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Research Article

Single-Machine Scheduling Problems with a Sum-of-Processing-Time-Based Learning Function

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Recently, learning scheduling problems have received increasing attention. However, the majority of the research assume that the actual job processing time is a function of its position. This paper deals with the single-machine scheduling problem with a sum-of-processing-time-based learning effect. By the effect of sum-of-processing-time-based learning, we mean that the processing time of a job is defined by total normal processing time of jobs in front of it in the sequence. We show that the single-machine makespan problem remains polynomially solvable under the proposed model. We show that the total completion time minimization problem for $a \geq 1$ remains polynomially solvable under the proposed model. For the case of $0 < a < 1$, we show that an optimal schedule of the total completion time minimization problem is V-shaped with respect to normal job processing times.

1. Introduction

Scheduling problems are very important in the fields of manufacturing systems. Hence numerous scheduling problems have been studied for many years. In the classical scheduling theory, most research assumes that processing times of jobs are constant over the entire planning horizon. However, in many realistic situations, the efficiency of the production facility (e.g., a machine or a worker) improves continuously over time [1–4]. As a result, the production time of a given product is shorter if it is scheduled (and so processed) later. The processing times of jobs may be subject to change due to learning phenomena by Pinedo [5].

During the last few years, learning effect has attracted growing attention in the scheduling community on account of its significance. There have been many attempts to formulate learning effect in a quantitative form as a function of learning variables, called a learning curve. Most of the concepts assume that the learning curve is a nonincreasing function which depends on the jobs already performed. For a survey on learning curves, the reader is refereed to Jaber and Bonney [6].

Biskup [7] and Cheng and Wang [8] were one of the pioneers who brought the concept of learning into the field of scheduling. Recently Biskup [9] discusses the questions why and when learning effects in scheduling environments might occur and should be regarded from a planning perspective. Afterwards he gives a concise overview on the literature on scheduling with learning effects. He classifies the learning models in scheduling into two types, namely position-based learning and sum-of-processing-times-based learning.

In the classical position-dependent learning effect model (Biskup [7]), the actual processing time p_{ir} of job J_i when it is scheduled in the rth position in a processing sequence is defined as $p_{jr} = p_j r^a$, where a is a nonpositive learning index and p_i denotes the normal processing time of job J_i. Biskup [7] indicated that the learning effect has been observed in numerous practical situations in different sectors of industry and for a variety of corporate activities. He proposed a position-based learning model and showed that two single-machine scheduling problems remain polynomially solvable. Since then, many researchers have paid more attention on the relatively young but very vivid area. Mosheiov [10] found that under Biskup's learning effect model the optimal schedules for some classical scheduling problems remain valid, but they require much greater computational effort to obtain. In addition, he gave some counterexamples to show that the optimal schedules for scheduling problems with learning effects may be different from those for the corresponding classical scheduling problems. Many researchers have studied such a learning effect model and its variants thereafter; a sample of these papers include [11–20].

Note that position-dependent learning effects neglect the processing times of the jobs already processed. If human interactions have significant impacts during the processing of jobs, the processing times will be added to the employees′ experience and thus cause learning effects. For situations like this it might be more appropriate to consider a time-dependent learning effect [9]. Kuo and Yang [21] considered a sum-of-job-processing-times-based learning effect model. They provided the optimal solution of the total completion time problem. For more papers about this time-dependent learning effect model, the reader is refereed to [22–27].

In this paper, we propose a sum-of-processing-time-based learning effect model where the learning effect is expressed as a function of the normal processing times of jobs already processed. This model is adopted from Cheng et al. [28] They consider some machine scheduling problems with deteriorating jobs and learning effect. The actual processing time of a job depends on not only the processing times of the jobs already processed but also its scheduled position. They derive polynomial-time optimal solutions for the problems to minimize makespan and total completion time in the single-machine case.

Specifically, we consider two single-machine scheduling problems with a sum-of-processing-time-based learning effect. The objectives are to minimize the makespan and the total completion time of all jobs, respectively. The rest of the paper is organized as follows. In the next section, we give the problem description. In Section 3, we consider two single-machine scheduling problems. The last section is the conclusion.

2. Problem Description

In this section, we first define the notation that is used throughout this paper, followed by the description of the problem. There are n jobs ready to be processed on a single machine. All jobs are available at time zero. Let p_j denote the normal processing time of job j for $j = 1, 2, \ldots, n$. In addition, let $p_{[r]}$ denote the normal processing time of the job scheduled in the

rth position in a job sequence. If job j is scheduled in the rth position in a sequence, then its actual processing time is

$$p_{jr} = p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a, \tag{2.1}$$

where $p_0 > 0$ is a given parameter, a denotes the learning rate where $a > 0$ and $\sum_{l=1}^{0} p_{[l]} = 0$. Under this learning effect model, the actual processing time of job j is affected by the previous $(r-1)$ jobs. In the remaining part of the paper, all the problems considered will be denoted using the three-field notation scheme $\alpha|\beta|\gamma$ introduced by Graham et al. [29]

3. Single-Machine Scheduling Problems

First, we give two lemmas; they are useful for the following theorems. The proofs of the lemmas can be obtained by differentiation.

Lemma 3.1. *One has* $[1-(1+\lambda x)^a] - \lambda[1-(1+x)^a] \geq 0$ *if* $\lambda \geq 1, x \geq 0, 0 < a < 1$.

Lemma 3.2. *One has* $[1-(1+\lambda x)^a] - \lambda[1-(1+x)^a] \leq 0$ *if* $\lambda \geq 1, x \geq 0, a \geq 1$.

Theorem 3.3. *For the problem* $1 \mid p_{jr} = p_j((p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l))^a \mid C_{\max}$,

(1) *if* $0 < a < 1$, *then an optimal schedule can be obtained by sequencing the jobs in nonincreasing order of* p_j *(the longest processing time (LPT) rule).*

(2) *if* $a \geq 1$, *then an optimal schedule can be obtained by sequencing the jobs in nondecreasing order of* p_j *(the shortest processing time (SPT) rule).*

Proof. The proof follows directly from the pairwise interchange analysis. Let π and π' be two job schedules where the difference between π and π' is a pairwise interchange of two adjacent jobs J_i and J_j, that is, $\pi = [S_1, J_i, J_j, S_2]$, $\pi' = [S_1, J_j, J_i, S_2]$, where S_1 and S_2 are partial sequences. Furthermore, we assume that there are $r-1$ jobs in S_1. Thus, J_i and J_j are the rth and $(r+1)$th jobs, respectively, in π. Likewise, J_j and J_i are scheduled in the rth and the $(r+1)$th positions in π'. To further simplify the notation, let A denote the completion time of the last job in S_1. Under π, the completion times of jobs J_i and J_j are

$$C_i(\pi) = A + p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a, \tag{3.1}$$

$$C_j(\pi) = A + p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \tag{3.2}$$

Under π', the completion times of jobs J_j and J_i are

$$C_j(\pi) = A + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a, \tag{3.3}$$

$$C_i(\pi) = A + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \tag{3.4}$$

After taking the difference between (3.2) and (3.4), it is obtained that

$$\begin{aligned} C_j(\pi) - C_i(\pi') = (p_i - p_j)\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a \\ - p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \end{aligned} \tag{3.5}$$

By substituting $t = (p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l)$, $\lambda = p_j/p_i$, $w = p_i/(p_0 + \sum_{l=1}^{n} p_l)$, and $x = w/t$ into (3.5), it is simplified to

$$C_j(\pi) - C_i(\pi') = p_i t^a\left\{\left[1 - (1 + \lambda x)^a\right] - \lambda\left[1 - (1 + x)^a\right]\right\}. \tag{3.6}$$

(1) Suppose $p_i \le p_j$, then we have $\lambda \ge 1$ and $x \ge 0$. From Lemma 3.1, we have $C_j(\pi) - C_i(\pi') \ge 0$. This completes the proof of (1).

(2) From case (1) and Lemma 3.2, the result can be easily obtained. □

Theorem 3.4. *The problem* $1 \mid p_{jr} = p_j((p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l))^a, a \ge 1 \mid \sum C_j$, *can be solved optimally by sequencing jobs in nondecreasing order of pj (the SPT rule).*

Proof. It is similar to the proof of Theorem 3.3 except what follows.

Suppose $p_i \le p_j$, then

$$\begin{aligned} C_i(\pi) + C_j(\pi) - C_i(\pi') - C_j(\pi') = 2(p_i - p_j)\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a \\ - p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \end{aligned} \tag{3.7}$$

By substituting $t = (p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l)$, $\lambda = p_j/p_i$, $w = p_i/(p_0 + \sum_{l=1}^{n} p_l)$ and $x = w/t$ into the (3.7), it is simplified to

$$C_i(\pi) + C_j(\pi) - C_i(\pi') - C_j(\pi') = p_i t^a\left\{1 - \lambda + \left[1 - (1 + \lambda x)^a\right] - \lambda\left[1 - (1 + x)^a\right]\right\}. \tag{3.8}$$

Since $\lambda \ge 1$, $x \ge 0$ and Lemma 3.2, (3.8) is nonpositive. Then we have

$$C_i(\pi) + C_j(\pi) \le C_i(\pi') + C_j(\pi'). \tag{3.9}$$

This completes the proof. □

However, in spite of increasing efforts to tackle the problem $1 \mid p_{jr} = p_j((p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l))^a, 0 < a < 1 \mid \sum C_j$, it cannot be solved optimally by sequencing jobs in nondecreasing order of p_j (the SPT rule) or by sequencing jobs in nonincreasing order of p_j (the LPT rule).

Example 3.5. $n = 3$, $p_0 = 1$, $p_1 = 1$, $p_2 = 2$, $p_3 = 3$, and the learning rate index $a = 0.5$. The SPT sequence is $\{J_1, J_2, J_3\}$, $\sum C_j(SPT) = 3.716$. The LPT sequence is $\{J_3, J_2, J_1\}$, $\sum C_j(LPT) = 3.572$. However, the optimal sequence is $\{J_3, J_1, J_2\}$, $\sum C_j(OPT) = 3.569$.

From Example 3.5, if $0 < a < 1$, we know that the classical SPT rule or LPT rule cannot give an optimal solution for the total completion time minimization problem. It remains an open problem. Now, we prove that the problem $1 \mid p_{jr} = p_j((p_0+\sum_{l=1}^{r-1} p_{[l]})/(p_0+\sum_{l=1}^{n} p_l))^a, 0 < a < 1 \mid \sum C_j$, has an important property, that is, the optimal schedule is V-shaped with respect to the normal job processing times. Before the proof, we first present the definition of the V-shaped policy.

Definition 3.6 (see [30]). Schedule is V-shaped with respect to normal job processing times if jobs are arranged in descending order if they are placed before the job with the smallest p_j but in ascending order if placed after it.

Theorem 3.7. *For the problem* $1 \mid p_{jr} = p_j((p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l))^a$, $0 < a < 1 \mid \sum C_j$, *an optimal schedule exists, which is V-shaped with respect to the normal job processing times.*

Proof. Consider a schedule π with three consecutive jobs, J_i, J_j, and J_j, that is, $\pi = [S_1, J_i, J_j, J_k, S_2]$, where S_1 and S_2 are partial sequences, such that $p_j > p_i$ and $p_j > p_k$. We show that an interchange between J_i and J_j or between J_j and J_k reduces the value of total completion time. Let π_1 be the schedule obtained from π by interchanging J_i and J_j, that is, $\pi_1 = [S_1, J_j, J_i, J_k, S_2]$. Similarly, let π_2 be the schedule obtained from π by interchanging J_j and J_k, that is, $\pi_2 = [S_1, J_i, J_k, J_j, S_2]$. Furthermore, we assume that there are $r - 1$ jobs in S_1. Thus, J_i, J_j, and J_k are the rth, $(r + 1)$th and $(r + 2)$th jobs, respectively, in π. Likewise, J_j, J_i, and $J_k(J_i$, J_k and $J_j)$ are scheduled in the rth, $(r + 1)$th, and $(r + 2)$th positions in $\pi_1(\pi_2)$. To further simplify the notation, let A denote the completion time of the last job in S_1. Then the contribution of the three jobs to the total completion time is

$$\begin{aligned} \Delta(\pi) = 3A &+ 3p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + 2p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a \\ &+ p_k\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \end{aligned} \tag{3.10}$$

Similar expressions are easily obtained for π_1 and π_2:

$$\begin{aligned} \Delta(\pi_1) = 3A &+ 3p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + 2p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a \\ &+ p_k\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a, \end{aligned}$$

$$\Delta(\pi_2) = 3A + 3p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + 2p_k\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i + p_k}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \tag{3.11}$$

It follows that

$$\Delta(\pi) - \Delta(\pi_1) = 3(p_i - p_j)\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]}}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + 2p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a - 2p_i\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a, \tag{3.12}$$

$$\Delta(\pi) - \Delta(\pi_2) = 2(p_j - p_k)\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i}{p_0 + \sum_{l=1}^{n} p_l}\right)^a + p_k\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i + p_j}{p_0 + \sum_{l=1}^{n} p_l}\right)^a - p_j\left(\frac{p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i + p_k}{p_0 + \sum_{l=1}^{n} p_l}\right)^a. \tag{3.13}$$

Let $\lambda = p_j/p_i$, $t = (p_0 + \sum_{l=1}^{r-1} p_{[l]})/(p_0 + \sum_{l=1}^{n} p_l)$, $w = p_i/(p_0 + \sum_{l=1}^{n} p_l)$, and $x = w/t$, then we have $\lambda \geq 1$ and $x \geq 0$. From (3.12) we have

$$\frac{\Delta(\pi) - \Delta(\pi_1)}{p_i t^a} = 3(1-\lambda) + 2\lambda(1+x)^a - 2(1+\lambda x)^a. \tag{3.14}$$

Let $\mu = p_j/p_k$, $s = (p_0 + \sum_{l=1}^{r-1} p_{[l]} + p_i)/(p_0 + \sum_{l=1}^{n} p_l)$, $v = p_k/(p_0 + \sum_{l=1}^{n} p_l)$, and $\theta = v/s$, then we have $\mu \geq 1$ and $\theta \geq 0$. From (3.13) we have

$$\frac{\Delta(\pi) - \Delta(\pi_2)}{p_k s^a} = 2(\mu - 1) + (1+\mu\theta)^a - \mu(1+\theta)^a. \tag{3.15}$$

Now, let $\Delta(\pi) - \Delta(\pi_1)$ be negative; from (3.14) and Lemma 3.1, we have

$$\begin{aligned} & 3(1-\lambda) + 2\lambda(1+x)^a - 2(1+\lambda x)^a < 0 \\ & \Longrightarrow 2(1-\lambda) + \lambda(1+x)^a - (1+\lambda x)^a + (1-\lambda) + \lambda(1+x)^a - (1+\lambda x)^a < 0 \\ & \Longrightarrow 2(1-\lambda) + \lambda(1+x)^a - (1+\lambda x)^a < 0 \\ & \Longrightarrow 2(\mu - 1) - \mu(1+\theta)^a + (1+\mu\theta)^a > 0. \end{aligned} \tag{3.16}$$

Hence we have $\Delta(\pi) - \Delta(\pi_2) > 0$.

Now, let $\Delta(\pi) - \Delta(\pi_2)$ be negative; from (3.15) and Lemma 3.1, we have

$$\begin{aligned}
&2(\mu - 1) - \mu(1+\theta)^a + (1+\mu\theta)^a < 0 \\
&\Longrightarrow 2(\mu - 1) - \mu(1+\theta)^a + (1+\mu\theta)^a + (\mu - 1) - \mu(1+\theta)^a + (1+\mu\theta)^a < 0 \\
&\Longrightarrow 3(\mu - 1) - 2\mu(1+\theta)^a + 2(1+\mu\theta)^a < 0 \\
&\Longrightarrow 3(1-\lambda) + 2\lambda(1+x)^a - 2(1+\lambda x)^a > 0.
\end{aligned} \tag{3.17}$$

Hence we have $\Delta(\pi) - \Delta(\pi_1) > 0$.

We conclude that an optimal schedule exists, which is V-shaped with respect to the normal job processing times. □

4. Conclusion

The main contribution of this paper is that we develop a learning effect model where the actual job processing time is a function of jobs already processed. We show that the single-machine makespan problem remains polynomially solvable under the proposed model. We also show that the total completion time minimization problem for $a \geq 1$ remains polynomially solvable under the proposed model. Moreover, for the case of $0 < a < 1$, we show that an optimal schedule of the total completion time minimization problem is V-shaped with respect to normal job processing times. It is suggested that, for future research to investigate this open problem, the sum-of-processing-time-based learning effect should be considered in the context of other scheduling problems, including multimachine and job-shops settings.

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