{g \in G} g\mathfrak{U}$ is a refinement of $\mathfrak{U}$ by G-open sets and hence it has a finite dimensional refinement by G-open sets and the same is a finite dimensional open refinement of $\mathfrak{U}$. Hence X is finitistic.

The above proposition is, roughly speaking, a consequence of the fact that for a finite group G, the set of all invariant open coverings of a G-space X is a cofinal set in the set of all open covers of X; and this, in turn, was possible because G is supposed to be finite. The above method therefore cannot be extended in infinite groups. In fact, it is not true also that the set of all invariant open covers is cofinal in the set of all open covers of X unless, of course, G is finite. To see this just take $G = S^1$ which acts on $X = S^1$ by translations. Then S^1 can be covered by small open intervals whereas X does not have an invariant open set except X itself. Nonetheless, we shall finally prove that even in case of a compact Lie group G, X is finitistic if and only if X is G-finitistic, an interesting result indeed. Let us now prove

PROPOSITION 3.2. Let X be a G-space. Then X is G-finitistic if and only if X/G is finitistic.

PROOF. Suppose X is G-finitistic. Let $\mathfrak{U} = \{U_\alpha\}$ be an open cover of X/G. Then $\{q^{-1}(U_\alpha)\}$ is a cover of X by G-open sets and so there exist a finite dimensional refinement $\{W_\beta \mid \beta \in B\}$ is an open refinement of $\mathfrak{U}$. Since

$$\dim(q(W_\beta) \mid \beta \in B\} \leq \dim\{W_\beta \mid \beta \in B\} ,$$

the open cover $\{q(W_\beta)\}$ is also finite dimensional. This proves the first part.

Conversely, suppose X/G is finitistic and let $\mathfrak{U} = \{U_\alpha\}$ be an open cover of X by G-open sets. Then $\{q(U_\alpha)\}$ is an open cover of X/G. Hence it has a finite dimensional open refinement $\{V_\beta \mid \beta \in B\}$. Then it is easily seen that $\{q^{-1}(V_\beta) \mid \beta \in B\}$ is a refinement of $\mathfrak{U}$ by G-open sets and since

$$\dim\{q^{-1}(V_\beta) \mid \beta \in B\} \leq \dim\{V_\beta \mid \beta \in B\}$$

it is also finite dimensional.

COROLLARY 3.3. Let G be a compact Lie group acting on a paracompact Hausdorff space X. Then X is finitistic if and only if X is G-finitistic. In particular, X is G-finitistic if and only if X is H-finitistic for any closed subgroup H of G.

PROOF. It has been proved in ([4] Thm. 3.8, [5] Thm. 4.5) that under the given hypotheses, X is finitistic if and only if X/G is finitistic. Hence the proof follows from Proposition 3.2.

REMARK. It is interesting to observe that the above corollary is in fact a deep result, but the depth can be measured only when one attempts a direct proof of it. In other words, the entire burden of the proof is relegated to the proof of corresponding theorem of [4] and [5] mentioned above. Corollary 3.3 also implies the following equivariant version of the characterization theorem for non finitistic spaces and for a compact Lie group G. This answers the question (i) raised in [5] page 165.

THEOREM. Let G be a compact Lie group. Then a paracompact G-space X is not finitistic if and only if there exists a discrete sequence $\{A_n\}$ of closed G-subspaces of X such that $\dim A_n > n$ for each $n \geq 1$.

Now we present an example to show that the concepts of finitisticness and G-finitisticness are, in general, not related.

EXAMPLE 3.4. There exists a G-finitistic space X which is not finitistic. Let us consider the standard free action of the discrete group Z of integers on the real line R. Let $G = \prod_{i=1}^{\infty} Z_i$ be the infinite product of groups $Z_i = Z$, $i = 1,2,\ldots$ and let $X = \prod_{i=1}^{\infty} R_i$ be the infinite product of real lines $R_i = R$, $i \geq 1$. Note that G is not compact and non-discrete and acts continuously on the metric space X. The orbit space X/G is homeomorphic to the infinite product $\prod_{i=1}^{\infty} S_i^1$ of 1-spheres S_i^1, $i \geq 1$, i.e., infinite dimensional torus. Thus X/G being compact is finitistic and so by Proposition 3.2, X is G-finitistic. However, it has been shown ([4], Example 2) that X is not finitistic.

One will then immediately ask as to what about the converse? Is it true that every finitistic G-space is G-finitistic? In this connection we raise the following:

QUESTION. Does there exist a paracompact Hausdorff space X and an action of a group G on X such that $(\dim X/G - \dim X)$ is larger than any given natural number n?

In other words can an orbit map $q: X \to X/G$ increase the covering dimension of X by arbitrary large numbers? There is an example due to R.F. Williams [9] of an n-dimensional space Y and action of the p-adic group A_p on Y such that $\dim Y/A_p = n+2$. But nothing better than this seems to be known. A stronger question would be asked if there exists an integer $k \geq 0$ and a sequence $\{X_n\}$ of G-spaces such that $\dim X_n \leq k$, but $\dim X_n/G \geq n$ for each natural number n.

Now we show that this stronger question is intimately related to asking whether there exists a finitistic G-space X which is not G-finitistic, at least for paracompact Hausdorff spaces. We have

PROPOSITION 3.3. Let G be a topological group. There exists a paracompact finitistic G-space X which is not G-finitistic if and only if there exists an integer k and a sequence $\{X_n\}$ of paracompact spaces such that $\dim X_n/G \geq n$ and $\dim X_n \leq k$ for each natural number n.

PROOF. We will make use of ([4] Thm. 3.6) to prove this result. Suppose X is a paracompact finitistic G-space which is not G-finitistic. Then, by Proposition 3.2, X/G is not finitistic. Hence by ([4] Thm. 3.6) there exists a relatively discrete sequence $\{A_n\}$ of closed subsets of X/G whose union is closed and $\dim A_n > n$ for each n. Then clearly $\{q^{-1}(A_n)\}$ is also a discrete family of closed sets whose union is closed. But X is finitistic and so the sequence $\{\dim q^{-1}(A_n)\}$ must be bounded, say $\dim q^{-1}(A_n) \leq k$ for all n. Then it is clear that $q^{-1}(A_{k+1})$, whose orbit spaces A_{k+j} have dimension $> k+j$.

Conversely, suppose we can find paracompact G-spaces $Y_1, Y_2, \ldots$ such that $\dim Y_n \leq k$ for all n, but $\{\dim Y_n/G\}$ is unbounded. Then, let $X = \bigcup_{m \geq 1} Y_n$ be the disjoint union of Y_ns and G act on X in an obvious manner. Then X is at most k-dimensional and paracompact and hence finitistic. However, X/G has a discrete family $\{Y_n/G\}$ of closed sets having closed union such that $\{\dim Y_n/G\}$ is unbounded. This means X/G is not finitistic. Hence by Proposition 3.2, X cannot be G-finitistic.

4. ANOTHER CHARACTERIZATION OF G-FINITISTIC SPACES.

The following theorem has been formulated by Hattori using one of the main results of [4].

THEOREM 4.1. A paracompact Hausdorff space X is finitistic if and only if there exists a compact subset K of X such that each closed subset F of X disjoint from K is finite dimensional.

Using Theorem 4.1 and Proposition 3.2, we can easily establish the following characterization theorem for G-finitistic spaces, for any compact group G.

THEOREM 4.2. A G-paracompact space X is G-finitistic if and only if there exists a G-compact subset K of X such that any G-closed F of X disjoint from K is finite G-dimensional.

PROOF. Suppose X is G-finitistic. Then X/G is paracompact and finitistic. Hence by Theorem 4.1 there exists a compact subset K of X/G such that any closed subset F of X/G which is disjoint from K is finite dimensional. This means $K^1 = q^{-1}(K)$ is G-compact. Now if F is any G-closed subset of X disjoint from K^1, then $q(K^1)$ is clearly a closed subset of X/G and is disjoint from K. This implies that $q(K^1)$ is finite dimensional and therefore K^1 is of finite G-dimension.

Conversely, suppose there exists a G-compact subset K of X satisfying the given conditions. It is sufficient to prove that X/G is finitistic and paracompact. Since X is G-paracompact X/G is paracompact by definition. Also because G is compact, K/G is compact. Now if F is any closed subset of X/G which is disjoint from K/G, then $q^{-1}(F)$ is G-closed and disjoint from K. Hence $q^{-1}(F)$ is of finite G-dimension which implies that F is finite dimensional. So, again by Theorem 4.1, X/G is finitistic.

Let us finally state.

DEFINITION 4.2. A G-space X will be called completely G-finitistic if each G-subspace of X is G-finitistic.

This definition combined with Proposition 3.2 guarantees the validity of the following:

PROPOSITION 4.3. A G-space X is completely G-finitistic if and only if X/G is completely finitistic.

THEOREM 4.4. A G-normal, G-Hausdorff G-finitistic space X is of finite G-dimension.

PROOF. We only note that X is G-normal makes X/G normal, X is G-Hausdorff makes X/G Hausdorff, and X is G-finitistic makes X/G finitistic. Therefore, we apply the main theorem of [2] and the above proposition to conclude the result.

The authors are thankful to the referee for his valuable suggestions which improved the presentation of the paper.

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