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Research Article

Relaxed Composite Implicit Iteration Process for Common Fixed Points of a Finite Family of Strictly Pseudocontractive Mappings

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We propose a relaxed composite implicit iteration process for finding approximate common fixed points of a finite family of strictly pseudocontractive mappings in Banach spaces. Several convergence results for this process are established.

1. Introduction and Preliminaries

Let E be a real Banach space, and let E^* be its dual space. Denote by J the normalized duality mapping from E into 2^{E^*} defined by

$$J(x) = \left\{\varphi \in E^* : \langle x, \varphi\rangle = \|x\|^2 = \|\varphi\|^2\right\}, \quad \forall x \in E, \tag{1.1}$$

where $\langle \cdot, \cdot\rangle$ is the generalized duality pairing between E and E^*. If E is smooth, then J is single valued and continuous from the norm topology of E to the weak* topology of E^*.

A mapping T with domain $D(T)$ and range $R(T)$ in E is called λ-strictly pseudocontractive in the terminology of Browder and Petryshyn [1], if there exists a constant $\lambda > 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \lambda\|x - y - (Tx - Ty)\|^2 \tag{1.2}$$

for all $x, y \in D(T)$ and all $j(x-y) \in J(x-y)$. Without loss of generality, we may assume $\lambda \in (0,1)$. If I denotes the identity operator, then (1.2) can be written in the form

$$\langle (I-T)x-(I-T)y, j(x-y)\rangle \geq \lambda \|(I-T)x-(I-T)y\|^2 \tag{1.3}$$

for all $x, y \in D(T)$ and all $j(x-y) \in J(x-y)$. In (1.2) and (1.3), the positive number $\lambda > 0$ is said to be a strictly pseudocontractive constant.

The class of strictly pseudocontractive mappings has been studied by several authors (see, e.g., [1–10]). It is shown in [4] that a strictly pseudocontractive map is L-Lipschitzian (i.e., $\|Tx-Ty\| \leq L\|x-y\|$, $\forall x, y \in D(T)$ for some $L > 0$). Indeed, it follows immediately from (1.3) that

$$\|x-y\| \geq \lambda \|(I-T)x-(I-T)y\| \geq \lambda \big(\|Tx-Ty\|-\|x-y\|\big), \tag{1.4}$$

and hence $\|Tx-Ty\| \leq L\|x-y\|$, $\forall x, y \in D(T)$ where $L = 1+1/\lambda$. It is clear that in Hilbert spaces the important class of nonexpansive mappings (mappings T for which $\|Tx-Ty\| \leq \|x-y\|$, $\forall x, y \in D(T)$) is a subclass of the class of strictly pseudocontractive maps.

Let K be a nonempty convex subset of E, and let $\{T_i\}_{i=1}^N$ be a finite family of nonexpansive self-maps of K. In [11], Xu and Ori introduced the following implicit iteration process; for any initial $x_0 \in K$ and $\{\alpha_n\}_{n=1}^\infty \subset (0,1)$, the sequence $\{x_n\}_{n=1}^\infty$ is generated as follows:

$$\begin{aligned}
x_1 &= \alpha_1 x_0 + (1-\alpha_1)T_1 x_1,\\
x_2 &= \alpha_2 x_1 + (1-\alpha_2)T_2 x_2,\\
&\vdots\\
x_N &= \alpha_N x_{N-1} + (1-\alpha_N)T_N x_N,\\
x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1})T_1 x_{N+1},\\
&\vdots
\end{aligned} \tag{1.5}$$

The scheme is expressed in a compact form as

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n x_n, \quad n \geq 1, \tag{1.6}$$

where $T_n = T_{n \bmod N}$. Moreover, they proved the following convergence theorem in a Hilbert space.

Theorem 1.1 (see [11]). *Let H be a Hilbert space, and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \varnothing$ where $F(T_i) = \{x \in K : T_i x = x\}$. Let $x_0 \in K$, and let $\{\alpha_n\}_{n=1}^\infty$ be a sequence in $(0,1)$, such that $\lim_{n\to\infty}\alpha_n = 0$. Then the sequence $\{x_n\}$ defined implicitly by (1.6) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

Subsequently, Osilike [12] extended their results from nonexpansive mappings to strictly pseudocontractive mappings and derived the following convergence theorems in Hilbert and Banach spaces.

Theorem 1.2 (see [12]). *Let H be a real Hilbert space, and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$. Let $x_0 \in K$, and let $\{\alpha_n\}_{n=1}^\infty$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty}\alpha_n = 0$. Then the sequence $\{x_n\}_{n=1}^\infty$ defined by* (1.6) *converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

Theorem 1.3 (see [12]). *Let E be a real Banach space, and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^\infty$ be a real sequence satisfying the conditions:*

(i) $0 < \alpha_n < 1$;

(ii) $\sum_{n=1}^\infty (1-\alpha_n) = +\infty$;

(iii) $\sum_{n=1}^\infty (1-\alpha_n)^2 < +\infty$.

Let $x_0 \in K$, and let $\{x_n\}_{n=1}^\infty$ be defined by (1.6)*. Then*

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all* $p \in F$;

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Let K be a nonempty closed convex subset of a real Banach space E. Very recently, Su and Li [13] introduced a new implicit iteration process for N strictly pseudocontractive self-maps $\{T_i\}_{i=1}^N$ of K:

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n) T_n y_n, \\ y_n &= \beta_n x_{n-1} + (1-\beta_n) T_n x_n, \quad n = 1, 2, \ldots, \end{aligned} \tag{1.7}$$

that is,

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n \left(\beta_n x_{n-1} + (1-\beta_n) T_n x_n\right), \quad n \geq 1, \tag{1.8}$$

where $T_n = T_{n \bmod N}$ and $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$. First, they established the following convergence theorem.

Theorem 1.4 ([13, Theorem 2.1]). *Let E be a real Banach space, and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^\infty, \{\beta_n\}_{n=1}^\infty \subset [0,1]$ be two real sequences satisfying the conditions:*

(i) $\sum_{n=1}^\infty (1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^\infty (1-\alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^\infty (1-\beta_n) < +\infty$;

(iv) $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, *where $L \geq 1$ is common Lipschitz constant of $\{T_i\}_{i=1}^N$.*

For $x_0 \in K$*, let* $\{x_n\}_{n=1}^{\infty}$ *be defined by* (1.8)*. Then*

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all* $p \in C$*;*

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Second, they derived the following result by using Theorem 1.4.

Theorem 1.5 ([13, Theorem 2.2]). *Let* K *be a nonempty closed convex subset of a real Banach space* E*, let* T *be a semicompact strictly pseudocontractive self-map of* K *such that* $F(T) \neq \emptyset$*, where* $F(T) = \{x \in K : Tx = x\}$*, and let* $\{\alpha_n\} \subset [0,1]$ *be a real sequence satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1-\alpha_n)^2 < +\infty$.

Then for $x_0 \in K$*, the sequence* $\{x_n\}$ *defined by Mann iterative process,*

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)Tx_{n-1}, \quad n \geq 1, \tag{1.9}$$

converges strongly to a fixed point of T*.*

On the other hand, Zeng and Yao [14] introduced a new implicit iteration scheme with perturbed mapping for approximation of common fixed points of a finite family of nonexpansive self-maps of a real Hilbert space H and established some convergence theorems for this implicit iteration scheme. To be more specific, let $\{T_i\}_{i=1}^{N}$ be a finite family of nonexpansive self-maps of H, and let $F : H \to H$ be a mapping such that for some constants $\kappa, \eta > 0$; F is a κ-Lipschitz and η-strongly monotone mapping. Let $\{\alpha_n\}_{n=1}^{\infty} \subset (0,1)$ and $\{\lambda_n\}_{n=1}^{\infty} \subset [0,1)$ and take a fixed number $\mu \in (0, 2\eta/\kappa^2)$. The authors proposed the following implicit iteration process with perturbed mapping F.

For an arbitrary initial point $x_0 \in H$, the sequence $\{x_n\}_{n=1}^{\infty}$ is generated as follows:

$$\begin{aligned}
x_1 &= \alpha_1 x_0 + (1-\alpha_1)\left[T_1 x_1 - \lambda_1 \mu F(T_1 x_1)\right],\\
x_2 &= \alpha_2 x_1 + (1-\alpha_2)\left[T_2 x_2 - \lambda_2 \mu F(T_2 x_2)\right],\\
&\ \vdots\\
x_N &= \alpha_N x_{N-1} + (1-\alpha_N)\left[T_N x_N - \lambda_N \mu F(T_N x_N)\right],\\
x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1})\left[T_1 x_{N+1} - \lambda_{N+1} \mu F(T_1 x_{N+1})\right],\\
&\ \vdots
\end{aligned} \tag{1.10}$$

The scheme is expressed in a compact form as

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n x_n - \lambda_n \mu F(T_n x_n)\right], \quad n \geq 1. \tag{1.11}$$

It is clear that if $\lambda_n \equiv 0$, then the implicit iteration scheme (1.11) with perturbed mapping reduces to the implicit iteration process (1.6).

Theorem 1.6 ([14, Theorem 2.1]). *Let H be a real Hilbert space, and let $F : H \to H$ be a mapping such that for some constants $\kappa, \eta > 0$; F is κ-Lipschitz and η-strongly monotone. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of H such that $C = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $\mu \in (0, 2\eta/\kappa^2)$, let $x_0 \in H$, $\{\lambda_n\}_{n=1}^\infty \subset [0,1)$, and let $\{\alpha_n\}_{n=1}^\infty \subset (0,1)$ satisfying the conditions: $\sum_{n=1}^\infty \lambda_n < \infty$ and $\alpha \leq \alpha_n \leq \beta$, $n \geq 1$, for some $\alpha, \beta \in (0,1)$. Then the sequence $\{x_n\}_{n=1}^\infty$ defined by (1.11) converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

The above Theorem 1.6 extends Theorem 1.1 from the implicit iteration process (1.6) to the implicit iteration scheme (1.11) with perturbed mapping.

Let E be a real Banach space, and let K be a nonempty convex subset of E. Recall that a mapping $F : K \to K$ is said to be δ-strongly accretive if there exists a constant $\delta \in (0,1)$ such that

$$\langle Fx - Fy, j(x-y) \rangle \geq \delta \|x-y\|^2 \tag{1.12}$$

for all $x, y \in K$ and all $j(x-y) \in J(x-y)$.

Proposition 1.7. *Let X be a real Banach space, and let $F : K \to K$ be a mapping:*

(i) *if F is λ-strictly pseudocontractive, then F is a Lipschitz mapping with constant $L = 1 + 1/\lambda$.*

(ii) *if F is both λ-strictly pseudocontractive and δ-strongly accretive with $\lambda + \delta \geq 1$, then $I - F$ is nonexpansive.*

Proof. It is easy to see that statement (i) immediately follows from the definition of strict pseudocontraction. Now utilizing the definitions of strict pseudocontraction and strong accretivity, we obtain

$$\lambda \|(I-F)x - (I-F)y\|^2 \leq \|x-y\|^2 - \langle Fx - Fy, j(x-y) \rangle \leq (1-\delta)\|x-y\|^2. \tag{1.13}$$

Since $\lambda + \delta \geq 1$,

$$\|(I-F)x - (I-F)y\| \leq \sqrt{\frac{1-\delta}{\lambda}} \|x-y\| \leq \|x-y\|, \tag{1.14}$$

and hence $I - F$ is nonexpansive. □

Let E be a real Banach space, and let K be a nonempty convex subset of E such that $K - K \subset K$. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K, and let $F : K \to K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. In this paper we introduce a general implicit iteration process as follows:

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right], \\ y_n &= \beta_n x_{n-1} + (1-\beta_n) T_n x_n, \quad n = 1, 2, \ldots, \end{aligned} \tag{1.15}$$

where $T_n = T_{n \bmod N}$, and $\{\alpha_n\}, \{\beta_n\}, \{\lambda_n\} \subset [0,1]$. In particular, whenever $\lambda_n \equiv 0$, it is easy to see that (1.15) reduces to (1.8).

Let $L \geq 1$ denote common Lipschitz constant of N strictly pseudocontractive self-maps $\{T_i\}_{i=1}^{N}$ of K. Since K is a nonempty convex subset of E such that $K - K \subset K$, for each $n \geq 1$, the operator

$$\begin{aligned}
S_n x &= \alpha_n x_{n-1} + (1-\alpha_n)\{T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - \lambda_n F T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x]\} \\
&= \alpha_n x_{n-1} + (1-\alpha_n)(I - \lambda_n F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\
&= \alpha_n x_{n-1} + (1-\alpha_n)[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x]
\end{aligned} \tag{1.16}$$

maps K into itself.

Utilizing Proposition 1.7, we have

$$\begin{aligned}
&\langle S_n x - S_n y, j(x-y)\rangle \\
&\quad= (1-\alpha_n)\langle[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\
&\qquad\qquad -[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y], j(x-y)\rangle \\
&\quad\leq (1-\alpha_n)\|[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] \\
&\qquad\qquad - [(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\
&\quad\leq (1-\alpha_n)\{(1-\lambda_n)\|T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\| \\
&\qquad\qquad +\lambda_n\|(I-F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - (I-F)T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\} \\
&\qquad \times \|x-y\| \\
&\quad\leq (1-\alpha_n)\|T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x] - T_n[\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\
&\quad\leq (1-\alpha_n)L\|\beta_n x_{n-1} + (1-\beta_n)T_n x - [\beta_n x_{n-1} + (1-\beta_n)T_n y]\|\|x-y\| \\
&\quad= (1-\alpha_n)(1-\beta_n)L\|T_n x - T_n y\|\|x-y\| \\
&\quad\leq (1-\alpha_n)(1-\beta_n)L^2\|x-y\|^2
\end{aligned} \tag{1.17}$$

for all $x, y \in K$. Thus, S_n is strongly pseudocontractive, if $(1-\alpha_n)(1-\beta_n)L^2 < 1$ for each $n \geq 1$. Since S_n is also Lipschitz mapping, it follows from [12, 15, 16] that S_n has a unique fixed point $x_n \in K$, that is, for each $n \geq 1$

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)[(1-\lambda_n)I + \lambda_n(I-F)]T_n[\beta_n x_{n-1} + (1-\beta_n)T_n x_n]. \tag{1.18}$$

Therefore, if $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, then the composite implicit iteration process (1.15) with perturbed mapping can be employed for the approximation of common fixed points of N strictly pseudocontractive self-maps of K.

The purpose of this paper is to investigate the problem of approximating common fixed points of strictly pseudocontractive mappings of Browder-Petryshyn in an arbitrary real Banach space by this general implicit iteration process (1.15). To this end, we need the following lemma and definition.

Lemma 1.8 (see [8]). *Let $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$, and $\{\epsilon_n\}_{n=1}^{\infty}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \leq (1+\epsilon_n)a_n + b_n, \quad n \geq 1. \tag{1.19}$$

If

$$\sum_{n=1}^{\infty} \epsilon_n < +\infty, \qquad \sum_{n=1}^{\infty} b_n < +\infty, \tag{1.20}$$

then $\lim_{n\to\infty} a_n$ *exists.*

The following definition can be found, for example, in [13].

Definition 1.9. Let D be a closed subset of a real Banach space E, and let $T : D \to D$ be a mapping. T is said to be semicompact if, for any bounded sequence $\{x_n\}$ in D such that $\|x_n - Tx_n\| \to 0\,(n \to \infty)$, there must exist a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $x_{n_i} \to x^* \in D$.

2. Main Results

We are now in a position to prove our main results in this paper.

Theorem 2.1. *Let E be a real Banach space, and let K be a nonempty closed convex subset of E such that $K - K \subset K$. Let $F : K \to K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. Let $\{T_i\}_{i=1}^{N}$ be N strictly pseudocontractive self-maps of K such that $C = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$, where $F(T_i) = \{x \in K : T_i x = x\}$, and let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$, and $\{\lambda_n\}_{n=1}^{\infty}$ be three real sequences in $[0,1]$ satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1-\alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^{\infty}(1-\beta_n) < +\infty$;

(iv) $\sum_{n=1}^{\infty}\lambda_n(1-\alpha_n) < +\infty$;

(v) $(1-\alpha_n)(1-\beta_n)L^2 < 1$, $\forall n \geq 1$, *where $L \geq 1$ is the common Lipschitz constant of $\{T_i\}_{i=1}^{N}$.*

For $x_0 \in K$, let $\{x_n\}_{n=1}^{\infty}$ be defined by

$$\begin{aligned} x_n &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right], \\ y_n &= \beta_n x_{n-1} + (1-\beta_n)T_n x_n, \quad n = 1,2,\ldots, \end{aligned} \tag{2.1}$$

where $T_n = T_{n \bmod N}$, then

(i) $\lim_{n\to\infty}\|x_n - p\|$ *exists for all $p \in C$;*

(ii) $\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0$.

Proof. First, since each strictly pseudocontractive mapping is a Lipschitz mapping, there exists a constant $L \geq 1$ such that

$$\|T_i x - T_i y\| \leq L\|x - y\|, \quad \forall x, y \in K,\ \forall i = 1,2,\ldots,N. \tag{2.2}$$

It is now well known (see, e.g., [15]) that

$$\|x+y\|^2 \le \|x\|^2 + 2\langle y, j(x+y)\rangle \tag{2.3}$$

for all $x, y \in E$ and all $j(x+y) \in J(x+y)$. Take $p \in C$ arbitrarily. Then it follows from (2.1) that

$$\begin{aligned}
x_n - p &= \alpha_n x_{n-1} + (1-\alpha_n)\left[T_n y_n - \lambda_n F(T_n y_n)\right] - p \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{\left[T_n y_n - \lambda_n F(T_n y_n)\right] - p\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{[(1-\lambda_n)I + \lambda_n(I-F)]T_n y_n - p\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - p) + \lambda_n\left[(I-F)T_n y_n - p\right]\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - T_n p) + \lambda_n\left[(I-F)T_n y_n - (I-F)T_n p\right]\right. \\
&\qquad\qquad \left. + \lambda_n\left[(I-F)T_n p - p\right]\right\} \\
&= \alpha_n(x_{n-1}-p) + (1-\alpha_n)\left\{(1-\lambda_n)(T_n y_n - T_n p) + \lambda_n\left[(I-F)T_n y_n - (I-F)T_n p\right]\right\} \\
&\quad - (1-\alpha_n)\lambda_n F(p).
\end{aligned} \tag{2.4}$$

Utilizing (2.3), we obtain

$$\begin{aligned}
\|x_n - p\|^2 &\le \alpha_n^2\|x_{n-1}-p\|^2 + 2(1-\alpha_n)(1-\lambda_n)\langle T_n y_n - T_n p, j(x_n - p)\rangle \\
&\quad + 2(1-\alpha_n)\lambda_n\langle (I-F)T_n y_n - (I-F)T_n p, j(x_n-p)\rangle \\
&\quad - 2(1-\alpha_n)\lambda_n\langle F(p), j(x_n-p)\rangle \\
&\le \alpha_n^2\|x_{n-1}-p\|^2 + 2(1-\alpha_n)(1-\lambda_n) \\
&\quad \times \left[\langle T_n y_n - T_n x_n, j(x_n-p)\rangle + \langle T_n x_n - T_n p, j(x_n-p)\rangle\right] \\
&\quad + 2(1-\alpha_n)\lambda_n\left[\langle (I-F)T_n y_n - (I-F)T_n x_n, j(x_n-p)\rangle\right. \\
&\qquad\qquad \left. + \langle (I-F)T_n x_n - (I-F)T_n p, j(x_n-p)\rangle\right] \\
&\quad - 2(1-\alpha_n)\lambda_n\langle F(p), j(x_n-p)\rangle.
\end{aligned} \tag{2.5}$$

Since each T_i, $i = 1, 2, \ldots, N$, is strictly pseudocontractive, there exists $\lambda \in (0,1)$ such that

$$\langle T_i x - T_i y, j(x-y)\rangle \le \|x-y\|^2 - \lambda\|x - T_i x - (y - T_i y)\|^2, \quad \forall x, y \in K. \tag{2.6}$$

Thus, utilizing Proposition 1.7(ii) we know from (2.5) that

$$\begin{aligned}
\|x_n - p\|^2 &\le \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n)(1-\lambda_n) \\
&\quad \times \left[L\|y_n - x_n\| \|x_n - p\| + \|x_n - p\|^2 - \lambda \|x_n - T_n x_n\|^2 \right] \\
&\quad + 2(1-\alpha_n)\lambda_n \left[L\|y_n - x_n\| \|x_n - p\| + L\|x_n - p\|^2 \right] \\
&\quad - 2(1-\alpha_n)\lambda_n \langle F(p), j(x_n - p) \rangle \\
&\le \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
&\quad \times \left[L\|y_n - x_n\| \|x_n - p\| + \|x_n - p\|^2 - \lambda \|x_n - T_n x_n\|^2 \right] \\
&\quad + 2(1-\alpha_n)\lambda_n L\|x_n - p\|^2 + 2(1-\alpha_n)\lambda_n \|F(p)\| \|x_n - p\| \\
&\le \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
&\quad \times \left[L\|y_n - x_n\| \|x_n - p\| + \|x_n - p\|^2 - \lambda \|x_n - T_n x_n\|^2 \right] \\
&\quad + 2(1-\alpha_n)\lambda_n L\|x_n - p\|^2 + (1-\alpha_n)\lambda_n \left(\|F(p)\|^2 + \|x_n - p\|^2 \right) \\
&\le \alpha_n^2 \|x_{n-1} - p\|^2 + 2(1-\alpha_n) \\
&\quad \times \left[L\|y_n - x_n\| \|x_n - p\| + \|x_n - p\|^2 - \lambda \|x_n - T_n x_n\|^2 \right] \\
&\quad + 3L(1-\alpha_n)\lambda_n \|x_n - p\|^2 + (1-\alpha_n)\lambda_n \|F(p)\|^2.
\end{aligned} \tag{2.7}$$

From (2.1), we also have that

$$\begin{aligned}
&\|y_n - x_n\| \\
&\quad \le \beta_n \|x_n - x_{n-1}\| + (1-\beta_n)\|x_n - T_n x_n\| \\
&\quad \le \beta_n (1-\alpha_n) \|T_n y_n - \lambda_n F(T_n y_n) - x_{n-1}\| + (1-\beta_n)\|x_n - T_n x_n\| \\
&\|T_n y_n - \lambda_n F(T_n y_n) - x_{n-1}\| \\
&\quad \le \|T_n y_n - \lambda_n F(T_n y_n) - p\| + \|x_{n-1} - p\| \\
&\quad = \|(1-\lambda_n)(T_n y_n - p) + \lambda_n((I-F)T_n y_n - p)\| + \|x_{n-1} - p\| \\
&\quad = \|(1-\lambda_n)(T_n y_n - p) + \lambda_n((I-F)T_n y_n - (I-F)p) - \lambda_n F(p)\| + \|x_{n-1} - p\| \\
&\quad \le (1-\lambda_n)\|T_n y_n - p\| + \lambda_n \|T_n y_n - p\| + \lambda_n \|F(p)\| + \|x_{n-1} - p\| \\
&\quad = \|T_n y_n - p\| + \lambda_n \|F(p)\| + \|x_{n-1} - p\| \\
&\quad \le L\|y_n - p\| + \lambda_n \|F(p)\| + \|x_{n-1} - p\| \\
&\quad \le (L\beta_n + 1)\|x_{n-1} - p\| + L^2(1-\beta_n)\|x_n - p\| + \lambda_n \|F(p)\|.
\end{aligned} \tag{2.8}$$

Since T_i is a Lipschitz mapping with constant L, we have

$$\|x_n - T_n x_n\| \le \|x_n - p\| + \|T_n x_n - p\| \le (L+1)\|x_n - p\|. \tag{2.9}$$

Substituting (2.8) and (2.9) into (2.7), we deduce that

$$\begin{aligned}
\|x_n - p\|^2 \le{}& \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&+ 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)(1-\beta_n)L(L+1)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)\|x_n - p\|^2 - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 \\
&+ 2L(1-\alpha_n)^2\lambda_n\beta_n\|F(p)\|\|x_n - p\| \\
&+ 3L(1-\alpha_n)\lambda_n\|x_n - p\|^2 + (1-\alpha_n)\lambda_n\|F(p)\|^2 \\
\le{}& \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&+ 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)(1-\beta_n)L(L+1)\|x_n - p\|^2 \\
&+ 2(1-\alpha_n)\|x_n - p\|^2 - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 \\
&+ 4L(1-\alpha_n)\lambda_n\|x_n - p\|^2 + 2L(1-\alpha_n)\lambda_n\|F(p)\|^2,
\end{aligned} \tag{2.10}$$

and hence

$$\begin{aligned}
&\left[1 - 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n) - 2(1-\alpha_n)(1-\beta_n)L(L+1) - 4L(1-\alpha_n)\lambda_n - 2(1-\alpha_n)\right]\|x_n - p\|^2 \\
&\quad\le \alpha_n^2\|x_{n-1} - p\|^2 + 2(1-\alpha_n)^2 L\beta_n(L\beta_n + 1)\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 + 2L(1-\alpha_n)\lambda_n\|F(p)\|^2.
\end{aligned} \tag{2.11}$$

Setting

$$b_n = 2(1-\alpha_n)^2 L^3\beta_n(1-\beta_n) + 2(1-\alpha_n)(1-\beta_n)L(L+1) + 4L(1-\alpha_n)\lambda_n, \tag{2.12}$$

we conclude from (2.11) that

$$\|x_n-p\|^2 \le \frac{\alpha_n^2}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|^2+\frac{2(1-\alpha_n)^2L\beta_n(L\beta_n+1)}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|\|x_n-p\| - \frac{2(1-\alpha_n)\lambda}{1-2(1-\alpha_n)-b_n}\|x_n-T_nx_n\|^2+\frac{(1-\alpha_n)\lambda_n}{1-2(1-\alpha_n)-b_n}2L\|F(p)\|^2. \tag{2.13}$$

Thus

$$\|x_n-p\|^2 \le \left[1+\frac{(1-\alpha_n)^2+b_n}{1-2(1-\alpha_n)-b_n}\right]\|x_{n-1}-p\|^2+\frac{2(1-\alpha_n)^2L\beta_n(L\beta_n+1)}{1-2(1-\alpha_n)-b_n}\|x_{n-1}-p\|\|x_n-p\| - 2(1-\alpha_n)\lambda\|x_n-T_nx_n\|^2+\frac{(1-\alpha_n)\lambda_n}{1-2(1-\alpha_n)-b_n}2L\|F(p)\|^2. \tag{2.14}$$

Since

$$1-2(1-\alpha_n)-b_n = 1-(1-\alpha_n)\left[2+2(1-\alpha_n)L^3\beta_n(1-\beta_n)+2(1-\beta_n)L(L+1)+4L\lambda_n\right] \tag{2.15}$$

and $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}, \{\lambda_n\}_{n=1}^{\infty} \subset [0,1]$, we get

$$\left[2+2(1-\alpha_n)L^3\beta_n(1-\beta_n)+2(1-\beta_n)L(L+1)+4L\lambda_n\right] \le 6L+2L^3+2L(L+1). \tag{2.16}$$

Setting $M_1 = 6L+2L^3+2L(L+1)$, it follows from condition (ii) that $\lim_{n\to\infty}(1-\alpha_n)=0$ and so there must exist a natural number N_1 such that for all $n \ge N_1$,

$$\frac{1}{1-2(1-\alpha_n)-b_n} < 2. \tag{2.17}$$

Therefore, it follows from (2.14) that

$$\|x_n-p\|^2 \le \left[1+2\left((1-\alpha_n)^2+b_n\right)\right]\|x_{n-1}-p\|^2 + 2\left[2(1-\alpha_n)^2L\beta_n(L\beta_n+1)\right]\|x_{n-1}-p\|\|x_n-p\| - 2(1-\alpha_n)\lambda\|x_n-T_nx_n\|^2+4L\|F(p)\|^2(1-\alpha_n)\lambda_n. \tag{2.18}$$

In order to consider the second term on the right-hand side of (2.18), we will prove that $\{x_n\}$ is bounded. Indeed, utilizing (2.8) and (2.9) and simplifying these inequalities, we have

$$
\begin{aligned}
&\|x_n - p\|^2 \\
&\quad = \langle x_n - p, j(x_n - p)\rangle \\
&\quad = \alpha_n\langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)\langle T_n y_n - \lambda_n F(T_n y_n) - p, j(x_n - p)\rangle \\
&\quad = \alpha_n\langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)(1-\lambda_n) \\
&\qquad \times \left[\langle T_n y_n - T_n x_n, j(x_n - p)\rangle + \langle T_n x_n - T_n p, j(x_n - p)\rangle\right] \\
&\qquad + (1-\alpha_n)\lambda_n\langle (I-F)T_n y_n - (I-F)T_n p, j(x_n - p)\rangle - (1-\alpha_n)\lambda_n\langle F(p), j(x_n - p)\rangle \\
&\quad \le \alpha_n\|x_{n-1} - p\|\|x_n - p\| + (1-\alpha_n)(1-\lambda_n)\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] \\
&\qquad + (1-\alpha_n)\lambda_n\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] + (1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\| \\
&\quad \le \alpha_n\|x_{n-1} - p\|\|x_n - p\| + (1-\alpha_n)\left[L\|y_n - x_n\|\|x_n - p\| + L\|x_n - p\|^2\right] \\
&\qquad + (1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\| \\
&\quad \le \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + \left[(1-\alpha_n)L + L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\qquad + L\beta_n(1-\alpha_n)^2\lambda_n\|F(p)\|\|x_n - p\| + (1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\| \\
&\quad \le \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + \left[(1-\alpha_n)L + L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\qquad + (L+1)(1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\|,
\end{aligned}
\tag{2.19}
$$

and hence

$$
\begin{aligned}
&\left[1 - (1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)\right]\|x_n - p\|^2 \\
&\quad \le \left[\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n + 1)\right]\|x_{n-1} - p\|\|x_n - p\| \\
&\qquad + (L+1)(1-\alpha_n)\lambda_n\|F(p)\|\|x_n - p\|.
\end{aligned}
\tag{2.20}
$$

This implies that

$$
\begin{aligned}
&\|x_n - p\| \\
&\quad \le \frac{\alpha_n + L(1-\alpha_n)^2\beta_n(L\beta_n+1)}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}\|x_{n-1}-p\| \\
&\qquad + \frac{(1-\alpha_n)\lambda_n}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}(L+1)\|F(p)\| \\
&\quad \le \left[1+\frac{L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1) + L(1-\alpha_n)^2\beta_n(L\beta_n+1)}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}\right]\|x_{n-1}-p\| \\
&\qquad + \frac{(1-\alpha_n)\lambda_n}{1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1)}(L+1)\|F(p)\|.
\end{aligned}
\tag{2.21}
$$

Now, we consider the second term on the right-hand side of (2.21). Since $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$, we have

$$
(1-\alpha_n)\left[L + L^3(1-\alpha_n)\beta_n(1-\beta_n) + L(1-\beta_n)(L+1)\right] \le (1-\alpha_n)\left[L + L^3 + L(L+1)\right]. \tag{2.22}
$$

Since $\lim_{n\to\infty}(1-\alpha_n) = 0$, there exists a natural number $N_2(\ge N_1)$ such that for all $n \ge N_2$,

$$
1-(1-\alpha_n)L - L^3(1-\alpha_n)^2\beta_n(1-\beta_n) - L(1-\alpha_n)(1-\beta_n)(L+1) \ge \frac{1}{2}. \tag{2.23}
$$

Again, it follows from condition $\{\alpha_n\}, \{\beta_n\} \subset [0,1]$ that

$$
\begin{aligned}
&L^3(1-\alpha_n)^2\beta_n(1-\beta_n) + L(1-\alpha_n)(1-\beta_n)(L+1) + L(1-\alpha_n)^2\beta_n(L\beta_n+1) \\
&\quad \le L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1).
\end{aligned}
\tag{2.24}
$$

Therefore, it follows from (2.21) that

$$
\begin{aligned}
\|x_n - p\| \le{}& \left\{1 + 2\left[L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1)\right]\right\}\|x_{n-1}-p\| \\
&+ 2(1-\alpha_n)\lambda_n(L+1)\|F(p)\|.
\end{aligned}
\tag{2.25}
$$

According to conditions (ii)–(iv), we can readily see that

$$
\begin{aligned}
&\sum_{n=1}^{\infty}\left\{2\left[L^3(1-\alpha_n)^2 + L(1-\beta_n)(L+1) + L(1-\alpha_n)^2(L+1)\right]\right\} < +\infty, \\
&\sum_{n=1}^{\infty}\left\{2(1-\alpha_n)\lambda_n(L+1)\|F(p)\|\right\} < +\infty.
\end{aligned}
\tag{2.26}
$$

Thus, in terms of Lemma 1.8 we deduce that $\lim_{n\to\infty}\|x_n - p\|$ exists, and hence $\{x_n\}$ is bounded.

Now, we consider the second term on the right-hand side of (2.18). Since $\{x_n\}$ is bounded, and $\{\beta_n\}_{n=1}^{\infty} \subset [0,1]$, there exists a constant $M_2 > 0$ and a natural number $N_3(\geq N_2)$ such that for all $n \geq N_3$,

$$2\left[2(1-\alpha_n)^2 L\beta_n(L\beta_n+1)\right]\|x_{n-1}-p\|\|x_n-p\| \leq 2(1-\alpha_n)^2 M_2. \tag{2.27}$$

Thus, it follows from (2.18) that

$$\begin{aligned}\|x_n-p\|^2 \leq {}& \left[1+2\left((1-\alpha_n)^2+b_n\right)\right]\|x_{n-1}-p\|^2 + 2(1-\alpha_n)^2 M_2 \\ & - 2(1-\alpha_n)\lambda\|x_n - T_n x_n\|^2 + 4L\|F(p)\|^2(1-\alpha_n)\lambda_n.\end{aligned} \tag{2.28}$$

Since $\{x_n\}$ is bounded, there exists a constant $M_3 > 0$ such that $\|x_n - p\|^2 \leq M_3$. It follows from (2.28) that

$$\begin{aligned}2\lambda\sum_{j=N+1}^{n}(1-\alpha_j)\|x_j - T_j x_j\|^2 \leq {}& \|x_N - p\|^2 + M_3\sum_{j=N+1}^{n}2\left[(1-\alpha_j)^2+b_j\right] \\ & + 2M_2\sum_{j=N+1}^{n}(1-\alpha_j)^2 + 4L\|F(p)\|^2\sum_{j=N+1}^{n}(1-\alpha_j)\lambda_j,\end{aligned} \tag{2.29}$$

and hence

$$\begin{aligned}2\lambda\sum_{n=N+1}^{\infty}(1-\alpha_n)\|x_n - T_n x_n\|^2 \leq {}& \|x_N - p\|^2 + M_3\sum_{n=N+1}^{\infty}2\left[(1-\alpha_n)^2+b_n\right] \\ & + 2M_2\sum_{n=N+1}^{\infty}(1-\alpha_n)^2 + 4L\|F(p)\|^2\sum_{n=N+1}^{\infty}(1-\alpha_n)\lambda_n.\end{aligned} \tag{2.30}$$

Utilizing conditions (ii)–(iv), we know from (2.30) that

$$2\lambda\sum_{n=1}^{\infty}(1-\alpha_n)\|x_n - T_n x_n\|^2 < +\infty. \tag{2.31}$$

Since $\sum_{n=1}^{\infty}(1-\alpha_n) = +\infty$, we have

$$\liminf_{n\to\infty}\|x_n - T_n x_n\| = 0. \tag{2.32}$$

This completes the proof of Theorem 2.1. □

The iterative scheme (1.15) becomes the explicit version as follows, whenever $\beta_n \equiv 1$:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n)[T_n x_{n-1} - \lambda_n F(T_n x_{n-1})], \quad n \geq 1. \tag{2.33}$$

In the case when $N = 1$, (2.33) is the Mann iteration process as follows:

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n)[T x_{n-1} - \lambda_n F(T x_{n-1})], \quad n \geq 1. \tag{2.34}$$

The conclusion of Theorem 2.1 remains valid for the iteration processes (2.33) and (2.34). Furthermore, we have the following result.

Theorem 2.2. *Let E be a real Banach space, and let K be a nonempty closed convex subset of E such that $K - K \subset K$. Let $F : K \to K$ be a perturbed mapping which is both δ-strongly accretive and λ-strictly pseudocontractive with $\delta + \lambda \geq 1$. Let T be a semicompact strictly pseudocontractive self-map of K such that $F(T) \neq \emptyset$, where $F(T) = \{x \in K : Tx = x\}$, and let $\{\alpha_n\}_{n=1}^{\infty}$ and $\{\lambda_n\}_{n=1}^{\infty}$ be two real sequences in $[0, 1]$ satisfying the conditions:*

(i) $\sum_{n=1}^{\infty}(1 - \alpha_n) = +\infty$;

(ii) $\sum_{n=1}^{\infty}(1 - \alpha_n)^2 < +\infty$;

(iii) $\sum_{n=1}^{\infty}\lambda_n(1 - \alpha_n) < +\infty$.

Then Mann iteration process (2.34) *converges strongly to a fixed point of T.*

Proof. Since

$$\liminf_{n \to \infty} \|x_n - T x_n\| = 0, \tag{2.35}$$

there exists a subsequence $\{n_k\}$ of $\{n\}$ such that

$$\lim_{k \to \infty} \|x_{n_k} - T x_{n_k}\| = 0. \tag{2.36}$$

By the semicompactness of T, there must exist a subsequence $\{x_{n_{k_i}}\}$ of $\{x_{n_k}\}$ such that

$$\lim_{i \to \infty} x_{n_{k_i}} = p_0. \tag{2.37}$$

It follows from (2.36) that $p_0 = T p_0$, and hence $p_0 \in F(T)$. Since $\lim_{n \to \infty}\|x_n - p_0\|$ exists, we have

$$\lim_{n \to \infty} \|x_n - p_0\| = \lim_{i \to \infty} \|x_{n_{k_i}} - p_0\| = 0. \tag{2.38}$$

This completes the proof of Theorem 2.2. □

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References

[1] F. E. Browder and W. V. Petryshyn, "Construction of fixed points of nonlinear mappings in Hilbert space," *Journal of Mathematical Analysis and Applications*, vol. 20, pp. 197–228, 1967.
[2] T. L. Hicks and J. D. Kubicek, "On the Mann iteration process in a Hilbert space," *Journal of Mathematical Analysis and Applications*, vol. 59, no. 3, pp. 498–504, 1977.
[3] Ş. Măruşter, "The solution by iteration of nonlinear equations in Hilbert spaces," *Proceedings of the American Mathematical Society*, vol. 63, no. 1, pp. 69–73, 1977.
[4] M. O. Osilike and A. Udomene, "Demiclosedness principle and convergence theorems for strictly pseudocontractive mappings of Browder-Petryshyn type," *Journal of Mathematical Analysis and Applications*, vol. 256, no. 2, pp. 431–445, 2001.
[5] M. O. Osilike, "Strong and weak convergence of the Ishikawa iteration method for a class of nonlinear equations," *Bulletin of the Korean Mathematical Society*, vol. 37, no. 1, pp. 153–169, 2000.
[6] B. E. Rhoades, "Comments on two fixed point iteration methods," *Journal of Mathematical Analysis and Applications*, vol. 56, no. 3, pp. 741–750, 1976.
[7] B. E. Rhoades, "Fixed point iterations using infinite matrices," *Transactions of the American Mathematical Society*, vol. 196, pp. 161–176, 1974.
[8] M. O. Osilike, S. C. Aniagbosor, and B. G. Akuchu, "Fixed points of asymptotically demicontractive mappings in arbitrary Banach spaces," *PanAmerican Mathematical Journal*, vol. 12, no. 2, pp. 77–88, 2002.
[9] L.-C. Zeng, G. M. Lee, and N. C. Wong, "Ishikawa iteration with errors for approximating fixed points of strictly pseudocontractive mappings of Browder-Petryshyn type," *Taiwanese Journal of Mathematics*, vol. 10, no. 1, pp. 87–99, 2006.
[10] L.-C. Zeng, N.-C. Wong, and J.-C. Yao, "Strong convergence theorems for strictly pseudocontractive mappings of Browder-Petryshyn type," *Taiwanese Journal of Mathematics*, vol. 10, no. 4, pp. 837–849, 2006.
[11] H.-K. Xu and R. G. Ori, "An implicit iteration process for nonexpansive mappings," *Numerical Functional Analysis and Optimization*, vol. 22, no. 5-6, pp. 767–773, 2001.
[12] M. O. Osilike, "Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps," *Journal of Mathematical Analysis and Applications*, vol. 294, no. 1, pp. 73–81, 2004.
[13] Y. Su and S. Li, "Composite implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps," *Journal of Mathematical Analysis and Applications*, vol. 320, no. 2, pp. 882–891, 2006.
[14] L.-C. Zeng and J.-C. Yao, "Implicit iteration scheme with perturbed mapping for common fixed points of a finite family of nonexpansive mappings," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 64, no. 11, pp. 2507–2515, 2006.
[15] S.-S. Chang, "Some problems and results in the study of nonlinear analysis," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 30, no. 7, pp. 4197–4208, 1997.
[16] Z.-H. Sun, "Strong convergence of an implicit iteration process for a finite family of asymptotically quasi-nonexpansive mappings," *Journal of Mathematical Analysis and Applications*, vol. 286, no. 1, pp. 351–358, 2003.