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Research Article

Value Distribution and Uniqueness Results of Zero-Order Meromorphic Functions to Their q-Shift

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We investigate value distribution and uniqueness problems of meromorphic functions with their q-shift. We obtain that if f is a transcendental meromorphic (or entire) function of zero order, and $Q(z)$ is a polynomial, then $af^n(qz) + f(z) - Q(z)$ has infinitely many zeros, where $q \in \mathbb{C} \setminus \{0\}$, a is nonzero constant, and $n \geq 5$ (or $n \geq 3$). We also obtain that zero-order meromorphic function share is three distinct values IM with its q-difference polynomial $P(f)$, and if $\limsup_{r\to\infty}(N(r,f)/T(r,f)) < 1$, then $f \equiv P(f)$.

1. Introduction and Main Results

A function $f(z)$ is called meromorphic function if it is analytic in the complex plane except at isolated poles. It is assumed that the reader is familiar with the standard symbols and fundamental results of Nevanlinna theory such as the characteristic function $T(r,f)$, proximity function $m(r,f)$, and the counting function $N(r,f)$, see [1–3]. Let us recall the definition of the order and the zeros exponent convergence of function f. The order of meromorphic function f is said by

$$\rho(f) = \limsup_{r\to\infty} \frac{\log T(r,f)}{\log r}. \tag{1.1}$$

The zeros of exponent convergence of meromorphic function f is said by

$$\lambda(f) = \limsup_{r\to\infty} \frac{\log N(r,f)}{\log r}. \tag{1.2}$$

In 1959, Hayman proved the following Theorems.

Theorem A (see [4], Theorem 8). *Let f be a transcendental entire function, and let $n \geq 3$ be an integer and a be a nonzero constant. Then $f'(z) - af(z)$ assumes all finite values infinitely often.*

Theorem B (see [4], Theorem 9). *Let f be a transcendental meromorphic function, and let $n \geq 5$ be an integer and a be a nonzero constant. Then $f'(z) - af(z)$ assumes all finite values infinitely often.*

Recently the difference variant of the Nevanlinna theory has been established independently in [5, 6]. Using these theories, value distribution theory uniqueness theory of difference polynomials of finite order transcendental meromorphic functions has been studied as well. We recall the following result by Liu and Laine.

Theorem C (see [7], Theorem 1.1). *Let f be a transcendental entire function of finite order not of period c, where c is a nonzero constant, and let $s(z)$ be a nonzero small function of f. Then the difference polynomial $f^n(z) + f(z+c) - f(z) - s(z)$ has infinitely many zeros in the complex plane provided that $n \geq 3$.*

In 2010, Chen considered the difference counterpart of Hayman's theorem and porved an almost direct difference analogue of Hayman's theorem.

Theorem D (see [8], Theorem 1.1). *Let f be a transcendental entire function of finite order not of period c, and let $a(\neq 0), b, c(\neq 0)$ be three complex numbers. Then $\Psi_n(z) = f(z+c) - f(z) - af^n(z)$ assumes all finite values infinitely often, provided that $n \geq 3$ and for every b one has $\lambda(\Psi_n(z) - b) = \rho(f)$.*

In this paper, we consider the value distribution of zero-order meromorphic functions with their q-shirt and prove the following results.

Theorem 1.1. *Let f be a transcendental meromorphic function of zero order, and let $Q(z)$ be a polynomial. If n is an integer and $n \geq 4$, then $af^n(qz) + f(z) - Q(z)$ has infinitely many zeros, where $q \in \mathbb{C} \setminus \{0\}$ and a is nonzero constant.*

Theorem 1.2. *Let f be a transcendental entire function of zero order, and let $Q(z)$ be a polynomial. If n is an integer and $n \geq 3$, then $af^n(qz) + f(z) - Q(z)$ has infinitely many zeros, where $q \in \mathbb{C} \setminus \{0\}$, and a is nonzero constant.*

It is well known that two meromorphic functions must be equal, if they share five distinct values. Recently, Heittokangas et al. research the uniqueness of meromorphic functions with their shifts in [6]. They got that if $f(z)$ and $f(z+c)$ share three distinct values, where $f(z)$ is finite order, then $f(z) = f(z+c)$. In this paper, we want to get some results on uniqueness of $f(z)$ and $f(qz)$, where $f(z)$ is zero order and $q \in \mathbb{C} \setminus \{0, 1\}$. Let us recall the notation of q-difference which is written by $\nabla_q = f(qz) - f(z)$.

Theorem 1.3. *Let f be a meromorphic function of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let $a_1, a_2, a_3 \in \mathbb{C}$ be three distinct values.*

(a) *If $f(z)$ and $f(qz)$ share a_1, a_2, a_3 CM, then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.*

(b) *If $f(z)$ and $f(qz)$ share a_1, a_2 CM, and if*

$$\operatorname*{Lim\,sup}_{r\to\infty} \frac{N(r,f)}{T(r,f)} < 1, \tag{1.3}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

Corollary 1.4. *Let f be an entire function of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let $a_1, a_2 \in \mathbb{C}$ be two distinct values.*

(a) *If $f(z)$ and $f(qz)$ share a_1, a_2 CM, then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.*

(b) *If $f(z)$ and $f(qz)$ share a_1 CM, and if*

$$\limsup_{r\to\infty} \frac{N(r,f)}{T(r,f)} < 1, \tag{1.4}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

Corollary 1.5. *Let f be a meromorphic function of zero order, and let $q \in \mathbb{C} \setminus \{0,1\}$. If $f(z)$ and $f(qz)$ share ∞ CM and a constant $a \in \mathbb{C}$ CM, and if there exists a constant $b \in \mathbb{C} \setminus \{a\}$ such that*

$$\limsup_{r\to\infty} \frac{N(r,1/(f-b))}{T(r,f)} < 1, \tag{1.5}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

Theorem 1.6. *Let f be a meromorphic function of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let*

$$P(f) = b_k(z) f\left(q^k z\right) + \cdots + b_1(z) f(qz) + b_0(z) f(z), \tag{1.6}$$

where $b_k, \ldots, b_0$ are constants. Let $n \in \{1, \ldots, k+1\}$ be the number of nonzero coefficients of the q-difference polynomial $P(f) - f$. If f and $P(f)$ share three distinct finite values a_1, a_2, a_3 IM, and if

$$\limsup_{r\to\infty} \frac{N(r,f)}{T(r,f)} < 1, \tag{1.7}$$

then $f \equiv P(f)$.

Corollary 1.7. *Let f be a meromorphic function of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let $a_1, a_2, a_3 \in \mathbb{C}$ be three distinct finite values.*

(a) *If $f(z)$ and $f(qz)$ share a_1, a_2, a_3 IM, and if*

$$\limsup_{r \to \infty} \frac{N(r,f)}{T(r,f)} < \frac{1}{2}, \tag{1.8}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

(b) *If $f(z)$ and $f(qz)$ share ∞ IM and two constants $a_1, a_2 \in \mathbb{C}$ IM, and if there exists a constant $b \in \mathbb{C} \setminus \{a_1, a_2\}$ such that*

$$\limsup_{r \to \infty} \frac{N(r,1/(f-b))}{T(r,f)} < \frac{1}{2}, \tag{1.9}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

Corollary 1.8. *Let f be an entire function of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let $a_1, a_2, a_3 \in \mathbb{C}$ be three distinct finite values.*

(a) *If $f(z)$ and $f(qz)$ share a_1, a_2, a_3 IM, then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.*

(b) *If $f(z)$ and $f(qz)$ share $a_1, a_2 \in \mathbb{C}$ IM, and if there exists a constant $b \in \mathbb{C} \setminus \{a_1, a_2\}$ such that*

$$\limsup_{r \to \infty} \frac{N(r,1/(f-b))}{T(r,f)} < \frac{1}{2}, \tag{1.10}$$

then $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

2. Auxiliary Results

The following auxiliary results will be instrumental in proving the theorems.

Lemma 2.1 (see [9], Theorem 1.2). *Let $f(z)$ be a nonconstant zero-order meromorphic function, and $q \in \mathbb{C} \setminus \{0\}$. Then*

$$m\left(r, \frac{f(qz)}{f(z)}\right) = S_q(r,f). \tag{2.1}$$

Lemma 2.2 (see [9], Theorem 3.1). *Let f be a non-constant meromorphic functions of zero order, let $q \in \mathbb{C} \setminus \{0,1\}$, and let $a_1, \ldots, a_p \in \mathbb{C}$, where $p \geq 2$, be distinct points. Then*

$$m(r,f) + \sum_{k=1}^{p} m\left(r, \frac{1}{f - a_k}\right) \leq 2T(r,f) - N_{\text{pair}}(r,f) + S_q(r,f), \tag{2.2}$$

where

$$N_{\text{pair}}(r,f) := 2N(r,f) - N(r,\nabla_q f) + N\left(r,\frac{1}{\nabla_q f}\right). \tag{2.3}$$

Lemma 2.3 (see [10], Theorem 1.1). *Let $f(z)$ be a non-constant zero-order meromorphic function, and $q \in \mathbb{C}\setminus\{0\}$. Then*

$$T(r,f(qz)) = T(r,f) + S_q(r,f). \tag{2.4}$$

Lemma 2.4 (see [10], Theorem 1.3). *Let $f(z)$ be a non-constant zero-order meromorphic function, and $q \in \mathbb{C}\setminus\{0\}$. Then*

$$N(r,f(qz)) = N(r,f) + S_q(r,f). \tag{2.5}$$

Lemma 2.5 (see [11], Lemma 4). *If $T : \mathbb{R}^+ \to \mathbb{R}^+$ is a piecewise continuous increasing function such that*

$$\lim_{r\to\infty}\frac{\log T(r)}{\log r} = 0, \tag{2.6}$$

then the set

$$E := \{r : T(C_1 r) \geq C_2 T(r)\} \tag{2.7}$$

has logarithmic density 0 for all $C_1 > 1$ and $C_2 > 1$.

3. Proof of Theorem 1.1

Let us put

$$\Psi(z) := af^n(qz) + f(z) - Q(z). \tag{3.1}$$

Hence, $\Psi(z)$ is not a constant identity. If not, let us suppose that $\Psi(z) \equiv c$, where c is a constant, and then

$$af^n(qz) \equiv Q(z) + c - f(z), \tag{3.2}$$

which give us

$$nT(r,f(qz)) = T(r,f) + O(\log r). \tag{3.3}$$

By using Lemma 2.3, we have $n \leq 1$, which contradicts the assumption $n \geq 5$. Hence $\Psi(z) \not\equiv c$. By taking logarithmic derivative on two sides of (3.1), we have

$$\frac{\Psi'(z)}{\Psi(z)} = \frac{\left(af^n(qz) + f(z) - Q(z)\right)'}{af^n(qz) + f(z) - Q(z)}. \tag{3.4}$$

If

$$\frac{\left(f^n(qz)\right)'}{f^n(qz)} - \frac{\Psi'(z)}{\Psi(z)} \equiv 0, \tag{3.5}$$

then by integrating two sides of which, we have $\Psi(z) = bf^n(qz)$, where b is a nonzero constant, and hence

$$(b-a)f^n(qz) = f(z) - Q(z). \tag{3.6}$$

If $b = a$, then $f(z) = Q(z)$, which is contradiction with $f(z)$, is transcendental function. If $b \neq a$, then Lemma 2.3 implies that $n = 1$, which is impossible. Therefore, we can write (3.4) as

$$af^n(qz) = \frac{(\Psi'(z)/\Psi(z))\left(f(z) - Q(z)\right) - \left(f(z) - Q(z)\right)'}{\left(f^n(qz)\right)'/f^n(qz) - \Psi'(z)/\Psi(z)}. \tag{3.7}$$

Let us put

$$\Psi_1(z) := \frac{\Psi'(z)}{\Psi(z)}\left(f(z) - Q(z)\right) - \left(f(z) - Q(z)\right)'. \tag{3.8}$$

Now we consider the poles of $\Psi_1(z)$. The poles of $\Psi_1(z)$ come from the zeros of $\Psi(z)$ and the poles of $f(qz)$, $f(z)$, and $Q(z)$. If z_0 is a zero of $\Psi(z)$ or a pole of $f(qz)$, but not a pole of $f(z)$, then z_0 is a simple pole of $\Psi(z)$. If z_0 is a common pole of $f(qz)$ and $f(z)$ with multiplicities of k and l, respectively, then z_0 is a pole of $\Psi_1(z)$ with multiplicity no more than $l+1$. If z_0 is a pole of $f(z)$ but not a pole of $f(qz)$, we obtain that z_0 is at most a simple pole of $\Psi_1(z)$ by using (3.7). Hence, Lemma 2.5 implies that

$$\begin{aligned} N(r,\Psi_1) &\leq \overline{N}\left(r,\frac{1}{\Psi}\right) + \overline{N}\left(r,f(qz)\right) + N\left(r,f(z)\right) + O(\log r) \\ &\leq \overline{N}\left(r,\frac{1}{\Psi}\right) + \overline{N}(r,f) + N\left(r,f(z)\right) + S_q(r,f). \end{aligned} \tag{3.9}$$

Let us put

$$\Psi_2(z) := \frac{\left(f^n(qz)\right)'}{f^n(qz)} - \frac{\Psi'(z)}{\Psi(z)}. \tag{3.10}$$

Now, let us consider the pole of $\Psi_2(z)$. The poles of $\Psi_2(z)$ come from the poles of $f(qz)$ and $f(z)$ and the zeros of $f(qz)$ and $\Psi(z)$. If z_0 is a zero of $\Psi(z)$, zero of $f(qz)$, or pole of $f(z)$, then z_0 is a simple pole of $\Psi_2(z)$. If z_0 is a pole of $f(qz)$ but not a pole of $f(z)$, by using Laurent series, we obtain that $\Psi_2(z)$ is analytic at z_0. Therefore, we have

$$\begin{aligned} N(r,\Psi_2) &\le \overline{N}\left(r,\frac{1}{\Psi}\right)+\overline{N}\left(r,\frac{1}{f(qz)}\right)+\overline{N}(r,f(z))+O(\log r) \\ &\le \overline{N}\left(r,\frac{1}{\Psi}\right)+\overline{N}\left(r,\frac{1}{f(z)}\right)+\overline{N}(r,f(z))+S_q(r,f), \end{aligned} \tag{3.11}$$

according to Lemma 2.5. In the coming (3.7) and Lemma 2.1, it implies that

$$\begin{aligned} nT(r,f) &= T(r,af^n(qz))+S_q(r,f) = T\left(r,\frac{\Psi_1}{\Psi_2}\right)+S_q(r,f) \\ &\le m(r,\Psi_1)+m(r,\Psi_2)+N(r,\Psi_1)+N(r,\Psi_2)+S_q(r,f) \\ &\le m(r,f)+m\left(r,\frac{\Psi'}{\Psi}-\frac{(f-Q)'}{f-Q}\right)+m(r,\Psi_2)+N(r,\Psi_1) \\ &\quad +N(r,\Psi_2)+S_q(r,f) \\ &\le 2\overline{N}\left(r,\frac{1}{\Psi}\right)+\overline{N}\left(r,\frac{1}{f}\right)+\overline{N}(r,f(z))+2N(r,f) \\ &\quad +m(r,f)+S_q(r,f). \end{aligned} \tag{3.12}$$

Therefore, we have

$$(n-3)T(r,f) \le 2\overline{N}\left(r,\frac{1}{\Psi}\right)+S_q(r,f), \tag{3.13}$$

which shows that $af^n(qz)+f(z)-Q(z)$ has infinite zeros by $n \ge 4$.

4. Proof of Theorem 1.2

In the same manner as in the proof of Theorem 1.1, we have (3.1)–(3.7), noting that

$$\Psi_1(z) := \frac{\Psi'(z)}{\Psi(z)}(f(z)-Q(z))-(f(z)-Q(z))'. \tag{4.1}$$

Now consider the poles of $\Psi_1(z)$. The poles of $\Psi_1(z)$ come from the zeros of $\Psi(z)$ and the poles of $Q(z)$. If z_0 is a zero of $\Psi(z)$, then z_0 is a simple pole of $\Psi(z)$. Hence, Lemma 2.4 implies that

$$N(r,\Psi_1) \le \overline{N}\left(r,\frac{1}{\Psi}\right)+O(\log r) \le \overline{N}\left(r,\frac{1}{\Psi}\right)+S_q(r,f). \tag{4.2}$$

Let us put

$$\Psi_2(z) := \frac{(f^n(qz))'}{f^n(qz)} - \frac{\Psi'(z)}{\Psi(z)}. \tag{4.3}$$

Now, let us consider the pole of $\Psi_2(z)$. The poles of $\Psi_2(z)$ come from the poles of $Q(z)$ and the zeros of $f(qz)$ and $\Psi(z)$. If z_0 is a zero of $\Psi(z)$, zero of $f(qz)$, then z_0 is a simple pole of $\Psi_2(z)$. Therefore, we have

$$\begin{aligned} N(r,\Psi_2) &\le \overline{N}\left(r,\frac{1}{\Psi}\right) + \overline{N}\left(r,\frac{1}{f(qz)}\right) + O(\log r) \\ &\le \overline{N}\left(r,\frac{1}{\Psi}\right) + \overline{N}\left(r,\frac{1}{f(z)}\right) + S_q(r,f), \end{aligned} \tag{4.4}$$

according to Lemma 2.5. In the coming (3.7) and Lemma 2.1, it implies that

$$\begin{aligned} nT(r,f) &= T(r,af^n(qz)) + S_q(r,f) = T\left(r,\frac{\Psi_1}{\Psi_2}\right) + S_q(r,f) \\ &\le m(r,\Psi_1) + m(r,\Psi_2) + N(r,\Psi_1) + N(r,\Psi_2) + S_q(r,f) \\ &\le m(r,f) + m\left(r,\frac{\Psi'}{\Psi} - \frac{(f-Q)'}{f-Q}\right) + m(r,\Psi_2) + N(r,\Psi_1) \\ &\quad + N(r,\Psi_2) + S_q(r,f) \\ &\le 2\overline{N}\left(r,\frac{1}{\Psi}\right) + \overline{N}\left(r,\frac{1}{f}\right) + m(r,f) + S_q(r,f). \end{aligned} \tag{4.5}$$

Therefore, we have

$$(n-2)T(r,f) \le 2\overline{N}\left(r,\frac{1}{\Psi}\right) + S_q(r,f), \tag{4.6}$$

which shows that $af^n(qz) + f(z) - Q(z)$ has infinite zeros by $n \ge 3$.

5. Proof of Theorem 1.3

(a) Suppose first that a_1, a_2, a_3 are three distinct values, and assume conversely to the assertion that $\nabla_q \not\equiv 0$. Then Lemma 2.2 yields

$$\sum_{k=1}^{3} m\left(r,\frac{1}{f-a_k}\right) \le 2T(r,f) + N(r,\nabla_q f) - 2N(r,f) - N\left(r,\frac{1}{\nabla_q f}\right) + S_q(r,f) \tag{5.1}$$

and so

$$T(r,f) \le \sum_{k=1}^{3} N\left(r, \frac{1}{f-a_k}\right) + N(r, f(qz)) - N(r,f) - N\left(r, \frac{1}{\nabla_q f}\right). \tag{5.2}$$

Since $f(z)$ and $f(qz)$ share a_1, a_2, and a_3 CM, it follows that

$$\sum_{k=1}^{3} N\left(r, \frac{1}{f-a_k}\right) \le N\left(r, \frac{1}{\nabla_q f}\right) \tag{5.3}$$

In addition, since f is zero-order meromorphic function, from Lemma 2.4, Lemma 2.5 and equations (5.2) and (5.3), we have

$$T(r,f) = S_q(r,f) \tag{5.4}$$

which is impossible. This contradiction is only avoided when $\nabla_q f \equiv 0$.

Suppose that $a_3 = \infty$ while a_1 and a_2 are distinct finite values. Similarly as above, $f(z) = f(qz)$ can be obtained. Therefore, $f(z) = f(qz)$ for all $z \in \mathbb{C}$.

(b) Assume that $\nabla_q f \not\equiv 0$. Similarly as above, Lemma 2.2 yields

$$\begin{aligned} m(r,f) + \sum_{k=1}^{2} m\left(r, \frac{1}{f-a_k}\right) &\le 2T(r,f) + N(r, \nabla_q f) \\ &\quad - 2N(r,f) - N\left(r, \frac{1}{\nabla_q f}\right) + S_q(r,f), \end{aligned} \tag{5.5}$$

and therefore $m(r,f) = S_q(r,f)$. This together with the condition results in a contradiction. Hence $\nabla_q f \not\equiv 0$.

6. Proof of Theorem 1.6

Assume on the contrary to the assertion that $f \not\equiv P(f)$. In what follows, $\varepsilon > 0$ is small enough and $R > 0$ is large enough. From the condition, we have

$$N(r,f) \le \frac{1-2\varepsilon}{n} T(r,f), \quad r \ge R, \tag{6.1}$$

and this together with Lemma 2.4 and Lemma 2.1 gives

$$
\begin{aligned}
N\left(r, \frac{1}{P(f)-f}\right) &\leq T(r, P(f)-f) + O(1) \\
&\leq m(r, P(f)-f) + N(r, P(f)-f) + O(1) \\
&= m\left(r, f\left(\frac{P(f)}{f}-1\right)\right) + nN(r,f) + S_q(r,f) \\
&\leq T(r,f) + (n-1)N(r,f) + S_q(r,f), \quad r \geq R.
\end{aligned} \tag{6.2}
$$

Therefore, by the sharing assumption,

$$
\begin{aligned}
\sum_{j=1}^{3} \overline{N}\left(r, \frac{1}{f-a_j}\right) &\leq N\left(r, \frac{1}{P(f)-f}\right) \\
&\leq T(r,f) + (n-1)N(r,f) + S_q(r,f), \quad r \geq R
\end{aligned} \tag{6.3}
$$

from above, it follows that

$$
\begin{aligned}
(2-\varepsilon)T(r,f) &\leq \overline{N}(r,f) + \sum_{j=1}^{3} \overline{N}\left(r, \frac{1}{f-a_j}\right) \\
&\leq (2-2\varepsilon)T(r,f) + S_q(r,f),
\end{aligned} \tag{6.4}
$$

which is impossible. This contradiction yields $f \equiv P(f)$.

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