

## *Research Article*

# **A Network Model of Credit Risk Contagion**

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A network model of credit risk contagion is presented, in which the effect of behaviors of credit risk holders and the financial market regulators and the network structure are considered. By introducing the stochastic dominance theory, we discussed, respectively, the effect mechanisms of the degree of individual relationship, individual attitude to credit risk contagion, the individual ability to resist credit risk contagion, the monitoring strength of the financial market regulators, and the network structure on credit risk contagion. Then some derived and proofed propositions were verified through numerical simulations.

## **1. Introduction**

In the financial market, behavioral factors of credit risk holders and financial market regulators have very important effect on credit risk contagion, especially the investor sentiment. The market behavioral approach recognizes that investors are not “rational” but “normal”, and that systematic biases in their beliefs induce them to trade on nonfundamental information, called “sentiment” [1]. Some financial economists also recognize that the market has mood swings. The link between asset valuation and investor sentiment soon became the subject of considerable deliberation among financial economists. Theories departing from rational asset pricing often posit the influence of investor sentiment [2], and investor sentiment will lead to fluctuate in prices, and generate risk. Some theoretical studies offer models to establish the relationship between investor sentiment and asset prices [2–4]. These models contain two types of investors: rational arbitrageurs who are sentiment free and irrational traders prone to exogenous sentiment [5]. Baker and Stein [6] find that total sentiment, particularly the global component of total sentiment, is a contrarian predictor of country-level market returns. Baker and Wurgler [5, 7] deem that broad waves of sentiment have greater effects on hard to arbitrage and hard to value stocks, and these stocks will exhibit high “sentiment

beta" [8]. Hence, sentiment to the extent influences valuation, taking a position opposite to prevailing market sentiment, can be both expensive and risky.

Some theoretical studies show that investor sentiment has very important effect on investor personal investment decisions [9]. Baker and Wurgler [7] point out that sentiment-based mispricing is based on an uninformed demand of some investors, the noise traders, and a limit to arbitrage. Since it is unknown how long buying or selling pressures from overly optimistic or pessimistic noise traders will persist, mispricing can be persistent. Similarly, there are many other significant studies in this area, such as Hertel et al. [10], Loewenstein et al. [11], Yuen and Lee [12], and Raghunathan and Corfman [13]. Recently, theoretical studies have found that investor sentiment is contagious across markets [14]. Actually, the effect of agent behaviors of credit risk holders and financial market regulator on credit risk contagion have been taken into account [15, 16]. Therefore, our paper will contribute to understanding the effect of behaviors of credit risk holders and financial market regulators on credit risk contagion across credit risk holders or markets.

Over the past few years, theoretical studies found that the network structure also have very important effect on risk contagion. Some theoretical studies have found some common structural characteristics, such as the small-world properties, the power-law degree distribution, the degree correlation, and individual heterogeneity [17–21]. Actually, the effects of these networked structural characteristics are significant on risk contagion. They can influence the complexity of risk contagion and the speed and efficiency of risk contagion among investors or markets. Allen and Gale [22] demonstrate that the spread of contagion risk depends crucially on the pattern of interconnectedness among banks. When the interbank connectedness is complete, the impact of a shock is readily attenuated, and there is no contagion risk. However, when the interbank connectedness is incomplete, the system is more fragile, and contagion risk is probably to occur.

With the development of network theory, modeling the complex structure of the financial system to analyze contagion risk in the financial system is a kind of trend. For random network in banking systems, Iori et al. [23] study banking systems with homogeneous and heterogeneous banks, respectively. With homogeneous banks, an interbank market unambiguously stabilizes the system. While with heterogeneous banks, knock-on effects become possible, but the stabilizing role of interbank lending remains, and consequently the interbank market plays an ambiguous role. For the tiered structure in banking systems, Teteryatnikova [24] constructs tiered banking networks and finds the advantages of tiering within the banking system in terms of both the resilience of the banking network to systemic shocks and the extent of necessary government intervention should a crisis evolve. Li [25] develops an analytical model of contagion risk in banking systems with tiered structure. He find that increasing interbank connections is conducive to handling banking crisis and reducing the effect of contagion risk, but its positive effect is limited; raising bank reserve ratio will enhance the stability of individual banks to a certain extent, but it may immediately lead to liquidity problems for banks that have less excess reserves, causing the occurrence of contagion risk; an excessive drive for risk assets with high return may bring high risk to banks and lead to instability of banking systems; the bank risk preference is crucial to the stability of banking systems, and the radicalness of it may lead to greater systemic instability. Similarly, there are many other significant studies in this area, such as Nier et al. [26], Canedo and Jaramillo [27], Georg and Poschmann [28], and Gai and Kapadia [29]. These theoretical studies look at contagion risk in banking systems via direct linkages between banks. However, there are other many studies analyzing contagion risk via indirect linkages. Dasgupta [30] discusses how linkages between banks represented by cross-holding deposits

can be a source of contagious breakdowns. De Vries [31] shows that there is interdependence between banks' portfolios given the fat tail property of the underlying assets, and it carries the potential of systemic breakdowns. Vivier-Lirimont [32] finds that only very dense networks, where banks are only a few links away from one another, are compatible with a Pareto optimal allocation. Babus [33] considers a model where banks form links with one another, whereby the contagion risk is greatly reduced. The network is formed endogenously and serves as an insurance mechanism. The prior literatures show that the network structure can affect risk contagion. With the development of network theory, network theory will be a new approach to studying credit risk contagion in the financial system, which will contribute to a better and deeper understanding to credit risk contagion.

In recent years, the contagion effect of credit risk again and again emerge, and that have serious effect on the financial market and economic development. Credit risk contagion is increasingly becoming a key factor that threatens the stability of the financial market. If we abstract credit risk holders for the network node and abstract complex relations between credit risk holders for the connections between nodes or edge, then many credit risk holders can constitute a complex network. We call it "social network" in the following. Among the social network, behaviors of credit risk holders and financial market regulators, the complex connections between credit risk holders and the structural characteristics of the social network play a key role in the process of credit risk contagion. Therefore, in this paper, we build a network model of credit risk contagion. We will likewise consider the effect of behaviors of credit risk holders and financial market regulators, the complex connections between credit risk holders, and the structural characteristics of the social network on the process of credit risk contagion. Our aim is to understand how the effect of the aforementioned factors on the process of credit risk contagion in the financial market.

The remainder of this paper is organized as follows. In Section 2, the network model of credit risk contagion is presented. In Section 3, we conduct a series of theoretical deducing and analysis. In Section 4, the effect of behaviors of credit risk holders and financial market regulator and the social network structure on credit risk contagion are discussed through numerical simulation. Finally, we conclude the paper in Section 5.

## **2. The Model**

### ***2.1. The Assumptions and Notation***

Consider the effect of behaviors of credit risk holders and financial market regulators, the complex connections between credit risk holders, and the structural characteristics of the social network on credit risk contagion, we have to define these factors which are involved in our model. As we know, the correlation of credit risk assets in the financial market is the basis to credit risk contagion and fluctuations in prices. According to the individual correlation in credit risk assets, we assume that the complex connections between credit risk holders can be divided into the direct linkages and the indirect linkages in the financial market, and this complex connection is divisible continuous variables. Meanwhile, we assume that the structure of the social network is constant. In other words, we do not consider the effect of node deleting in this paper. Therefore, we abstract credit risk holders for a node on complex network and abstract complex relations between credit risk holders for the connections between nodes or edge. Moreover, we assume that the number of individuals  $N$  is limited in the social network,  $N = 1, 2, \dots, n$ .

Let

- (i)  $\lambda$  be the contagion rate of credit risk in the financial market, and  $\lambda \in [0, 1]$ ;
- (ii)  $\beta_k(t)$  be individuals rate that are infected credit risk at time  $t$ ;
- (iii)  $\mu$  be the monitoring strength of the financial market regulators, and  $\mu \in [1, +\infty)$ ;
- (iv)  $\delta$  be the connection degree between credit risk holders in the financial market, and  $\delta \in (0, +\infty)$ . Moreover,  $0 < \delta < 1$  represents the correlation degree of indirect linkages between two individuals, and  $\delta \geq 1$  represents the correlation degree of direct linkages between two individuals. Meanwhile, the greater the  $\delta$ , the stronger the correlation degree;
- (v)  $\xi$  be the effect strength of credit event, and  $\xi \in [0, 1]$ ;
- (vi)  $\gamma(\xi)$  be the mental attitude or sentiment to the effect of credit risk contagion, and  $0 \leq \gamma(\xi)$ ,  $\gamma'(\xi) > 0$ , and  $\gamma''(\xi) > 0$ ;
- (vii)  $\theta$  be the individual ability to resist credit risk contagion in financial market, and  $\theta \in [1, +\infty)$ .

## 2.2. Definition of the Model

Generally speaking, credit risk tends to infect these nodes that the degree of node is bigger, and the connection degree is higher to these individuals that have been infected credit risk. This leads to the speed of credit risk contagion that is much faster in the financial market. Let  $P(k)$  represent degree distribution that the degree of nodes is equal to  $k$  in the social network, then  $\langle k \rangle = \sum_k kP(k)$ , where  $0 < k < n$ . Therefore, the probability  $\Theta(k', t)$ , that individuals are infected credit risk by other individuals that have been infected credit risk, and their degrees that are equal to  $k'$  can be described by the following equation:

$$\Theta(k', t) = P(k' | k) = \frac{\sum_{k'} k' P(k') \beta_{k'}(t)}{\langle k \rangle}, \quad (2.1)$$

where  $\beta_{k'}(t)$  is the share of individual that the degree is equal to  $k'$  in the social network. Equation (2.1) implies the contagion probability  $\Theta(k', t)$  depends on the social network structure, and it is monotone increasing with the increase in  $k'$ ,  $P(k')$ , and  $\beta_{k'}(t)$ . According to Eboli [34] and Lopez [35], credit risk contagion could be treated as a Markov process in the social network. Hence, the increasing rate of  $\beta_k(t)$  can be described by the following differential equation:

$$\frac{d\beta_k(t)}{dt} = \frac{\lambda k \delta \gamma(\xi) [1 - \beta_k(t)] \sum_k k P(k) \beta_k(t)}{\mu \theta \langle k \rangle} - \eta \beta_k(t), \quad (2.2)$$

where  $\eta$  is the speed that individuals restore to the health status after being infected credit risk. Let  $\alpha = \lambda/\eta$  represent the effectively contagion rates of credit risk contagion. Therefore, when  $d\beta_k(t)/dt = 0$ , we can obtain the rate that individuals are infected credit risk, and their

degrees are equal to  $k$  under the equilibrium state of the system of credit risk contagion as follows:

$$\beta_k(t) = \frac{\alpha(k\delta\gamma(\xi)/\mu\theta)(\sum_k kP(k)\beta_k(t)/\langle k \rangle)}{1 + \alpha(k\delta\gamma(\xi)/\mu\theta)(\sum_k kP(k)\beta_k(t)/\langle k \rangle)} \quad (2.3)$$

or

$$\beta_k(t) = \frac{\alpha(k\delta\gamma(\xi)/\mu\theta)\Theta(k,t)}{1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta(k,t)}. \quad (2.4)$$

According to the complex network theory and the finance theory, we know  $0 \ll \beta_k(t) \ll 1$  in the incomplete financial market. In other words, any credit events can cause credit risk contagion in the financial market, but the situation is never occurring that all of credit risk holders are infected credit risk. Thus, we plug (2.1) into (2.4) and can get an autonomous equation

$$\Theta(k,t) = \frac{\alpha(\delta\gamma(\xi)/\mu\theta)}{\langle k \rangle} \sum_k \frac{k^2 P(k)\Theta(k,t)}{1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta(k,t)}. \quad (2.5)$$

Equation (2.5) describes the influencing factors of the probability  $\Theta(k,t)$  under the equilibrium state of the system of credit risk contagion. Let  $\Theta = \Theta(k,t)$ . Obviously,  $\Theta = 0$  is trivial solution of (2.5). However, the trivial solution is not focus of our attention in our study of credit risk contagion, our attention is nontrivial solution, namely,  $\Theta \neq 0$ . According to (2.5), we can get the nonzero threshold of the system of credit risk contagion as follow:

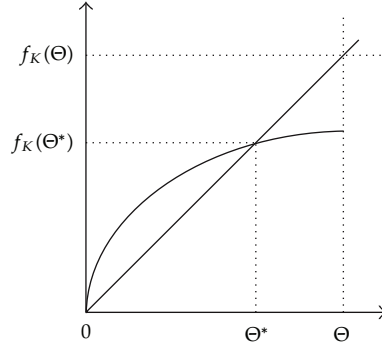
$$\alpha_c = \frac{\langle k \rangle}{\langle (\delta\gamma(\xi)/\mu\theta)k^2 \rangle} \quad \text{or} \quad \alpha_c = \frac{\langle k \rangle}{(\delta\gamma(\xi)/\mu\theta)\langle k^2 \rangle}. \quad (2.6)$$

According to the complex network theory, we know  $\langle k \rangle \ll \langle k^2 \rangle$  in the social network. As we know,  $0 \ll \delta\gamma(\xi)/\mu\theta \ll 1$  under the incomplete financial market, then  $\langle k \rangle \ll (\delta\gamma(\xi)/\mu\theta)\langle k^2 \rangle \ll \langle k^2 \rangle$ . Therefore,  $0 < \alpha_c = \langle k \rangle / (\delta\gamma(\xi)/\mu\theta)\langle k^2 \rangle \rightarrow 0$ . This can explain why only parts of credit event can cause credit risk contagion in the financial market, but is not all. Meanwhile, which imply why an influential credit event is very easy to cause credit risk contagion, and trigger "butterfly effect".

### 3. The Theoretical Analysis of Model

#### 3.1. The Impact Analysis of Behavioral Factors

The effects of the complex connection between credit risk holders and its relationship degree are very important on credit risk contagion. Let  $\Theta^*$  and  $\beta_k^*$ , respectively, denote the probability that individuals are infected credit risk, and the share of individuals that they are infected credit risk and their degrees are equal to  $k$  under the equilibrium state of credit risk contagion; and  $\Theta^* \in (0, 1)$ ;  $\beta_k^* \in (0, 1)$ .



**Figure 1:** The equilibrium relationship graph of credit risk contagion system.

**Theorem 3.1.** For any network that its degree is equal to  $K$ , when the system of credit risk contagion is in equilibrium status, the system of credit risk contagion emerge the following properties.

- (1) Under the incomplete market, the equilibrium probability  $\Theta^* > 0$  of the system of credit risk contagion is unique.
- (2) Under the incomplete market, the effect scale of credit risk contagion is monotonic increasing concave function on the degree  $\delta$  of individual relationship, the effect strength  $\xi$  of credit event, and individual attitude  $r(\xi)$  to credit risk contagion. Meanwhile, it is also monotonic diminishing convex function on the monitoring strength  $\mu$  of the financial market regulators and the individual ability  $\theta$  to resist credit risk contagion.

*Proof.* Let

$$f_K(\Theta) = \sum_k \frac{\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)\Theta}{\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta)}. \quad (3.1)$$

We can get  $f_K'(\Theta) = \partial f_K(\Theta)/\partial\Theta = \sum_k (\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)/\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta)^2) > 0$ ,  $f_K''(\Theta) = \partial^2 f_K(\Theta)/\partial\Theta^2 = -\sum_k (2\alpha^2[\delta\gamma(\xi)/\mu\theta]^2k^3P(k)/\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta)^3) < 0$ . Thus,  $f_K(\Theta)$  is a monotonic increasing concave function on  $\Theta$ . Because  $f_K(1) = \sum_k (\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)/\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k)) < \sum_k (\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)/\langle k \rangle\alpha(\delta\gamma(\xi)/\mu\theta)k) = 1$ , and  $f_K(0) = 0$ . Hence,  $\Theta = \sum_k (\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)\Theta/\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta))$  have a fixed point at least and have two fixed point in the interval  $[0,1]$  at most, as Figure 1. Meanwhile, for the network of credit risk holders under the incomplete market, we can get  $0 < \langle k \rangle/\langle k^2 \rangle \rightarrow 0$ , and  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle/\langle k^2 \rangle$ . Thus  $f_K'(\Theta)|_{\Theta=0} > 1$ . According to Figure 1, we know that  $\Theta = \sum_k (\alpha(\delta\gamma(\xi)/\mu\theta)k^2P(k)\Theta/\langle k \rangle(1 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta))$  have unique equilibrium value  $\Theta^*$  in the interval  $[0,1]$ , and  $\Theta^* > 0$ . According to (2.4), we can get  $\partial\beta_k(t)/\partial\delta = \alpha(k\gamma(\xi)/\mu\theta)\Theta/[1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta]^2 > 0$ , and  $\partial^2\beta_k(t)/\partial\delta^2 = -(2\alpha[(k\gamma(\xi)/\mu\theta)\Theta]^2/[1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta]^3) < 0$ . Thus  $\beta_k(t)$  is monotonic increasing concave function on the degree  $\delta$  of individual relationship.

Similarly, we have  $\partial\beta_k(t)/\partial\gamma(\xi) > 0$ ,  $\partial^2\beta_k(t)/\partial(\gamma(\xi))^2 < 0$ ,  $\partial\beta_k(t)/\partial\xi > 0$ ,  $\partial^2\beta_k(t)/\partial\xi^2 < 0$ ,  $\partial\beta_k(t)/\partial\mu < 0$ ,  $\partial^2\beta_k(t)/\partial\mu^2 > 0$ ,  $\partial\beta_k(t)/\partial\theta < 0$ , and  $\partial^2\beta_k(t)/\partial\theta^2 > 0$ . Thus, the effect scale of credit risk contagion is monotonic increasing concave function on the effect strength  $\xi$  of credit event and individual attitude  $r(\xi)$  to credit risk contagion, and it is also monotonic



diminishing convex function on the monitoring strength  $\mu$  of the financial market regulators and the individual ability  $\theta$  to resist credit risk contagion.  $\square$

From Theorem 3.1, we can find that under the incomplete market, individual attitude to credit risk contagion, the degree of individual relationship, and the effect strength of credit event can increase the speed and effect scale of credit risk contagion and cause "butterfly effect". But the monitoring strength of the financial market regulators and the individual ability to resist credit risk contagion can reduce or restrain the speed and effect scale of credit risk contagion. Therefore, Theorem 3.1 has important theoretical and practical value to the financial market regulators that they can make effective monitoring strategy to credit risk contagion.

### 3.2. Effect of Network Structure

We use first- and second-order stochastic dominance of degree distribution, respectively, to denote the change of the average degree and the heterogeneity of network (e.g., [36]). Let  $P$  and  $P'$ , respectively, represent the degree distributions of two networks. If  $P$  first-order stochastic dominates  $P'$ , which is equivalent to having  $\sum_k F(k)P(k) > \sum_k F(k)P'(k)$  for any monotonic decreasing function  $F(k)$ , this implies  $\langle k \rangle > \langle k' \rangle$ . If  $P$  second-order stochastic dominates  $P'$ , which is equivalent to having  $\sum_k F(k)P(k) > \sum_k F(k)P'(k)$  for any concave function  $F(k)$ , this implies that  $P'$  has a higher difference than  $P$ . Therefore, we assume that the degree distributions of network  $K$  and  $K'$  are, respectively,  $P$  and  $P'$ , and their average degrees are, respectively,  $\langle k \rangle$  and  $\langle k' \rangle$ . According to the stochastic dominance standards of the social networks, we could get the following propositions.

**Theorem 3.2.** *When  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle / \langle k^2 \rangle$ , if the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of the network  $K$ ,  $\Theta^{*'} is greater than  $\Theta^*$  under the equilibrium state of credit risk contagion.$*

*Proof.* We assume that Theorem 3.2 cannot be tenable. In other words, if the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of the network  $K$ ,  $\Theta^{*'} \leq \Theta^*$ . According to (2.5) and (3.1), we can get

$$\Theta^* \geq \Theta^{*' = f_{K'}(\Theta^*). \quad (3.2)$$

According to Theorem 3.1, we know that the equilibrium probability  $\Theta^* > 0$  is unique when  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle / \langle k^2 \rangle$ .

Let

$$F(k) = \frac{\alpha(\delta\gamma(\xi)/\mu\theta)k^2\Theta}{\langle k \rangle(1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta)}. \quad (3.3)$$

We can get  $\partial F(k)/\partial k = \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta(2 + \alpha(\delta\gamma(\xi)/\mu\theta)k\Theta)/\langle k \rangle(1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta)^2 > 0$ , thus  $F(k)$  is a monotonic decreasing function on  $k$ . According to Jackson [36], if the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of the network  $K$ , then  $P$  first order-stochastic dominates  $P'$ . Therefore, if the average degree  $\langle k' \rangle$  of network  $K'$  is greater than the average degree  $\langle k \rangle$  of network  $K$ , which is equivalent to

$\sum_k F(k)P'(k) > \sum_k F(k)P(k)$ , this implies that we have  $f_{K'}(\Theta) > f_K(\Theta)$  for any  $\Theta > 0$ . Thus we can get  $f_{K'}(\Theta^*) > f_K(\Theta^*)$  for all  $\Theta^* > 0$ . In contrast to (3.2), we find  $\Theta^* \geq f_{K'}(\Theta^*) > f_K(\Theta^*)$ . This conflicts with  $\Theta^* = f_K(\Theta^*)$ . Thus our hypothesis is tenable; namely, Theorem 3.2 is true.  $\square$

**Corollary 3.3.** *When  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle / \langle k^2 \rangle$ , if the heterogeneity of network  $K'$  is higher than the heterogeneity of network  $K$ ,  $\Theta^*$  is greater than  $\Theta^{*'}$  under the equilibrium state of credit risk contagion.*

*Proof.* We assume that Corollary 3.3 cannot be tenable. In other words, if the heterogeneity of network  $K'$  is higher than the heterogeneity of network  $K$ ,  $\Theta^* \leq \Theta^{*'}$  under the equilibrium state of credit risk contagion. According to (2.5) and (3.1), we can get

$$\Theta^* \leq \Theta^{*'} = f_{K'}(\Theta^*). \quad (3.4)$$

According to Theorem 3.1, we know that  $\Theta^* > 0$  is unique when  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle / \langle k^2 \rangle$ . According to (3.3), we have  $\partial^2 F(k) / \partial k^2 > 0$ , thus  $F(k)$  is a concave function on  $k$ . According to Jackson [36], if the heterogeneity of the network  $K'$  is higher than the heterogeneity of the network  $K$ , then  $P$  second-order stochastic dominates  $P'$ . Therefore, if the heterogeneity of network  $K'$  is higher than the heterogeneity of network  $K$ , which is equivalent to having  $\sum_k F(k)P(k) > \sum_k F(k)P'(k)$ , this implies that we have  $f_K(\Theta) > f_{K'}(\Theta)$  for any  $\Theta > 0$ . Thus we can get  $f_K(\Theta^*) > f_{K'}(\Theta^*)$  for all  $\Theta^* > 0$ . In contrast to (3.2), we can get  $\Theta^* \leq f_{K'}(\Theta^*) < f_K(\Theta^*)$ . This conflicts with  $\Theta^* = f_K(\Theta^*)$ . Thus our hypothesis is tenable; namely, Corollary 3.3 is true.  $\square$

**Corollary 3.4.** *When  $\alpha(\delta\gamma(\xi)/\mu\theta) > \langle k \rangle / \langle k^2 \rangle$ , if the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of the network  $K$ ,  $\beta_k^{*'}$  is greater than  $\beta_k^*$  under the equilibrium state of credit risk contagion.*

*Proof.* According to Theorem 3.2, we can get  $\Theta^{*'} > \Theta^*$  when the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of the network  $K$ . Which is equivalent to  $\beta^{*'}(k) > \beta^*(k)$  and  $\sum_k \beta^{*'}(k) > \sum_k \beta^*(k)$  for any  $k > 0$ . Thus we can get

$$\beta_k^{*'} > \sum_k \beta^*(k)P'(k). \quad (3.5)$$

According to (2.4), we can get  $\partial\beta_k(t)/\partial k = \alpha(\delta\gamma(\xi)/\mu\theta)\Theta / (1 + \alpha(k\delta\gamma(\xi)/\mu\theta)\Theta)^2 > 0$ . This implies that  $\beta_k(t)$  is a monotonic decreasing function on  $k$ . According to Jackson [36], if the average degree  $\langle k' \rangle$  of the network  $K'$  is greater than the average degree  $\langle k \rangle$  of network  $K$ , we can get  $\sum_k \beta^{*'}(k)P'(k) > \sum_k \beta^*(k)P(k) = \beta_k^*$ . Combining (3.5), we have  $\beta_k^{*'} > \beta_k^*$ .  $\square$

From Theorem 3.2 and Corollary 3.4, we find that the denser the network, the stronger the contagion effect of credit risk. The main reason is that the denser the network, the higher the interactive frequency between credit risk holders, and the stronger the behavior consistency and the convergence effect between credit risk holders in the social network. These will enhance the contagion speed and effect scale of credit risk contagion. According to Corollary 3.3, we find that the higher the heterogeneity of individuals in the social network, the less the contagion probability of credit risk. This main reason is that the higher the heterogeneity network, the greater the difference of the degree distributions of nodes, and



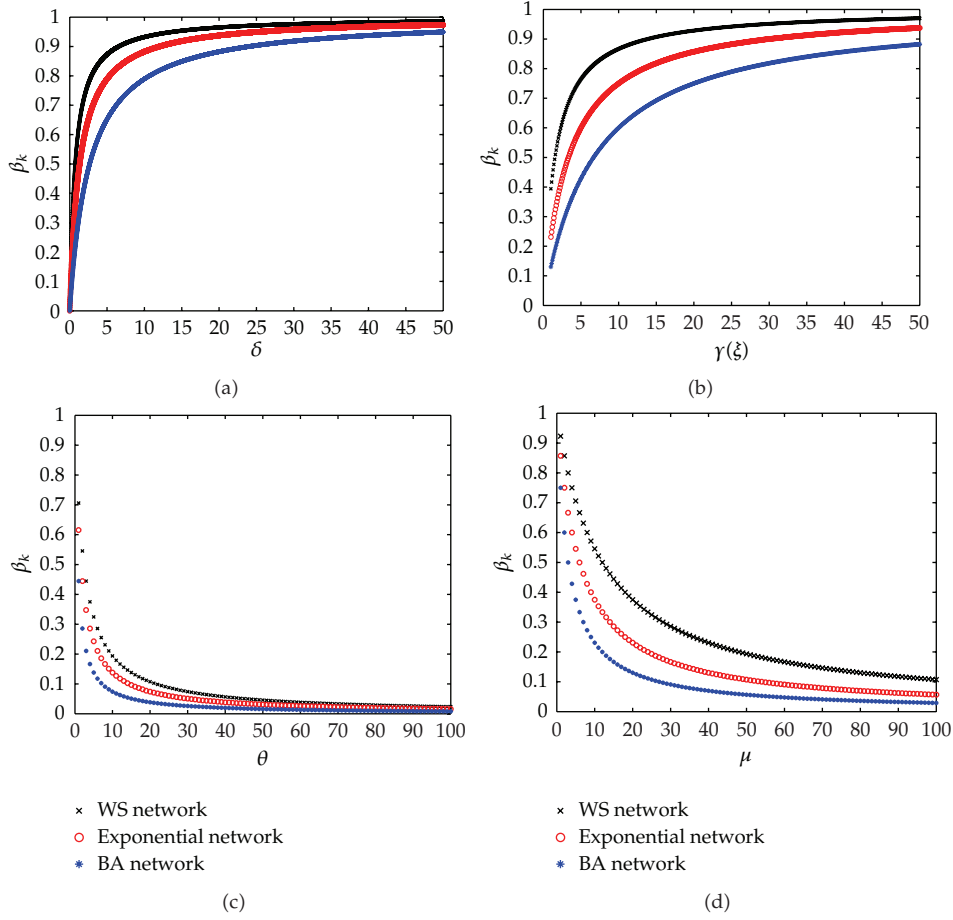


Figure 2: Effect of behavior factors to credit risk contagion.

the stronger the communication obstacles between credit risk holders, and the larger the resistance that information spread in the social network. Thus these can reduce the contagion probability of credit risk in the social network.

#### 4. Numerical Simulation Analysis

To conduct numerical simulation analysis of credit risk contagion, it is necessary to assign a value to parameters of model. First, we choose WS network, exponential network, and BA network to conduct numerical simulation, where the probability of long distance connection of nodes in WS network is equal to 0.05, the degree distributions of exponential network is  $P(k) \propto e^{-ek/2m}$ , and the degree distributions of BA network is  $P(k) \propto 2m^2/k^3$  (e.g., [37, 38]). Second, let  $N = 1000$ , and let  $m_0 = m = 5$ . Therefore, we can observe the effect of the degree of individual relationship, the effect strength of credit event, individual attitude to credit risk contagion, the monitoring strength of the financial market regulators, the individual ability to resist credit risk contagion, and the network structure of credit risk contagion.

Figure 2 shows the effect of behaviors of credit risk holders and financial market regulator on the scale of credit risk contagion. In Figures 2(a) and 2(b), we find that the

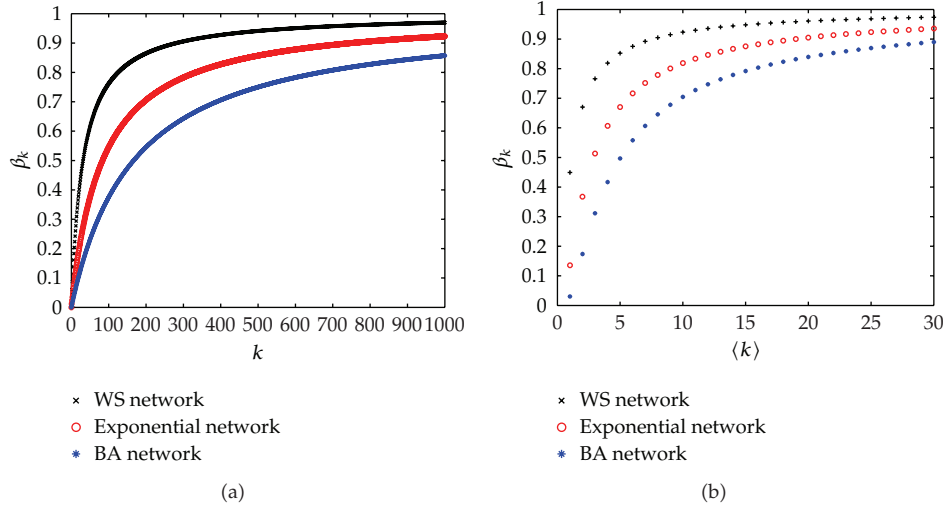


Figure 3: Effect of the network structure on credit risk contagion.

contagion speed of credit risk is significantly increasing with the increase in the degree of individual relationship and individual attitude to credit risk contagion. Meanwhile, the contagion speed of credit risk is the fastest in WS network, but is the slowest in BA network. This reason is that the direct correlation edges between credit risk holders are much more in WS network, thus the probability of the formation is greater on “psychological group,” and the probability of the behavior convergence effect is also higher. However, the direct correlation edges between credit risk holders are much less in BA network, thus the speed of information dissemination and the probability of the communication between individuals are lower, and the inhibitory action to credit risk contagion is stronger. These will emerge that the contagion speed of credit risk is faster in WS network than in BA network. In Figures 2(c) and 2(d), we find that the monitoring strength of the financial market regulators and the individual ability to resist credit risk contagion have strong inhibitory action to credit risk contagion, and this kind of inhibitory action in BA network is more significant than in WS network. The reason is that the direct correlation edges between credit risk holders are much less in BA network, and the difference of the degree distributions of nodes is higher. But these features are the opposite to WS network. Meanwhile, the inhibitory action of the individual ability to resist credit risk contagion is more significant than the monitoring strength of the financial market regulators. In addition, Figure 2 also shows that the lower the heterogeneity of the network, the higher the effect scale of credit risk contagion. This reason is that the lower heterogeneity of correlation edge is conducive to information dissemination in WS network, but the higher heterogeneity of correlation edge restrains and hinders information dissemination in BA network.

Figure 3 shows the effect of the network structure on the scale of credit risk contagion. In Figure 3(a), we find that the larger the degree of node, the faster the contagion speed of credit risk, and the higher the rate that individuals are infected credit risk. The reason is that the larger the degree of node, the more closely between credit risk holders. Thus the frequency of communication and the speed of information spread is higher, forming “psychological group” is the easier, and the convergent effect between credit risk holders is stronger. Therefore, the probability and speed of credit risk contagion is also higher. In Figure 3(b),

**Table 1:** Sensitivity analysis of the effect scale on credit risk contagion.

Behavior		The effect scale $\beta_k$ of credit risk contagion								
		WS network			Exponential network			BA network		
Factors	Variables	$\langle k \rangle = 5$	$\langle k \rangle = 10$	$\langle k \rangle = 15$	$\langle k \rangle = 5$	$\langle k \rangle = 10$	$\langle k \rangle = 15$	$\langle k \rangle = 5$	$\langle k \rangle = 10$	$\langle k \rangle = 15$
$\delta$	0.1	0.0253	0.0348	0.0457	0.0207	0.0298	0.0401	0.0186	0.0271	0.0369
	0.5	0.0923	0.1378	0.1839	0.0857	0.1252	0.1713	0.0614	0.1006	0.1435
	0.8	0.1012	0.1429	0.1891	0.0902	0.1287	0.1749	0.0798	0.1098	0.1581
	1	0.1286	0.1901	0.2714	0.1269	0.1782	0.2534	0.0964	0.1473	0.2193
	5	0.1709	0.2635	0.3927	0.1439	0.2301	0.3427	0.1293	0.2167	0.3286
	10	0.2216	0.3164	0.4279	0.1825	0.2742	0.3841	0.1587	0.2573	0.3654
$\theta$	1	0.2216	0.3164	0.4279	0.1825	0.2742	0.3741	0.1587	0.2573	0.3654
	5	0.2023	0.2892	0.4018	0.1527	0.2374	0.3486	0.1279	0.2289	0.3376
	10	0.1892	0.2594	0.3742	0.1203	0.2081	0.3149	0.0937	0.1989	0.3096
	15	0.1687	0.2281	0.3563	0.1076	0.1769	0.2843	0.0824	0.1671	0.2789
	20	0.1434	0.2074	0.3287	0.0947	0.1476	0.2571	0.0697	0.1294	0.2491
$r(\xi)$	0.1	0.2023	0.2892	0.4018	0.1527	0.2374	0.3486	0.1279	0.2289	0.3376
	0.5	0.2217	0.3044	0.4179	0.1639	0.2509	0.3627	0.1387	0.2432	0.3601
	1	0.2308	0.3203	0.4351	0.1766	0.2658	0.3767	0.1496	0.2594	0.3761
	2	0.2481	0.3389	0.4527	0.1907	0.2831	0.3923	0.1621	0.2763	0.3943
	5	0.2643	0.3576	0.4876	0.2174	0.3017	0.4129	0.1889	0.2944	0.4037

we find that the denser the network, the stronger the contagion effect of credit risk. This is consistent with Corollary 3.4.

To more clearly depict the sensitivity how the effect of behavior factors of credit risk holders on credit risk contagion. Let  $\mu = 10$  and  $\alpha = 0.4$ , we conduct numerical calculation, and the results are as Table 1. According to Table 1, we find that the results of sensitivity analysis are consistent with simulation analysis. This further verified the correctness of the aforementioned propositions.

### 5. Conclusion

In this paper, we discussed the effects of behavioral factors of credit risk holders and financial market regulators, the social network structure on credit risk contagion through the theoretical analysis and numerical simulation, and we obtain a series of important conclusions through numerical simulations and sensitivity analysis as follows.

- (1) Under the incomplete market, the equilibrium contagion probability of credit risk is unique, its value is a positive, and the effect scale of credit risk contagion is a monotonic increasing concave function on the degree of individual relationship, the effect strength of credit event, and individual attitude to credit risk contagion. Meanwhile, it is also a monotonic diminishing convex function on the monitoring strength of the financial market regulators and the individual ability to resist credit risk contagion.
- (2) The greater the average degree of the network, the greater the contagion probability and effect scale of credit risk contagion.

- (3) The greater the heterogeneity of the network, the less the contagion probability and effect scale of credit risk contagion.

These conclusions have important theoretical and practical value in the credit risk management. Certainly, there are still much works worth further research. For example, the weakening of assumptions, considering the effect of node deleting, and so forth. Therefore, our future study will focus on these issues.

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