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Research Article

Fixed Point Results for Multivalued Maps in Cone Metric Spaces

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We prove some fixed point theorems for multivalued maps in cone metric spaces. We improve and extend a number of known fixed point results including the corresponding recent fixed point results of Feng and Liu (1996) and Chifu and Petrusel (1997). The remarks and example provide improvement in the mentioned results.

1. Introduction

The well-known Banach contraction principle and its several generalizations in the setting of metric spaces play a central role for solving many problems of nonlinear analysis. For example, see [1–5]. Using the concept of the Hausdorff metric, Nadler [6] obtained a multivalued version of the Banach contraction principle. Without using the concept of the Hausdorff metric, recently, Feng and Liu [7] obtained a new fixed point theorem for nonlinear contractions in metric spaces, extending Nadler's result. Recently, Chifu and Petrusel obtained a fixed point result [18, Theorem 2.1] which contains [7, Theorem 3.1].

In 1980, Rzepecki [8] introduced a generalized metric by replacing the set of real numbers with normal cone of the Banach space. In 1987, Lin [9] introduced the notion of K-metric spaces by replacing the set of real numbers with cone in the metric function. Zabrejko [10] studied new revised version of the fixed point theory in K-metric and K-normed linear spaces. Most recently, Huang and Zhang [11] announced the notion of cone metric spaces, replacing the set of real numbers by an ordered Banach spaces. They proved some basic properties of convergence of sequences and also obtained various fixed point theorems for contractive single-valued maps in such spaces. For more fixed point results in cone metric spaces, see [12–17].

In this paper, first we prove a useful lemma in the setting of cone metric spaces. Then, we prove some results on the existence of fixed points for multivalued maps in cone metric spaces. Consequently, our results improve and extend a number of known fixed point results including the corresponding recent main fixed point results of Chifu and Petrusel [18, Theorems 2.1 and 2.5].

2. Preliminaries

Let E be a real Banach space and P a subset of E. P is called a *cone* if and only if

(i) P is closed, nonempty, and $P \neq \{0\}$;

(ii) $a, b \in \mathbb{R}, a, b \geq 0$, and $x, y \in P$ imply that $ax + by \in P$;

(iii) $x \in P$ and $-x \in P$ imply that $x = 0$.

For a given cone $P \subset E$, we define partial ordering $\preceq$ on E with respect to P by the following: for $x, y \in E$, we say that $x \preceq y$ if and only if $y - x \in P$. Also,we write $x \ll y$ if $y - x \in \text{int}P$, where $\text{int}\, P$ denotes the interior of P.

The cone P is called *normal* if there is a constant $K > 0$ such that for all $x, y \in E$

$$0 \preceq x \preceq y \text{ implies } \|x\| \leq K\|y\|. \tag{2.1}$$

The least positive number K satisfying the above inequality is called the normal constant of P; for details see ([3, 11]).

In the sequel, E is a real Banach space, P is a cone in E, and $\preceq$ is partial ordering with respect to P.

Definition 2.1. Let X be a nonempty set. Suppose that the map $d : X \times X \to E$ satisfies

(i) $0 \preceq d(x, y)$ for all $x, y \in X$, and $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(iii) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a *cone metric* on X, and (X, d) is called a *cone metric space* ([11]).

Example 2.2 (see [11]). Let $E = \mathbb{R}^2, P = \{(x, y) \in E : x, y \geq 0\} \subset \mathbb{R}^2$, $X = \mathbb{R}$, and $d : X \times X \to E$ defined by $d(x, y) = (|x - y|, \alpha|x - y|)$, where $\alpha \geq 0$ is a constant. Then (X, d) is a cone metric space.

Example 2.3 (see [16]). Let $E = \ell^1, P = \{\{x_n\}_{n\geq 1} \in E : x_n \geq 0, \text{ for all } n\}$, (X, ρ) a metric space, and $d : X \times X \to E$ defined by $d(x, y) = \{\rho(x, y)/2^n\}_{n\geq 1}$. Then (X, d) is a cone metric space.

Clearly, the above examples show that class of cone metric spaces contains the class of metric spaces.

Now, we recall some basic definitions of sequences in cone metric spaces (see, [11, 17]. Let (X, d) be a cone metric space and $\{x_n\}$ a sequence in X. Then

(i) $\{x_n\}$ converges to $x \in X$ whenever for every $c \in E$ with $0 \ll c$, there is a natural number N such that $d(x_n, x) \preceq c$ for all $n \geq N$; we denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$;

(ii) $\{x_n\}$ is a Cauchy sequence whenever for every $c \in E$ with $0 \ll c$, there is a natural number N such that $d(x_n, x_m) \preceq c$ for all $n, m \geq N$;

(iii) (X, d) is said to be complete space if every Cauchy sequence in X is convergent in X;

(iv) A set $A \subseteq X$ is said to be closed if for any sequence $\{x_n\} \subset A$ converges to x, we have $x \in A$;

(v) A map $f : X \to \mathbb{R}$ is called *lower semicontinuous* if for any sequence $\{x_n\} \subset X$ such that $x_n \to x \in X$, we have $f(x) \leq \liminf_{n\to\infty} f(x_n)$.

Lemma 2.4 (see [11]). *Let (X, d) be a cone metric space, and let P be a normal cone with normal constant K. Let $\{x_n\}$ be any sequence in X. Then*

(a) *$\{x_n\}$ converges to $x \in X$ if and only if $d(x_n, x) \to 0$, as $n \to \infty$;*

(b) *$\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \to 0$, as $n, m \to \infty$.*

Let (X, d) be a cone metric space. We denote 2^X as a collection of nonempty subsets of X, and $Cl(X)$ as a collection of nonempty closed subsets of X. An element $x \in X$ is called a fixed point of a multivalued map $T : X \to 2^X$ if $x \in T(x)$. Denote $\mathrm{Fix}(T) = \{x \in X : x \in T(x)\}$.

For $T : X \to \mathrm{Cl}(X)$, and $x \in X$, one denotes

$$D(x, Tx) = \{d(x, z) : z \in Tx\}. \tag{2.2}$$

For $c \in E$ with $0 \ll c$, one denotes

$$\widetilde{B}(x, c) = \{y \in X : d(x, y) \preceq c\}. \tag{2.3}$$

The set $\widetilde{B}(x, c)$ is closed [16, Lemma 2.3].

3. The Results

First, we prove our key lemma.

Lemma 3.1. *Let (X, d) be a cone metric space and let P be a normal cone with normal constant K. If there exist a sequence $\{x_n\}$ in X and a real number $\gamma \in (0, 1)$ such that for every $n \in \mathbb{N}$,*

$$d(x_{n+1}, x_n) \preceq \gamma d(x_n, x_{n-1}), \tag{3.1}$$

then $\{x_n\}$ is a Cauchy sequence.

Proof. Let $x_0 \in X$ be an arbitrary but fixed. Note that

$$d(x_{n+1}, x_n) \preceq \gamma^n d(x_1, x_0). \tag{3.2}$$

Now, for all $m > n$,

$$\begin{aligned} d(x_n, x_m) &\preceq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &\preceq \left(\gamma^n + \gamma^{n+1} + \cdots + \gamma^{m-1}\right) d(x_1, x_0) \\ &\preceq \frac{\gamma^n}{1-\gamma} d(x_1, x_0). \end{aligned} \tag{3.3}$$

Since P is a normal cone with the normal constant K, we have

$$\|d(x_n, x_m)\| \leq K \frac{\gamma^n}{1-\gamma} \|d(x_1, x_0)\|; \tag{3.4}$$

taking limit as $n, m \to \infty$, we get $d(x_n, x_m) \to 0$. Thus $\{x_n\}$ is a Cauchy sequence.
Applying Lemma 3.1, we prove the following result. □

Theorem 3.2. *Let (X, d) be a complete cone metric space, P a normal cone with normal constant K, and $T : X \to \mathrm{Cl}(X)$. Suppose that the following hold for arbitrary but fixed $x_0 \in X, u_0 \in D(x_0, Tx_0)$, and $c \in E$ with $c \gg 0$:*

(i) *there exist constants $a, b \in (0, 1]$ with $a < b$ such that for each $x \in \widetilde{B}(x_0, c)$ and for any $u \in D(x, Tx)$ there exist $y \in Tx$ and $v \in D(y, Ty)$ satisfying*

$$\begin{aligned} b d(x, y) &\preceq u, \\ v &\preceq a d(x, y); \end{aligned} \tag{3.5}$$

(ii) *$u_0 \preceq (1 - a/b) bc$;*

(iii) *the function $f : X \to \mathbb{R}$ defined by $f(x) = \inf_{y \in Tx} \|d(x, y)\|$ is lower semicontinuous.*

Then $\mathrm{Fix}(T) \cap \widetilde{B}(x_0, c) \neq \emptyset$.

Proof. Since $x_0 \in \widetilde{B}(x_0, c)$ and $u_0 \in D(x_0, Tx_0)$, it follows from (i) and (ii) that there exist $x_1 \in Tx_0$ and $u_1 \in D(x_1, Tx_1)$ satisfying

$$b d(x_0, x_1) \preceq u_0 \preceq \left(1 - \frac{a}{b}\right) bc, \tag{3.6}$$

$$u_1 \preceq a d(x_0, x_1). \tag{3.7}$$

Note that

$$d(x_0, x_1) \preceq \left(1 - \frac{a}{b}\right) c \preceq c, \tag{3.8}$$

and thus $x_1 \in \widetilde{B}(x_0, c)$. From (3.6) and (3.7) it follows that

$$u_1 \preceq \frac{a}{b} u_0. \tag{3.9}$$

Now, since $x_1 \in \widetilde{B}(x_0, c)$ and $u_1 \in D(x_1, Tx_1)$, there exist $x_2 \in Tx_1$ and $u_2 \in D(x_2, Tx_2)$ such that

$$bd(x_1, x_2) \preceq u_1, \tag{3.10}$$

$$u_2 \preceq ad(x_1, x_2). \tag{3.11}$$

Using (3.9), (3.10) (3.11) we obtain

$$u_2 \preceq \left(\frac{a}{b}\right)^2 u_0. \tag{3.12}$$

From (3.7), (3.8) and (3.10) it follows that

$$d(x_1, x_2) \preceq \frac{a}{b} d(x_0, x_1) \preceq \frac{a}{b}\left(1 - \frac{a}{b}\right)c. \tag{3.13}$$

Note that

$$\begin{aligned} d(x_0, x_2) &\preceq d(x_0, x_1) + d(x_1, x_2) \\ &\preceq \left(1 - \frac{a}{b}\right)c + \frac{a}{b}\left(1 - \frac{a}{b}\right)c \\ &\preceq \left(1 - \left(\frac{a}{b}\right)^2\right)c \\ &\preceq \mathrm{c}, \end{aligned} \tag{3.14}$$

and so $x_2 \in \widetilde{B}(x_0, c)$. Continuing this process, we obtain $x_n \in \widetilde{B}(x_0, c)$ and $u_n \in D(x_n, Tx_n)$ such that $x_{n+1} \in Tx_n$ and $u_{n+1} \in D(x_{n+1}, Tx_{n+1})$ satisfying

$$bd(x_n, x_{n+1}) \preceq u_n \preceq ad(x_{n-1}, x_n), \tag{3.15}$$

and we get

$$d(x_n, x_{n+1}) \preceq \frac{a}{b} d(x_{n-1}, x_n) \quad \text{for } n \in \mathbb{N}. \tag{3.16}$$

Thus by Lemma 3.1, $\{x_n\}$ is a Cauchy sequence in the closed set $\widetilde{B}(x_0, c) \subset X$. Due to the completeness of $\widetilde{B}(x_0, c)$, there exists $x^* \in \widetilde{B}(x_0, c)$ such that $\lim_{n \to \infty} x_n = x^*$. Note that

$$u_{n+1} \preceq ad(x_n, x_{n+1}) \preceq \frac{a}{b} u_n \preceq \cdots \preceq \left(\frac{a}{b}\right)^{n+1} u_0, \tag{3.17}$$

and thus

$$0 \preceq u_{n+1} \preceq \left(\frac{a}{b}\right)^{n+1} u_{0.} \tag{3.18}$$

From (3.18) and the fact the cone P is normal, we have

$$\|u_{n+1}\| \leq K\left(\frac{a}{b}\right)^{n+1} \|u_0\|, \tag{3.19}$$

and thus $\|u_{n+1}\| \to 0$ as $n \to \infty$; it follows that $\{u_n\}$ is convergent to 0. Since $u_n \in D(x_n, Tx_n)$, there exists a sequence $\{z_n\}$ such that $z_n \in Tx_n$ and $u_n = d(x_n, z_n)$. Now by the convergence of the sequence $\{u_n\}$ and by assumption (iii) we obtain

$$\inf_{y \in Tx^*} \|d(x^*, y)\| \leq \liminf_{n \to \infty} \inf_{y \in Tx_n} \|d(x_n, y)\| \leq \liminf_{n \to \infty} \|d(x_n, z_n)\| = 0. \tag{3.20}$$

Thus

$$\inf_{y \in Tx^*} \|d(x^*, y)\| = 0. \tag{3.21}$$

From (3.21), it follows that there exists a sequence $\{y_n\} \subset Tx^*$ such that $\lim_{n\to\infty} \|d(x^*, y_n)\| = 0$, and thus $d(x^*, y_n) \to 0$ as $n \to \infty$. Hence, $y_n \to x^*$. Since Tx^* is closed, we get $x^* \in Tx^*$. Thus, $\operatorname{Fix}(T) \cap \widetilde{B}(x_0, c) \neq \emptyset$. □

Remark 3.3. Our Theorem 3.2 extends the main fixed point result of Chifu and Petrusel [18, Theorem 2.1] to the setting of cone metric spaces, and thus the result of Feng and Liu [7, Theorem 2.1] follows from our Theorem 3.2 as well. Theorem 3.2 also extends some results from [2, 5, 6].

Another fixed point result is the following.

Theorem 3.4. *Let (X, d) be a complete cone metric space, P a normal cone with normal constant K, and $T : X \to \mathrm{Cl}(X)$. Suppose that the following hold for arbitrary but fixed $x_0 \in X, u_0 \in D(x_0, Tx_0)$, and $c \in E$ with $c \gg 0$:*

(i) *there exist $a, b, s \in (0, 1]$ with $a + sb < b$ such that for each $x \in \widetilde{B}(x_0, c)$ and for any $u \in D(x, Tx)$ there exist $y \in Tx$ and $v \in D(y, Ty)$ satisfying*

$$\begin{gathered} bd(x,y) \preceq u, \\ v \preceq ad(x,y) + su; \end{gathered} \tag{3.22}$$

(ii) *$u_0 \preceq [1 - (a/b + s)]bc$;*

(iii) *the function $f : X \to \mathbb{R}$ defined by $f(x) = \inf_{y \in Tx} \|d(x, y)\|$ is lower semicontinuous.*

Then $\operatorname{Fix}(T) \cap \widetilde{B}(x_0, c) \neq \emptyset$.

Proof. Since $x_0 \in \widetilde{B}(x_0, c)$ and $u_0 \in D(x_0, Tx_0)$, there exist $x_1 \in Tx_0$ and $u_1 \in D(x_1, Tx_1)$ satisfying

$$bd(x_0, x_1) \preceq u_0, \tag{3.23}$$

$$u_1 \preceq ad(x_0, x_1) + su_0. \tag{3.24}$$

From (3.23) and (3.24) we have

$$u_1 \preceq \left(\frac{a}{b} + s\right) u_0. \tag{3.25}$$

Using (3.23) and (ii), we get

$$d(x_0, x_1) \preceq \left[1 - \left(\frac{a}{b} + s\right)\right] c \preceq c, \tag{3.26}$$

and so $x_1 \in \widetilde{B}(x_0, c)$. Therefore, there exist $x_2 \in Tx_1$ and $u_2 \in D(x_2, Tx_2)$ satisfying

$$bd(x_1, x_2) \preceq u_1 \tag{3.27}$$

$$u_2 \preceq ad(x_1, x_2) + su_1. \tag{3.28}$$

Using (3.25), (3.27), and (3.28), we get

$$u_2 \preceq \left(\frac{a}{b} + s\right) u_1 \preceq \left(\frac{a}{b} + s\right)^2 u_0. \tag{3.29}$$

Now, using (3.23), (3.24), (3.27), and (ii), we have

$$d(x_1, x_2) \preceq \frac{1}{b}\left(\frac{a}{b} + s\right) u_0 \preceq \left(\frac{a}{b} + s\right)\left(1 - \left(\frac{a}{b} + s\right)\right) c. \tag{3.30}$$

Note that

$$\begin{aligned} d(x_0, x_2) &\preceq d(x_0, x_1) + d(x_1, x_2) \\ &\preceq \left(1 - \left(\frac{a}{b} + s\right)\right) c + \left(\frac{a}{b} + s\right)\left(1 - \left(\frac{a}{b} + s\right)\right) c \\ &\preceq \left(1 - \left(\frac{a}{b} + s\right)^2\right) c \\ &\preceq c. \end{aligned} \tag{3.31}$$

Thus, $x_2 \in \widetilde{B}(x_0, c)$. Continuing this process, we get $x_n \in \widetilde{B}(x_0, c)$ and $u_n \in D(x_n, Tx_n)$ such that $x_{n+1} \in Tx_n$ and $u_{n+1} \in D(x_{n+1}, Tx_{n+1})$ satisfying

$$bd(x_n, x_{n+1}) \preceq u_n \preceq \left(\frac{a}{b} + s\right)^n u_0. \tag{3.32}$$

Thus, we get

$$d(x_n, x_{n+1}) \preceq \frac{1}{b} u_n \preceq \left(\frac{a}{b} + s\right)^n u_0. \tag{3.33}$$

Now, for $m > n$, we have

$$\begin{aligned} d(x_n, x_m) &\preceq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &\preceq \frac{1}{b}\left(t^n + t^{n+1} + \cdots + t^{m-1}\right) u_0 \\ &\preceq \frac{1}{b} \frac{t^n}{1-t} u_0, \end{aligned} \tag{3.34}$$

where $t = a/b + s < 1$. Since P is normal, we have

$$\|d(x_n, x_m)\| \leq K\left(\frac{t^m}{b(1-t)}\right)\|u_0\|, \tag{3.35}$$

and hence $\{x_n\}$ is a Cauchy sequence. Due to the completeness of $\widetilde{B}(x_0, c)$, there exists $x^* \in \widetilde{B}(x_0, c)$, such that $\lim_{n\to\infty} x_n = x^*$. Also, note that

$$\|u_{n+1}\| \leq K\left(\frac{a}{b} + s\right)^{n+1} \|u_0\|, \tag{3.36}$$

and thus,

$$\lim_{n\to\infty} \|u_{n+1}\| = 0. \tag{3.37}$$

□

The rest of the proof runs as the proof of Theorem 3.2, and hence we get $\mathrm{Fix}(T) \cap \widetilde{B}(x_0, c) \neq \emptyset$.

Remark 3.5. Theorem 3.4 extends the fixed point result of Chifu and Petrusel [18, Theorem 2.5] to cone metric spaces.

Most recently, Asadi et al. [13, Lemma 2.1] proved the closedness of the set $\mathrm{Fix}(T)$ in complete cone metric spaces without the normality assumption. In the following remark, we obtain the same conclusion without normality and completeness assumptions.

Remark 3.6. Let (X, d) be a cone metric space, and let $T : X \to \mathrm{Cl}(X)$ be any multivalued map. If the function $f : X \to \mathbb{R}$ defined by $f(x) = \inf_{y\in Tx}\|d(x, y)\|$ is lower semicontinuous, then the set $\mathrm{Fix}(T)$ is closed.

Indeed, let $z_n \in \mathrm{Fix}(T)$ be such that $z_n \to z^*$ as $n \to \infty$. Clearly, $f(z_n) = \inf_{y\in Tz_n}\|d(z_n,y)\| = 0$ because $z_n \in Tz_n$. Using the lower semicontinuity of the function f, we get

$$\inf_{y\in Tz^*}\|d(z^*,y)\| \leq \liminf_{n\to\infty}\inf_{y\in Tz_n}\|d(z_n,y)\| = 0. \tag{3.38}$$

Thus

$$\inf_{y\in Tz^*}\|d(z^*,y)\| = 0. \tag{3.39}$$

So, there exists a sequence $\{y_n\} \subset Tz^*$ such that $\lim_{n\to\infty}\|d(z^*,y_n)\| = 0$. Hence $y_n \to z^* \in T(z^*)$.

Example 3.7. Let $X = [0,1]$, $E = \mathbb{R}^2$ a Banach space with the maximum norm, and $P = \{(x,y) \in E : x,y \geq 0\}$ a normal cone. Define $d : X\times X \to E$ by

$$d(x,y) = \left(|x-y|, \beta|x-y|\right), \quad \beta \in (0,1). \tag{3.40}$$

Then the pair (X,d) is a complete cone metric space. Now, define the map $T : X \to \mathrm{Cl}(X)$ by

$$T(x) = \begin{cases} \{0,1\}, & x = 0, \\ \left\{\dfrac{1}{2}x\right\}, & x \in (0,1), \\ \left\{\dfrac{1}{2},1\right\}, & x = 1. \end{cases} \tag{3.41}$$

Note that the map

$$f(x) = \inf_{y\in Tx}\|d(x,y)\| = \begin{cases} \dfrac{1}{2}x, & x \in (0,1), \\ 0, & x \in \{0,1\} \end{cases} \tag{3.42}$$

is lower semicontinuous. Now, if we take $c = (1/2,1/2) \in E$, $x_0 = 0 \in X$, we get

$$\widetilde{B}(x_0,c) = \{x \in X, d(x_0,x) \leq c\} = \left[0,\frac{1}{2}\right]. \tag{3.43}$$

Now, for the case $x \in (0,1/2]$ and $y = (1/2)x \in T(x)$, we obtain

$$\begin{aligned} D(x,Tx) &= \left\{\left(\frac{1}{2}x, \beta\frac{1}{2}x\right)\right\}, \\ D(y,Ty) &= \left\{\left(\frac{1}{4}x, \beta\frac{1}{4}x\right)\right\}. \end{aligned} \tag{3.44}$$

Now, taking $a = 1/2$ and $b = 1$, we get

$$bd(x,y) \leq u, \quad \text{for each } u \in D(x,Tx),$$
$$v \leq ad(x,y), \quad \text{for } v = \left(\frac{1}{4}x, \beta\frac{1}{4}x\right) \in D(y,Ty). \tag{3.45}$$

Now, for the case $x = 0, y = 0 \in T(x)$, we have

$$D(x,Tx) = \{(0,0),(1,\beta)\},$$
$$D(y,Ty) = \{(0,0),(1,\beta)\}. \tag{3.46}$$

And also, for this case we get

$$bd(x,y) \leq u, \quad \text{for each } u \in D(x,Tx),$$
$$v \leq ad(x,y), \quad \text{for } v = 0 \in D(y,Ty). \tag{3.47}$$

Further, for $u_0 = (0,0) \in D(x_0, Tx_0)$, we have

$$u_0 \leq \left(1 - \frac{a}{b}\right)bc. \tag{3.48}$$

Therefore, all the assumptions of Theorem 3.2 are satisfied, and note that $\text{Fix}(T) \cap \widetilde{B}(x_0, c) = \{0\}$.

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