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# ON THE DISSIPATION LAYER OF RADIAL BEARINGS

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ABSTRACT. The dissipation boundary layer of certain radial bearings is identified. It is shown that, under certain conditions, the temperature outside this layer is constant.

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## 1. INTRODUCTION.

In paper [1], we discussed the solution to the coupled momentum and energy equations

$$\frac{\partial\phi}{\partial t} = \Pr \frac{\partial}{\partial y}\left(\exp\left(\frac{\lambda_1}{\theta+\lambda_2}\right) \frac{\partial\phi}{\partial y}\right), \tag{1.1}$$

$$\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} - \left(\frac{\partial\theta}{\partial y}\right)^2 + \Delta \exp\left(\theta+\frac{\lambda_1}{\theta+\lambda_2}\right)\left(\frac{\partial\phi}{\partial y}\right)^2 \tag{1.2}$$

where $\lambda_1$, $\lambda_2$ and $\Delta$ are constants.

The boundary and initial conditions are

$$\frac{\partial\phi}{\partial y}(0,t) = -\frac{B}{\sqrt{t}}\cdot \phi(0,t) \quad \text{given,} \quad \phi(\infty,t) = 0 \tag{1.3}$$

$$\phi(y,0) = 0 \tag{1.4}$$

$$\theta(0,t) = \theta(\infty,t) = 0 \tag{1.5}$$

$$\theta(y,0) = 0 \tag{1.6}$$

The above equations arose from investigations on the flow of thin lubricating oils.

Earlier in [2], we discussed the thermal runaway of variable viscosity flows between concentric cylinders - there is a wide gap between the cylinders and we show that, under certain conditions which affect the Peclet number, the reduced Reynolds number and the Nahme-Griffith number, the width of the thermal boundary layer is $O(R)$ where $R$ is the radius of the inner cylinder. Under the same assumptions, there exists a dissipation boundary layer of width $O(R/G^2)$ where $G$ is the Nahme-Griffith number.

In the case of radial bearings, the width, h, of the lubricant is small and thus if $R_1$, $R_2$ are the radii of the shaft and the bearing then

$$R_i \gg h. \quad i = 1, 2 \tag{1.7}$$

(1.7) implies that as a first approximation, we may regard the shaft and the bearing as flat surfaces, that is $R_i \to \infty$ when $h$ is the standard measure. We therefore need to investigate the boundary layer of the lubrication problem although $O(R/G^2)$ is adequate for [2].

## 2. DISSIPATION BOUNDARY LAYER.

Shampine [3] investigated the problem

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(c)\frac{\partial c}{\partial x}\right), \tag{2.1}$$

$$c(0,t) = 1, \quad \frac{\partial c}{\partial x}(0,t) \text{ , given} \tag{2.2}$$

He showed that if

$$-D(1)\ \frac{\partial c}{\partial t}(0,t) = \frac{1}{2} + \max D(c),$$

then there is an $\eta_1 < 1$ such that $c(\eta_1) = 0$, where $\eta = x/\sqrt{t}$ . For the proof of this, see [3].

In this paper as in [1], we have

$$\Pr \exp\left(\frac{\lambda_1}{\theta+\lambda_2}\right)$$

in the plane of $D(c)$ above, but a careful study of the proof advanced by Shampine showed that his result is true for (1.1). That is, there exists an $\eta_1 < 1$ such that if

$$B = 1 + \frac{1}{2}\, e^{-\frac{\lambda_1}{\lambda_2}} \tag{2.3}$$

then there exists an $\eta_1 < 1$ such that $\phi(\eta_1) = 0$ and $\phi(\eta) \equiv 0$ for $\eta \geq \eta_1$. This leads to

THEOREM. Let (1.1), (1.2) (1.3) - (1.6), and (2.3) hold. Then there exists an $\eta_2 \leq 1$ such that $\theta(\eta_2) = 0$ and $\theta(\eta\ ) \equiv 0$ and $\eta \geq \eta_2$.

PROOF. $\phi(\eta_1) \equiv 0$ for $\eta \geq \eta_1$

implies $\frac{\partial \phi}{\partial y} \equiv 0$ for $\eta > \eta_1$.

That is, $\frac{\partial \phi}{\partial y} \equiv 0$ for $\eta \geq \eta_2 = \eta_1^+$

Hence $\theta \equiv 0$ for $\eta \geq \eta_2$ .

## 3. PHYSICAL INTERPRETATION.

The above theorem shows that at time $t$ the heat generated has only penetrated to $y = \eta_2\sqrt{t}$ . This identifies the dissipation layer. The viscosity of the lubricant outside this layer is not affected by heat under the conditions assumed in this problem.

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